



# ***Understand water-like anomalies with two-scale isotropic interactions***

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*International School of Neutron Science and Instrumentation 3<sup>rd</sup>  
Course: Water and the water systems*

*Erice-Sicily, July 2016*

# PERIODIC TABLE OF THE ELEMENTS

1 <b>H</b> 1.0079																	2 <b>He</b> 4.0026
3 <b>Li</b> 6.941	4 <b>Be</b> 9.012											5 <b>B</b> 10.811	6 <b>C</b> 12.011	7 <b>N</b> 14.007	8 <b>O</b> 16.00	9 <b>F</b> 19.00	10 <b>Ne</b> 20.179
11 <b>Na</b> 22.99	12 <b>Mg</b> 24.30											13 <b>Al</b> 26.98	14 <b>Si</b> 28.09	15 <b>P</b> 30.974	16 <b>S</b> 32.06	17 <b>Cl</b> 35.453	18 <b>Ar</b> 39.948
19 <b>K</b> 39.10	20 <b>Ca</b> 40.08	21 <b>Sc</b> 44.96	22 <b>Ti</b> 47.90	23 <b>V</b> 50.94	24 <b>Cr</b> 52.00	25 <b>Mn</b> 54.938	26 <b>Fe</b> 55.85	27 <b>Co</b> 58.93	28 <b>Ni</b> 58.69	29 <b>Cu</b> 63.55	30 <b>Zn</b> 65.39	31 <b>Ga</b> 69.72	32 <b>Ge</b> 72.59	33 <b>As</b> 74.92	34 <b>Se</b> 78.96	35 <b>Br</b> 79.90	36 <b>Kr</b> 83.80
37 <b>Rb</b> 85.47	38 <b>Sr</b> 87.62	39 <b>Y</b> 88.91	40 <b>Zr</b> 91.22	41 <b>Nb</b> 92.91	42 <b>Mo</b> 95.94	43 <b>Tc</b> (98)	44 <b>Ru</b> 101.1	45 <b>Rh</b> 102.91	46 <b>Pd</b> 106.42	47 <b>Ag</b> 107.87	48 <b>Cd</b> 112.41	49 <b>In</b> 114.82	50 <b>Sn</b> 118.71	51 <b>Sb</b> 121.75	52 <b>Te</b> 127.60	53 <b>I</b> 126.91	54 <b>Xe</b> 131.29
55 <b>Cs</b> 132.91	56 <b>Ba</b> 137.33	*57 <b>La</b> 138.91	72 <b>Hf</b> 178.49	73 <b>Ta</b> 180.95	74 <b>W</b> 183.85	75 <b>Re</b> 186.21	76 <b>Os</b> 190.2	77 <b>Ir</b> 192.2	78 <b>Pt</b> 195.08	79 <b>Au</b> 196.97	80 <b>Hg</b> 200.59	81 <b>Tl</b> 204.38	82 <b>Pb</b> 207.2	83 <b>Bi</b> 208.98	84 <b>Po</b> (209)	85 <b>At</b> (210)	86 <b>Rn</b> (222)
87 <b>Fr</b> (223)	88 <b>Ra</b> 226.02	†89 <b>Ac</b> 227.03	104 <b>Unq</b> (261)	105 <b>Unp</b> (262)	106 <b>Unh</b> (263)	107 <b>Uns</b> (262)	108 <b>Uno</b> (265)	109 <b>Une</b> (266)									

Alkali metals  
Alkaline earth metals  
Transition metals  
Actinide series  
Other metals  
Nonmetals  
Noble gases  
Lanthanide series

*Lanthanide Series:	58 <b>Ce</b> 140.12	59 <b>Pr</b> 140.91	60 <b>Nd</b> 144.24	61 <b>Pm</b> (145)	62 <b>Sm</b> 150.4	63 <b>Eu</b> 151.97	64 <b>Gd</b> 157.25	65 <b>Tb</b> 158.93	66 <b>Dy</b> 162.50	67 <b>Ho</b> 164.93	68 <b>Er</b> 167.26	69 <b>Tm</b> 168.93	70 <b>Yb</b> 173.04	71 <b>Lu</b> 174.97
†Actinide Series:	90 <b>Th</b> 232.04	91 <b>Pa</b> 231.04	92 <b>U</b> 238.03	93 <b>Np</b> 237.05	94 <b>Pu</b> (244)	95 <b>Am</b> (243)	96 <b>Cm</b> (247)	97 <b>Bk</b> (247)	98 <b>Cf</b> (251)	99 <b>Es</b> (252)	100 <b>Fm</b> (257)	101 <b>Md</b> (258)	102 <b>No</b> (259)	103 <b>Lr</b> (260)

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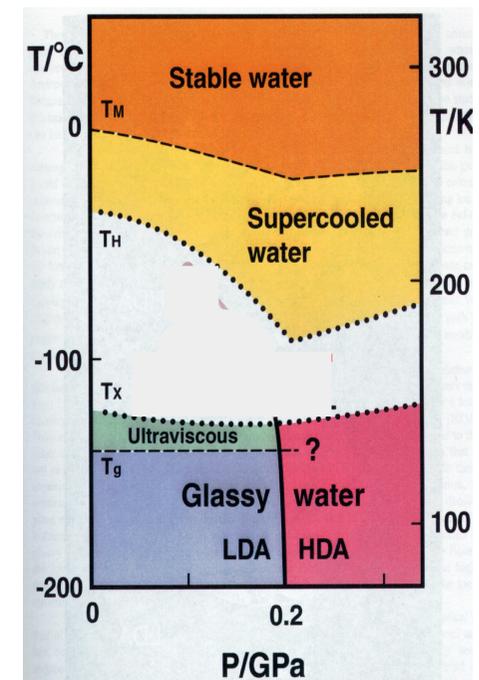
Tetrahedral structured: H<sub>2</sub>O, SiO<sub>2</sub>, BeF<sub>2</sub>, etc

Compounds: Y<sub>2</sub>O<sub>3</sub>-Al<sub>2</sub>O<sub>3</sub>, Fe-Co, Ce-Al, etc

# Materials share similar anomalous properties

- Negatively-sloped melting line

$$\frac{dP}{dT} = \frac{\Delta S}{\Delta V} = \frac{L}{T \Delta V}$$



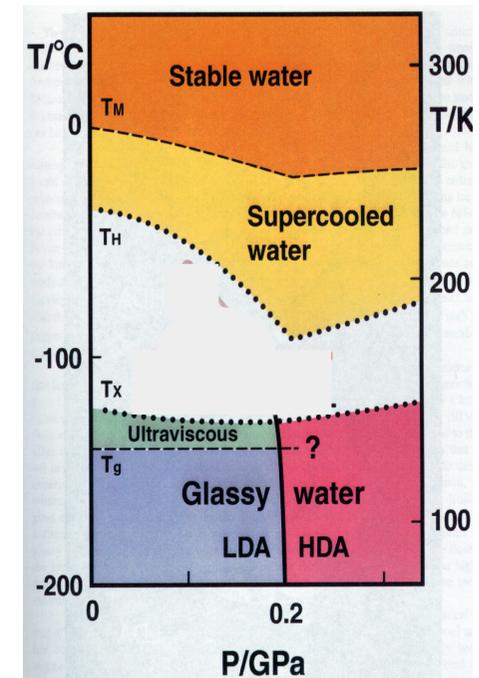
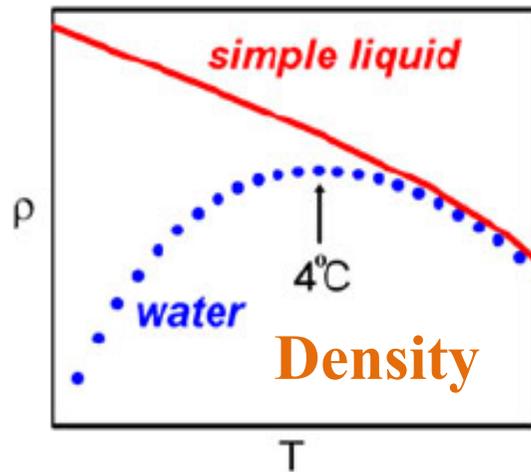
Poole et. al, Nature (1992)

Mishima and Stanley, Nature (1998)

# Materials share similar anomalous properties

- Negatively-sloped melting line
- Density anomaly

$$\frac{dP}{dT} = \frac{\Delta S}{\Delta V} = \frac{L}{T \Delta V}$$



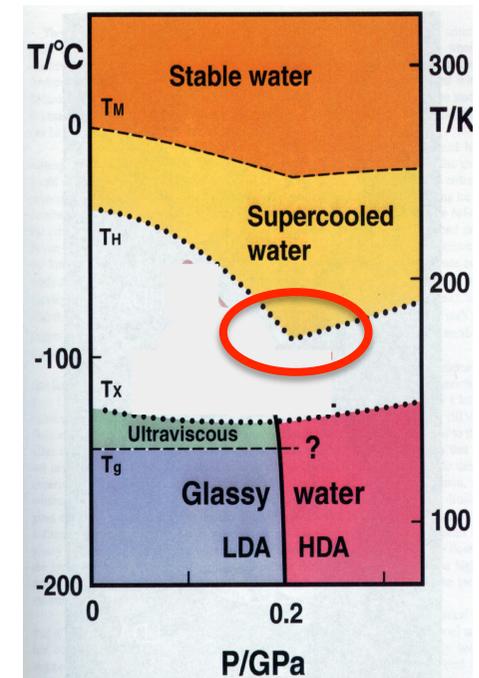
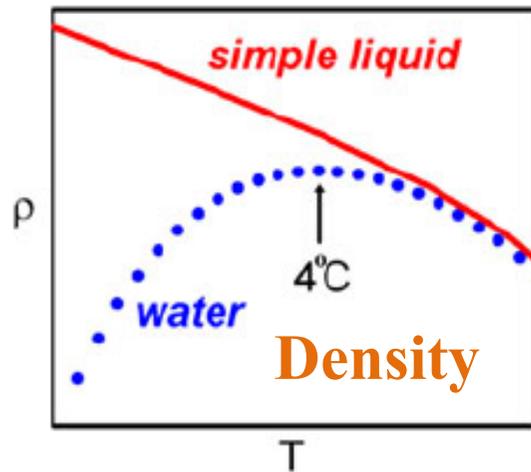
Poole et. al, Nature (1992)

Mishima and Stanley, Nature (1998)

# Materials share similar anomalous properties

- Negatively-sloped melting line
- Density anomaly
- Extrema in melting curve

$$\frac{dP}{dT} = \frac{\Delta S}{\Delta V} = \frac{L}{T \Delta V}$$

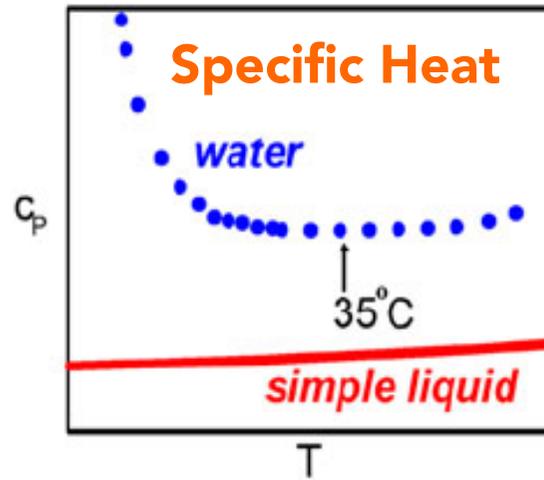
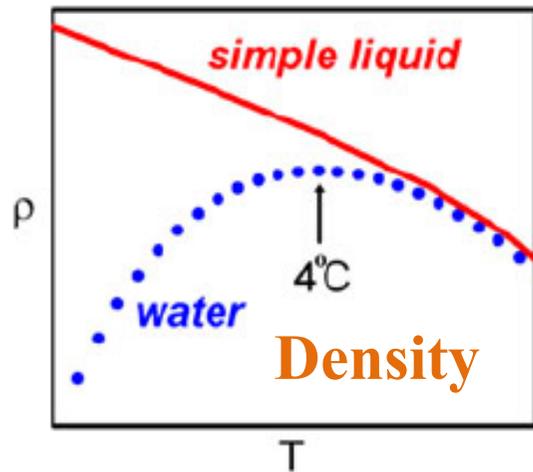


Poole et. al, Nature (1992)  
Mishima and Stanley, Nature (1998)

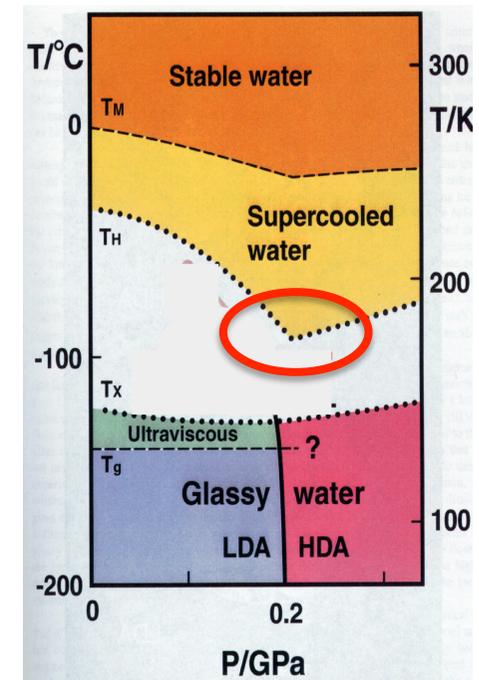
# Materials share similar anomalous properties

- Negatively-sloped melting line
- Density anomaly
- Extrema in melting curve
- Response function anomaly

$$\frac{dP}{dT} = \frac{\Delta S}{\Delta V} = \frac{L}{T \Delta V}$$



$$\langle (\delta S)^2 \rangle \sim N k_B C_p$$



Poole et. al, Nature (1992)  
Mishima and Stanley, Nature (1998)

# Understanding water anomalies

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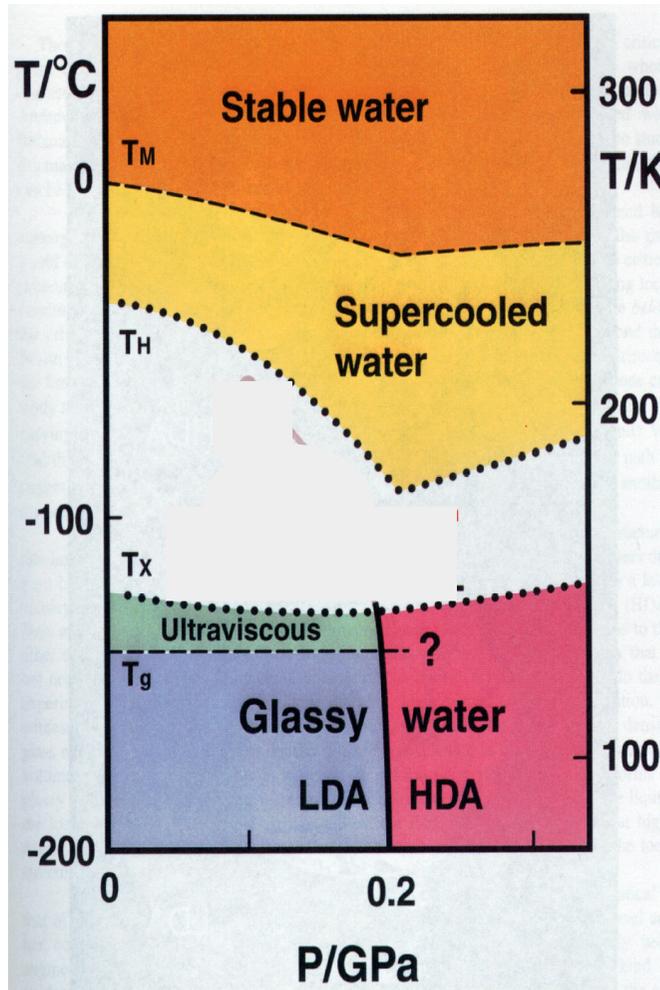
## Scenarios and views

(with or without second critical point)

- Liquid-liquid critical point (Stanley)
- Two-state model (Anisimov)
- Two-order parameter model (Tanaka)
- Spinodal reentrant (Angell & Speedy)
- Singularity-free (Sastry & Debenedetti)

# Challenges remains to detect experimentally

## Liquid-liquid hypothesis



Poole et. al, Nature (1992)

Mishima and Stanley, Nature (1998)

**Difficulty:** not easily to testify the location of liquid phases in deep supercooled region due to crystallization

**Anders Nilsson's & Thomas Loerting's group lowering the limit from above and pushing the limit up**

- ❑ Can we construct a simple model that shows **stable** LLCPC and captures water-like anomalies?
- ❑ How to understand properties of water in terms of isotropic two-scale interactions
- ❑ How confinement and surface chemistry affect the anomalous properties and phase behaviors?

□ Can we construct a simple model that shows **stable** LLCPC and captures water-like anomalies?

□ How to understand properties of water in terms of isotropic two-scale interactions

□ How confinement and surface chemistry affect the anomalous properties and phase behaviors?

# Characteristics of model

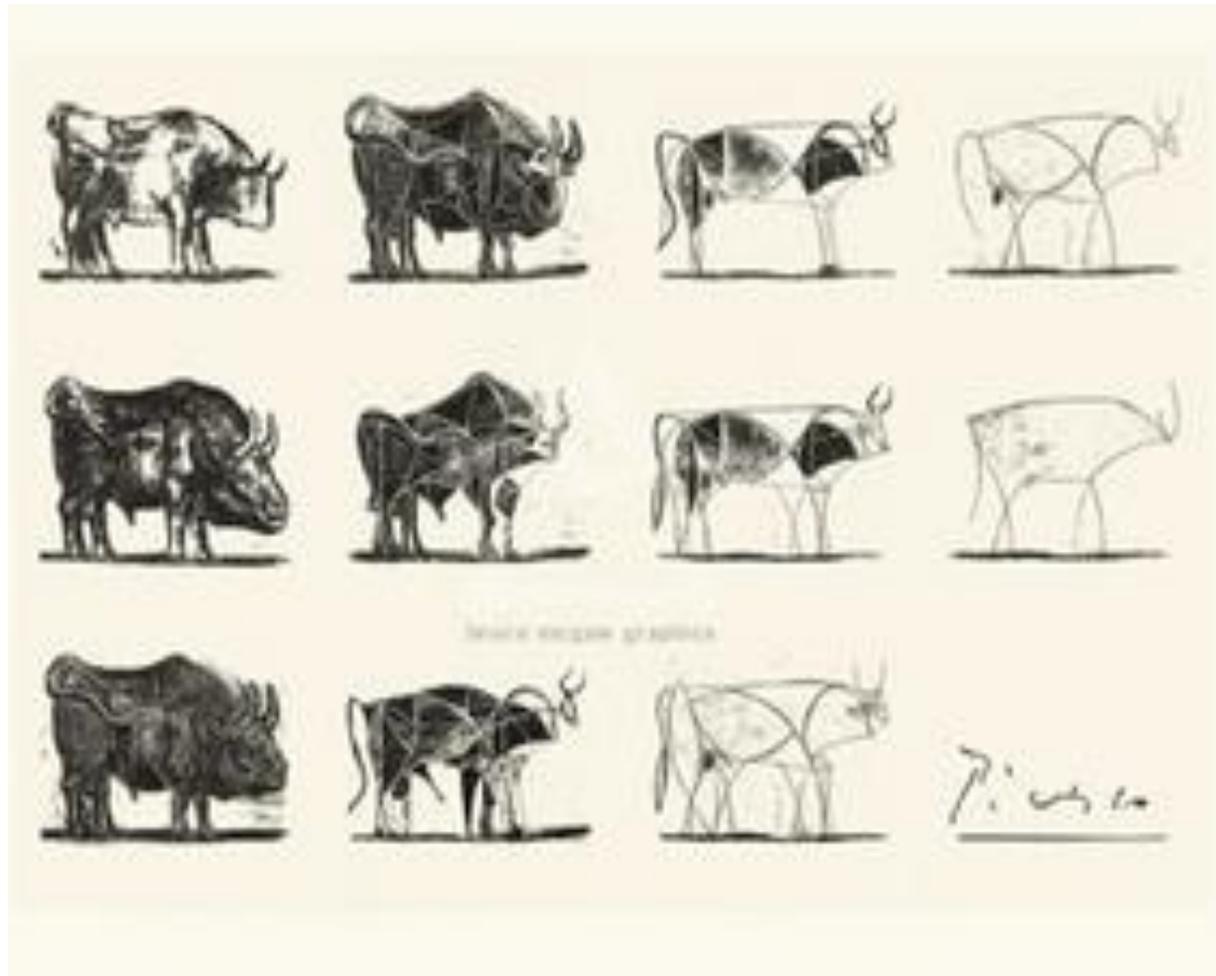
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➤ Accessible liquid-liquid critical point  
(Testify the liquid-liquid phase transition hypothesis)

➤ Water-like anomalies  
(Map simulation result to experimental results)

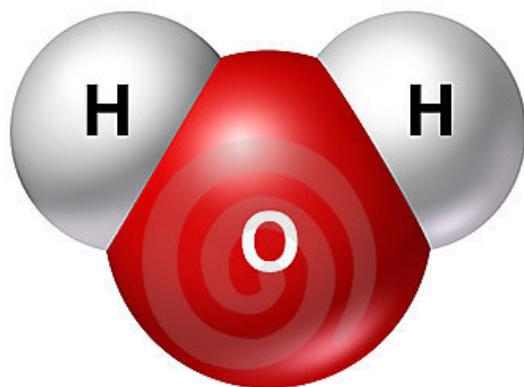
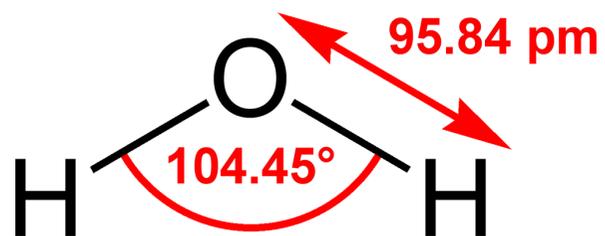
# What makes water water

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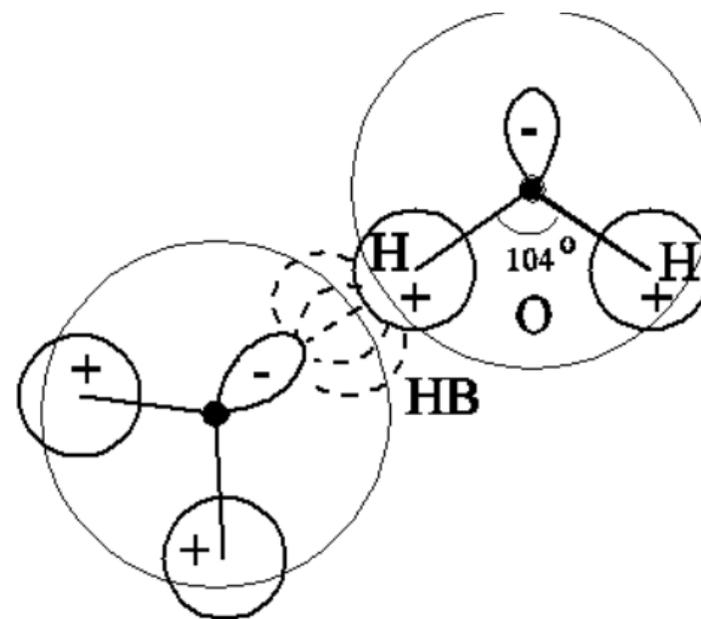
# Simulation approach

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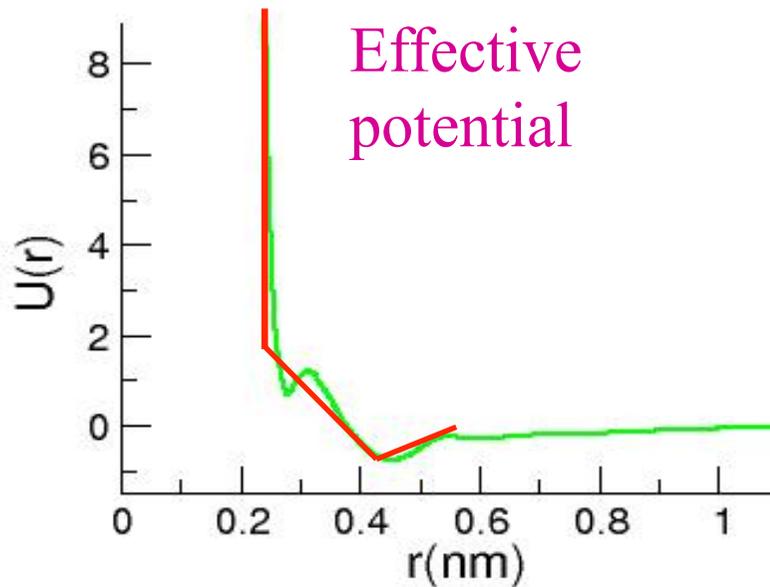


**WATER  
MOLECULE**

dreamstime.com



# Spherically symmetric potentials



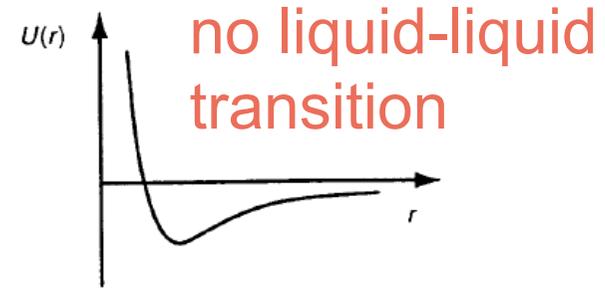
T. Head-Gordon and F. H. Stilinger.

J. Chem. Phys. 98, 3313 (1993)

$$U(r) \sim \ln g(r)$$

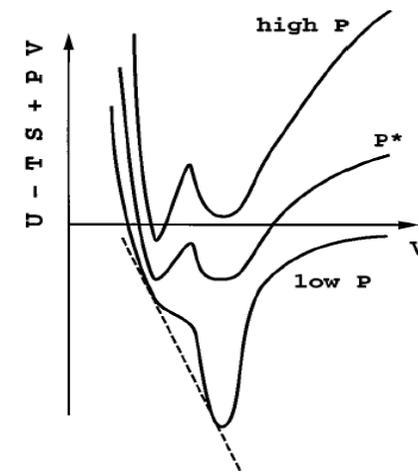
## Characteristics:

- Coarse-grained spherical symmetrical potential
- Two-length scale: hardcore & softcore



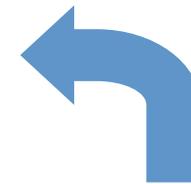
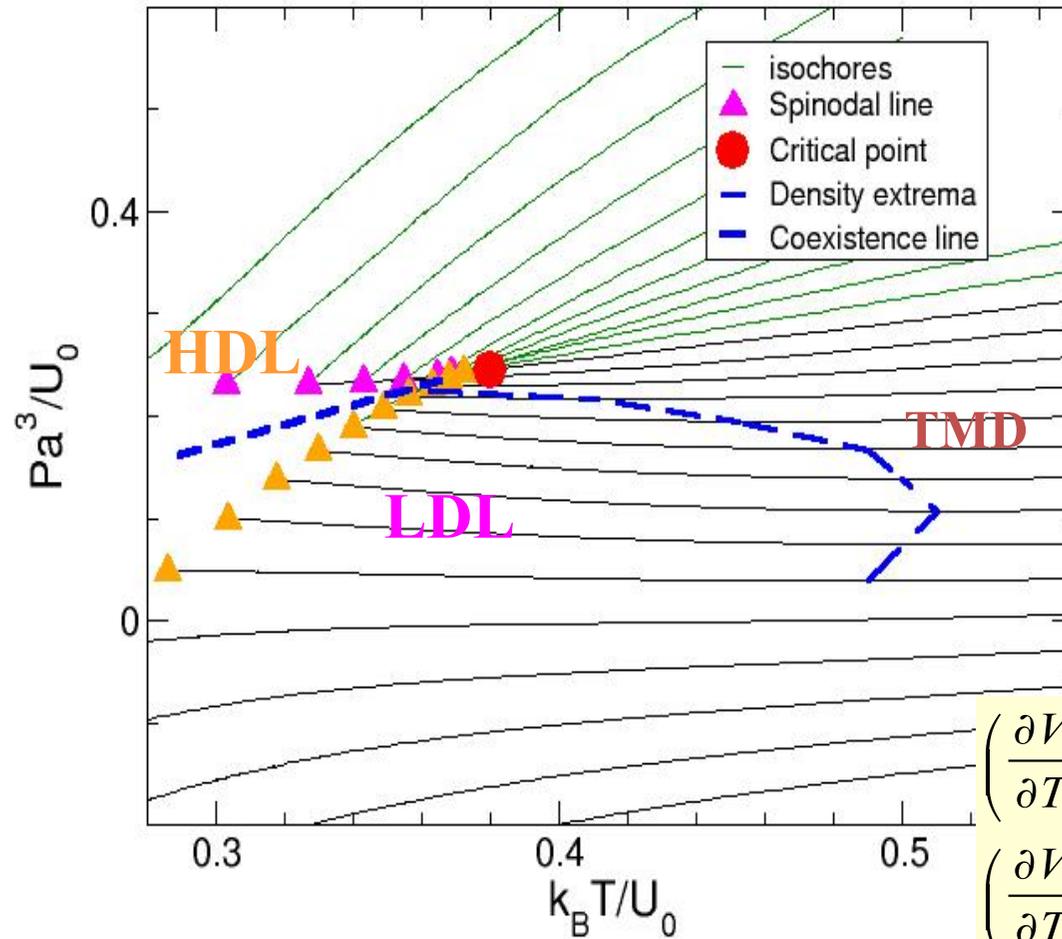
with liquid-liquid transition

$$G(P, T) \equiv \min_V \{ U + PV - TS \}$$



L. Xu et al., Phys. Rev. E 74, (2006)

# Equation of State

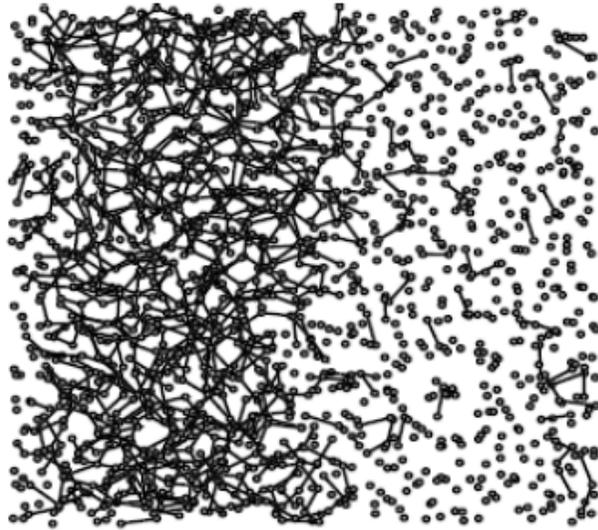


$$\left(\frac{\partial V}{\partial T}\right)_P = 0$$

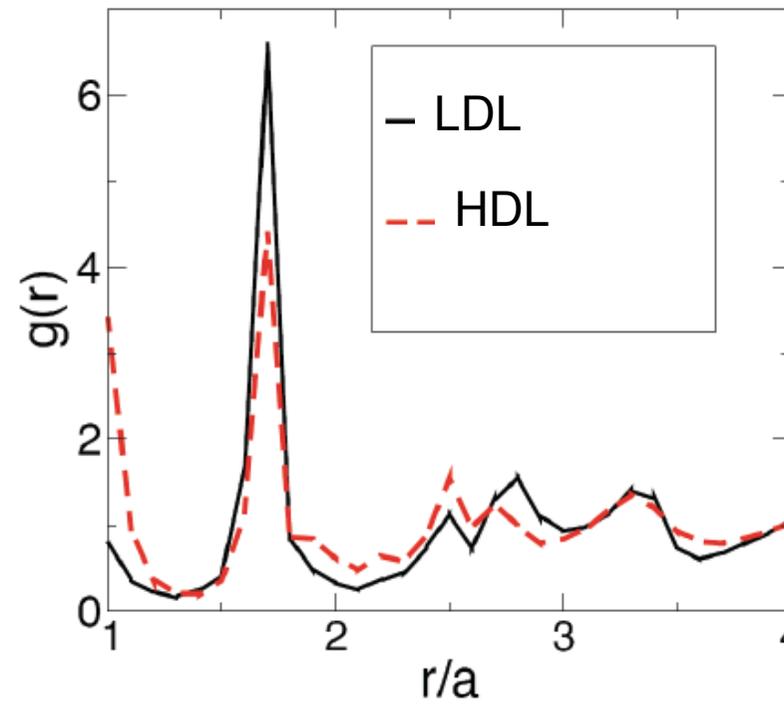
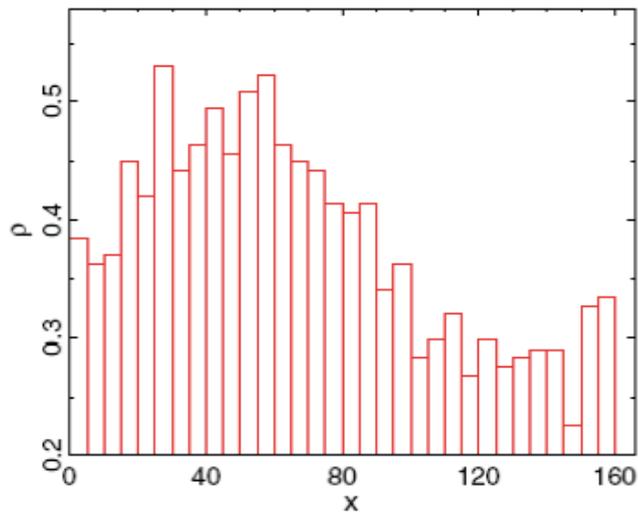
$$\left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V \left(\frac{\partial P}{\partial V}\right)_T = -1$$

$$\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial P}\right)_T = VK_T \left(\frac{\partial P}{\partial T}\right)_V$$

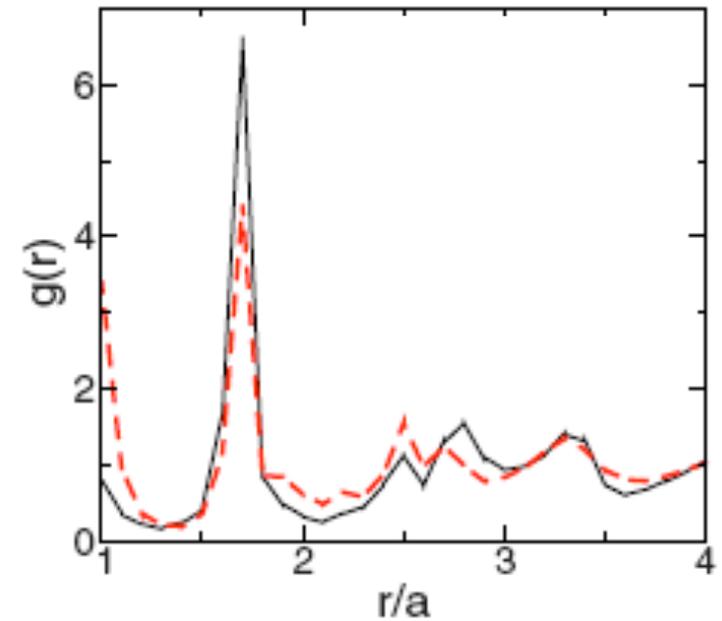
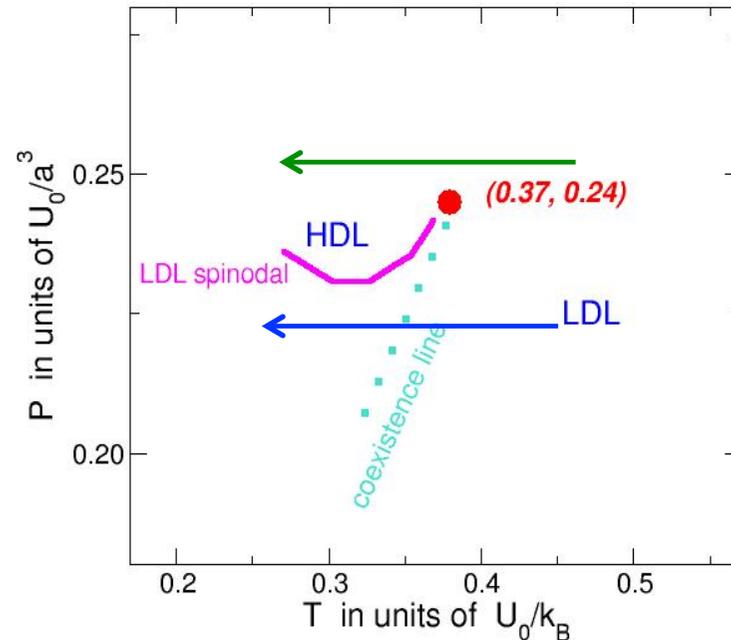
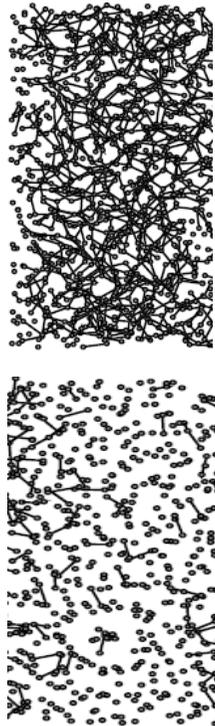
# Liquid polyamorphism



Phase Segregation  
at coexistence line



# Two glasses upon cooling

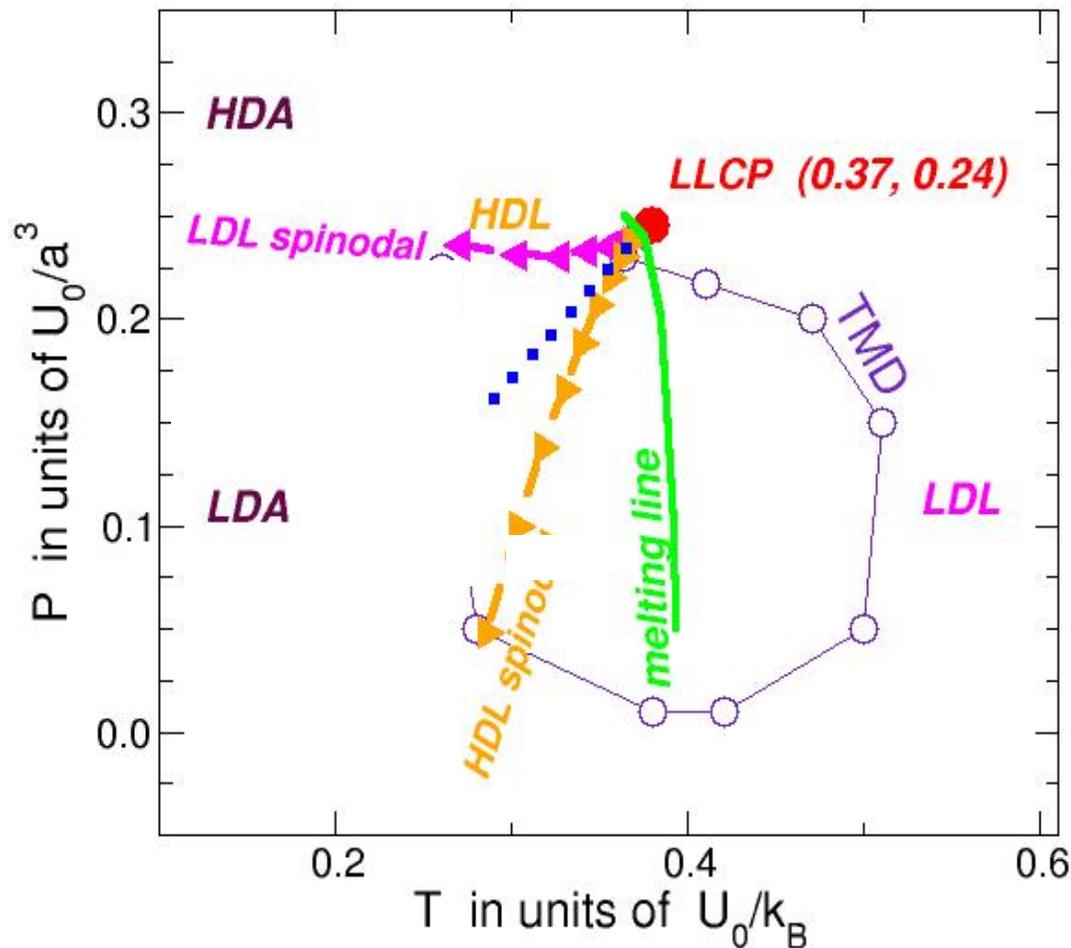


## Two glass states obtained upon cooling

Low density liquid → Low density amorphous (LDA)

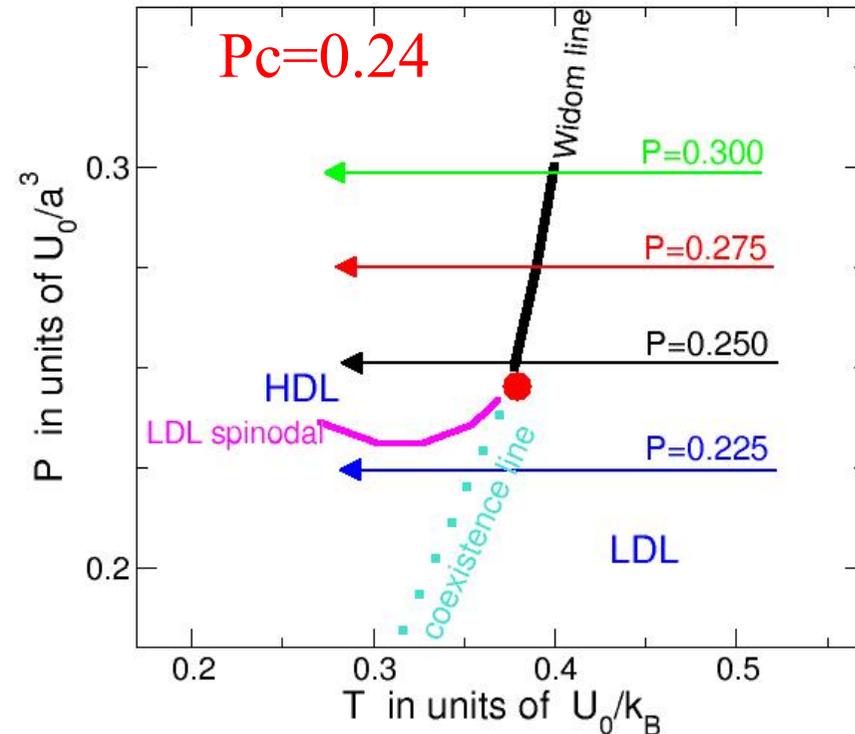
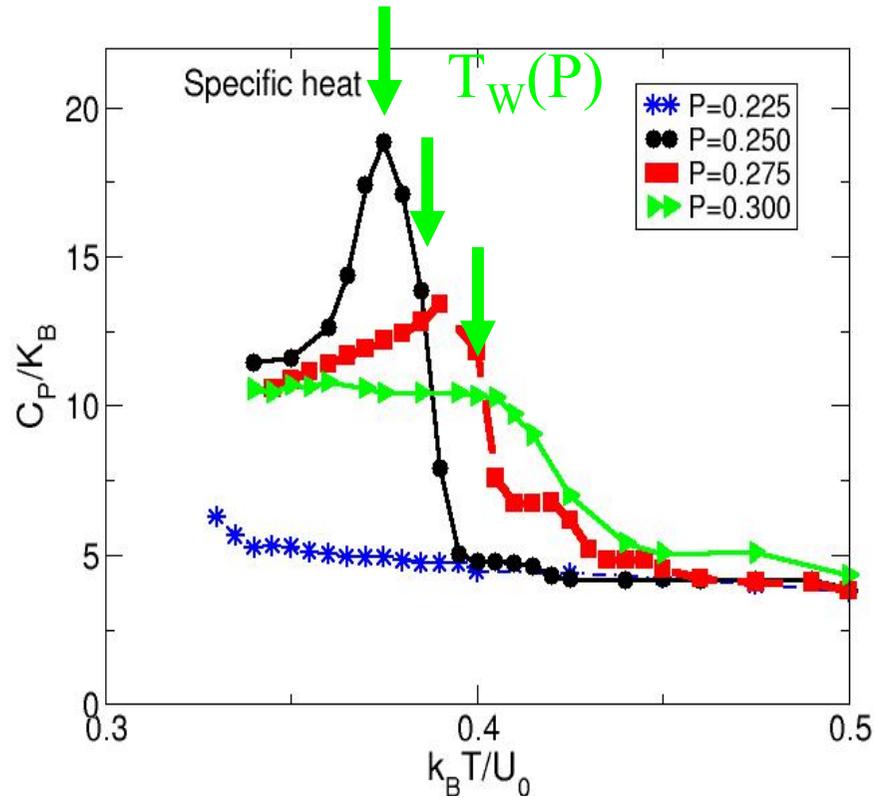
High density liquid → High density amorphous (HDA)

# Phase diagram



- Stable liquid-liquid critical point (LLCP)
- Density anomaly (TMD and TmD)
- LDA and HDA

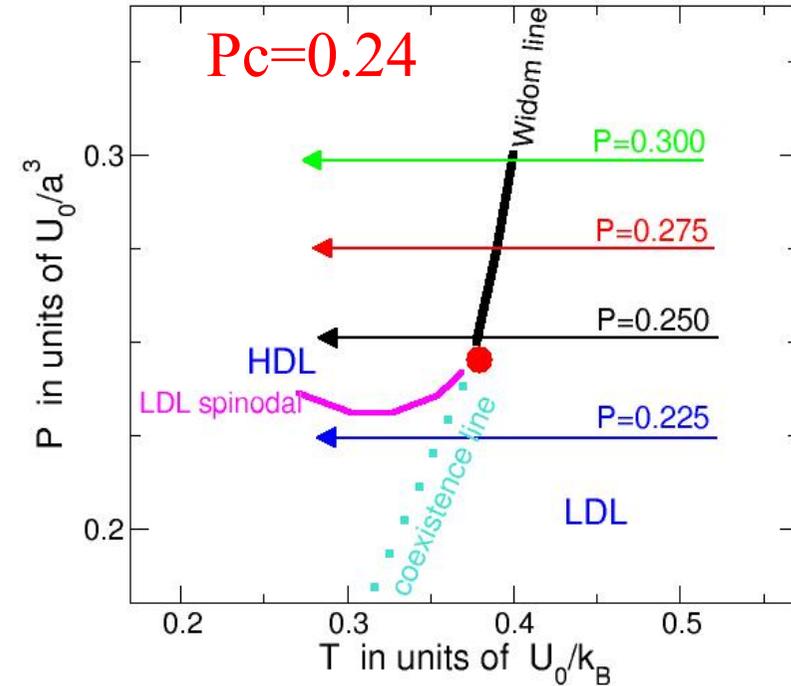
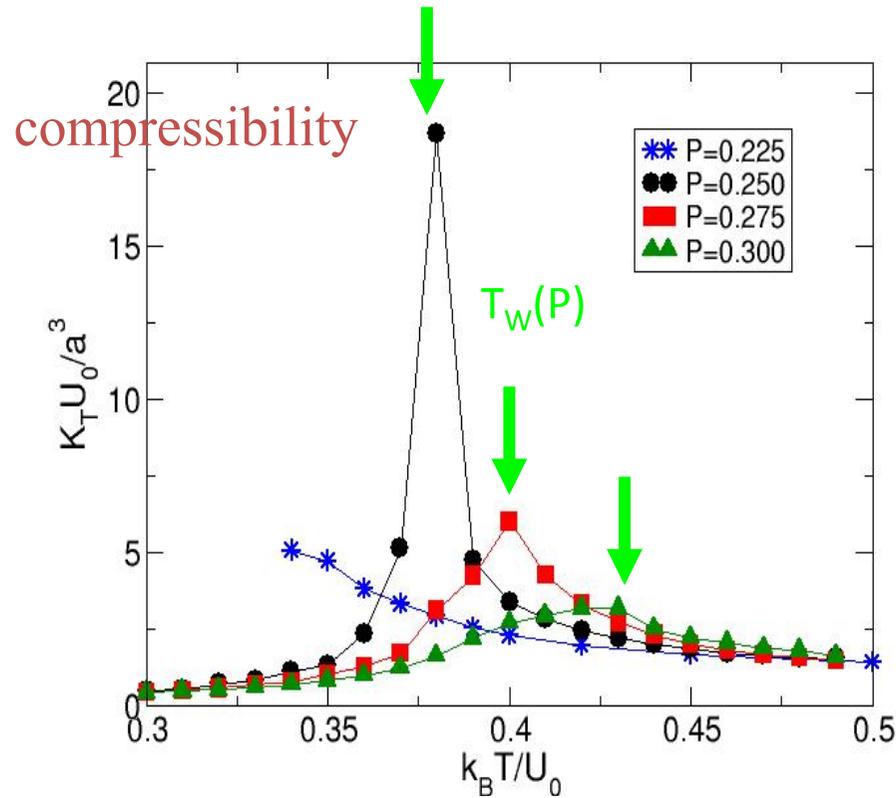
# Changes in thermal response functions



- $P < P_c$ : No anomalous behaviour! (Metastability)
- $P > P_c$ :  $C_p$  show peaks (Widom line –locus of the  $C_p^{\max}$ )

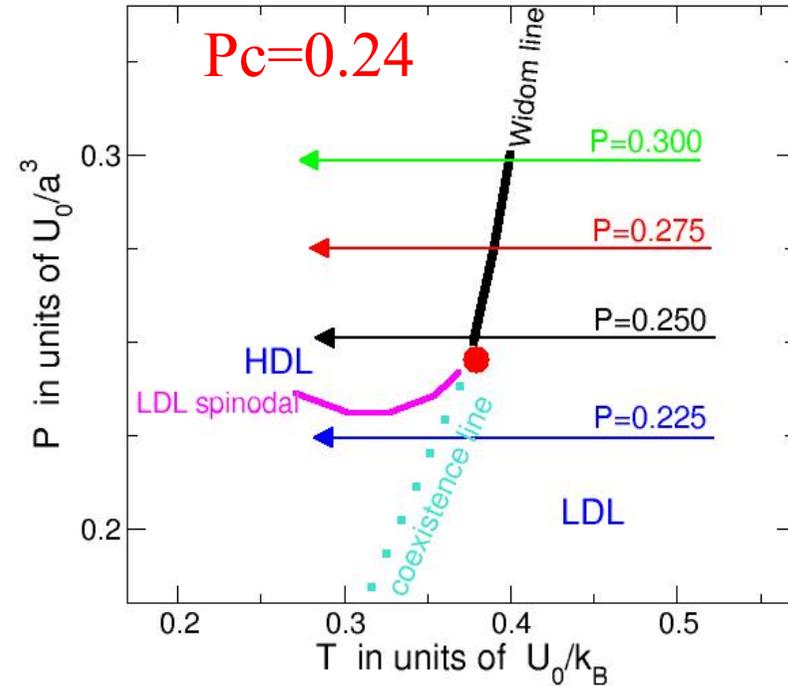
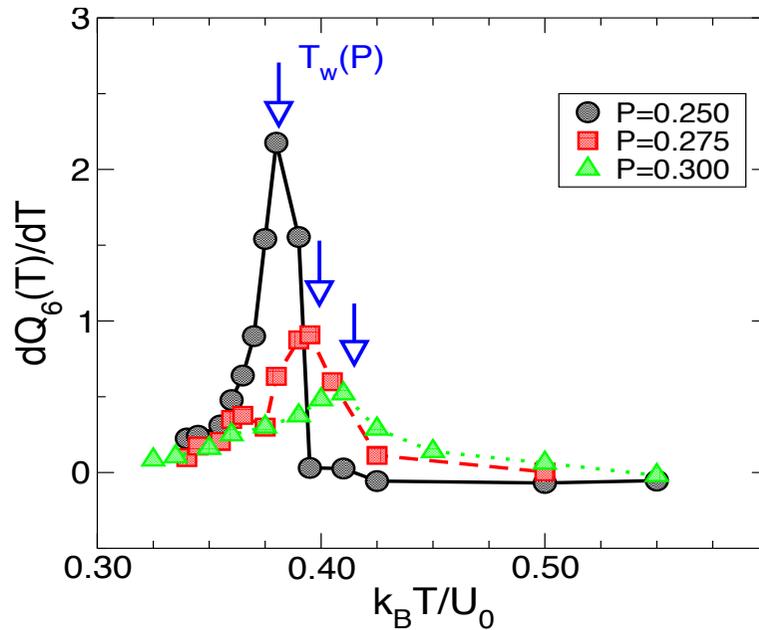
The Widom line terminates at the liquid-liquid critical point

# Changes in compressibility



- $P < P_c$  : No anomalous behaviour (Metastability)
- $P > P_c$  : Response functions show peaks. The location of the peaks decreases approaching to the critical pressure

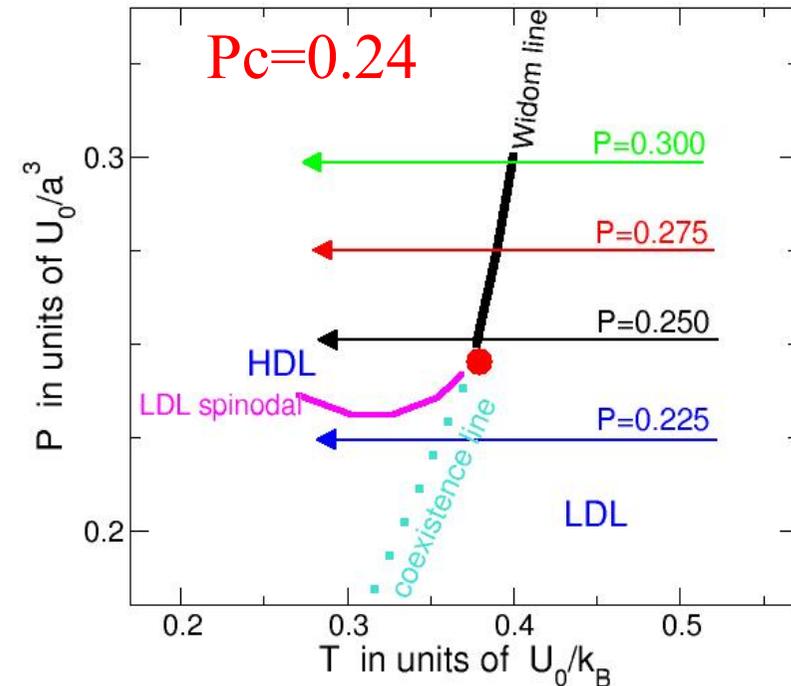
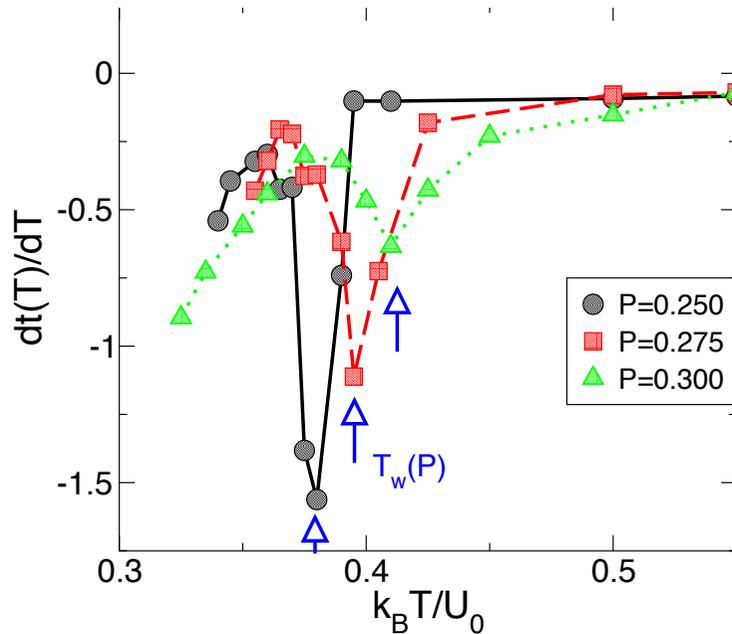
# Changes in structures: orientational order



Orientalional order:

$$Q_l = \left[ \frac{4\pi}{2l+1} \sum_{m=-l}^{m=l} |Y_{l,m}(\theta, \varphi)|^2 \right]^{1/2}$$

# Changes in structures: translational order

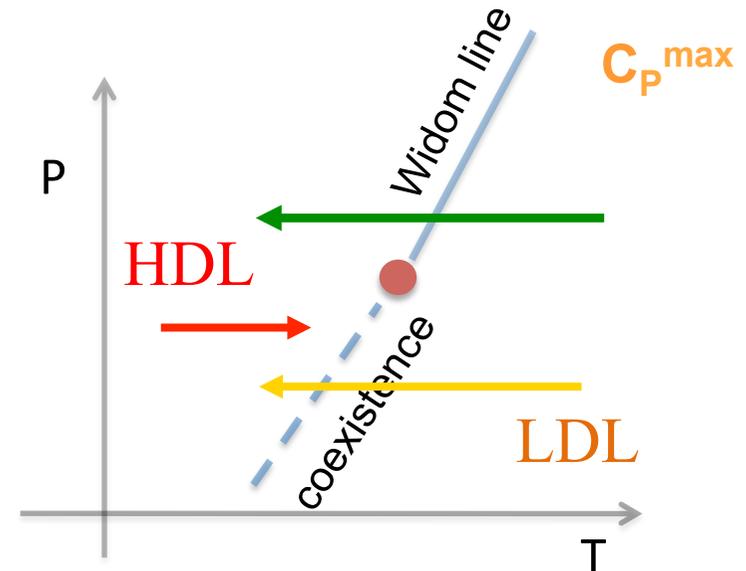
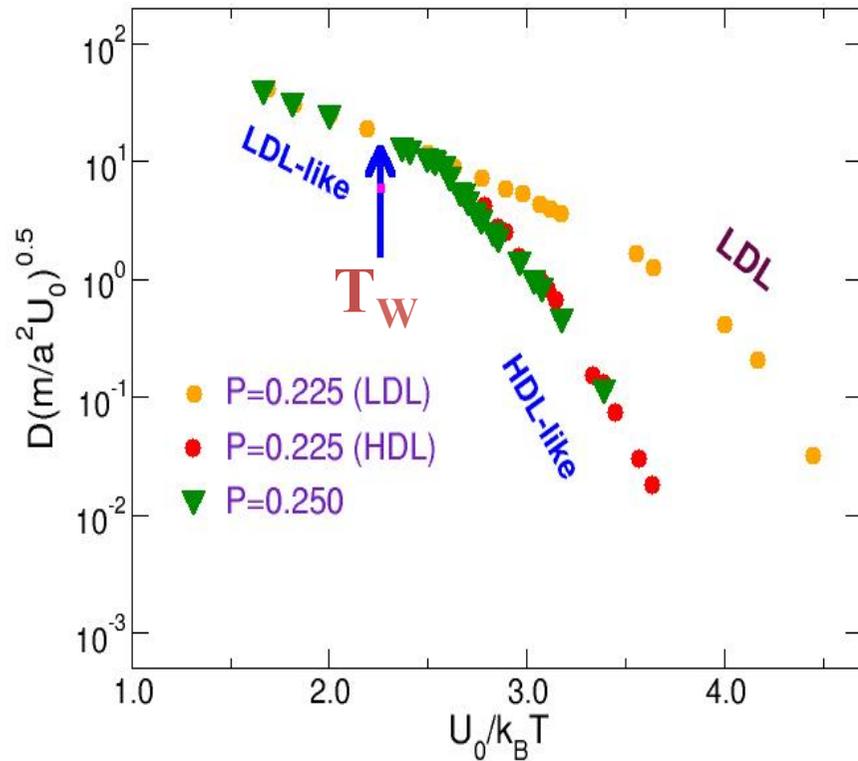


Translational order parameter:

$$t \equiv \int_0^{r_c} |g(r) - 1| dr$$

Upon crossover the Widom line, structure change is a maximum

# Changes in dynamics



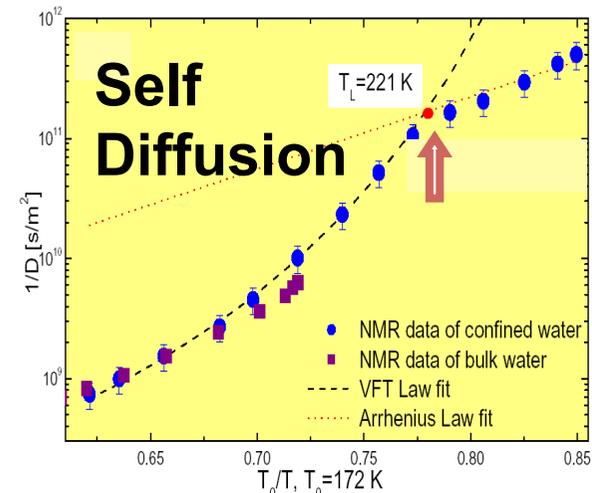
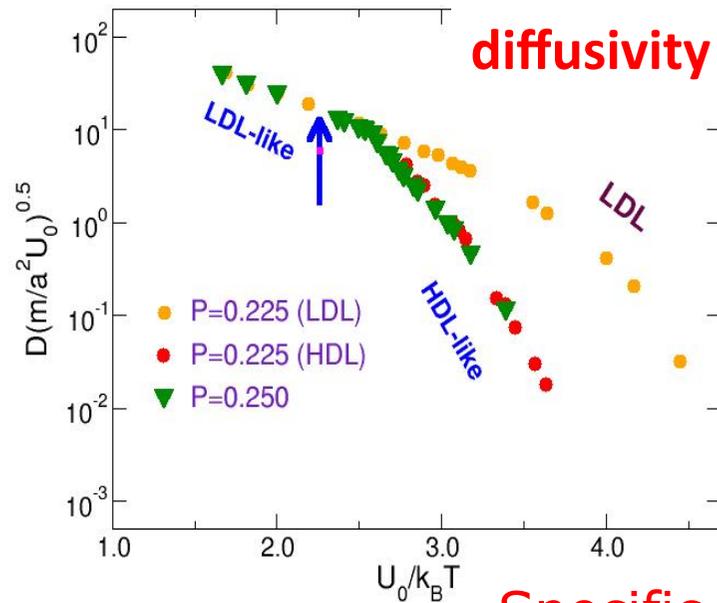
Upon crossover the Widom line, a kink in  $D$  occurs near  $T_W$

- ❑ Can we construct a simple model that shows **stable** LLCPC and captures water-like anomalies?

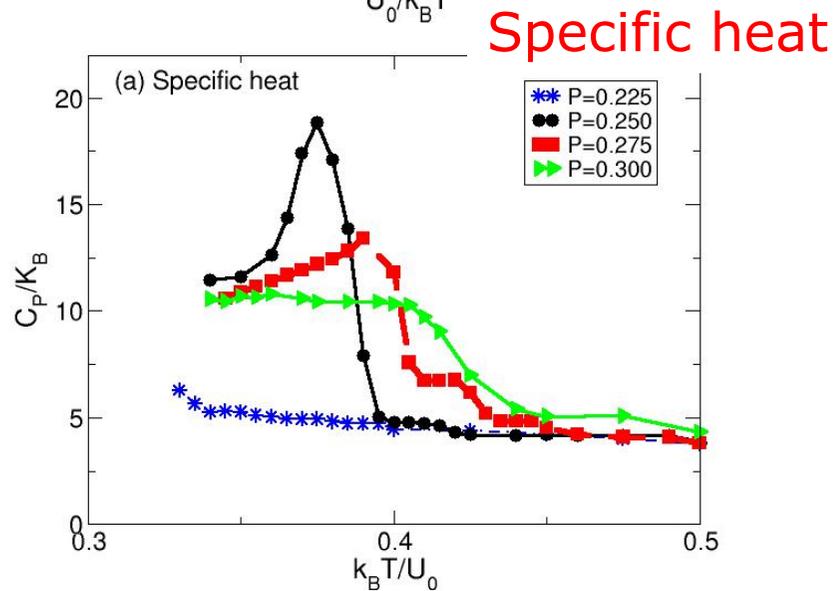
- ❑ How to understand properties of water in terms of isotropic two-scale interactions

- ❑ How confinement and surface chemistry affect the anomalous properties and phase behaviors?

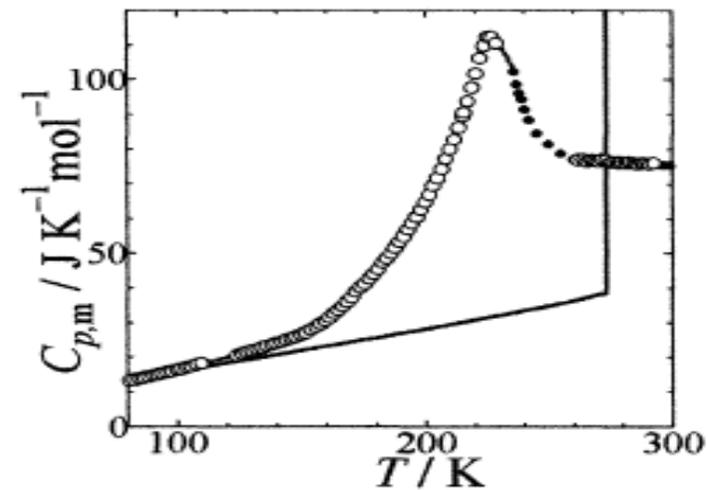
# Compare computation results with experiment



Mallamace et al, JCP 124, 161102 (2006)

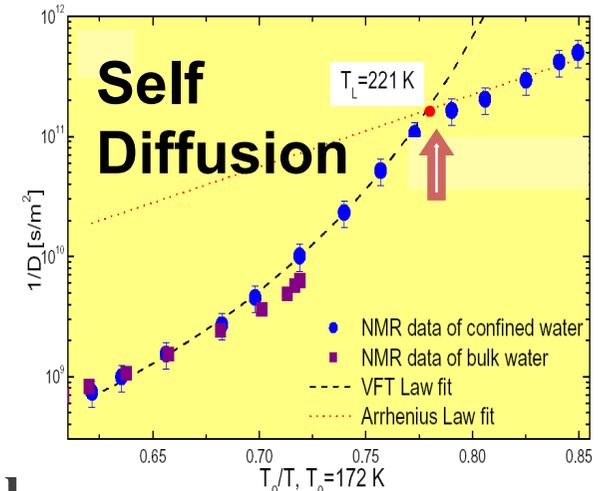
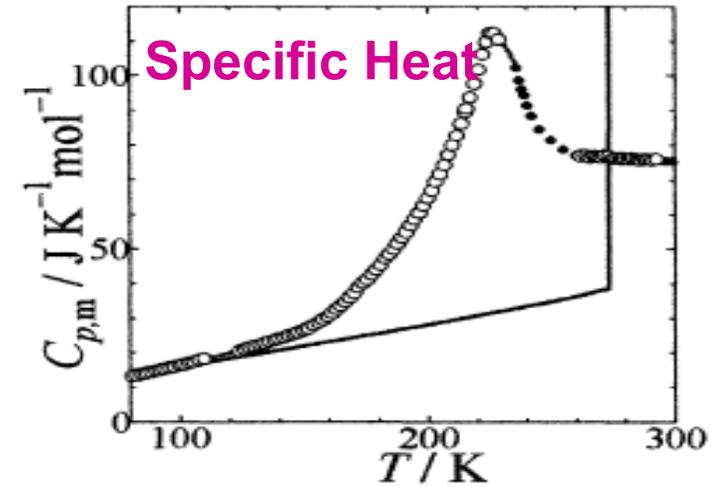
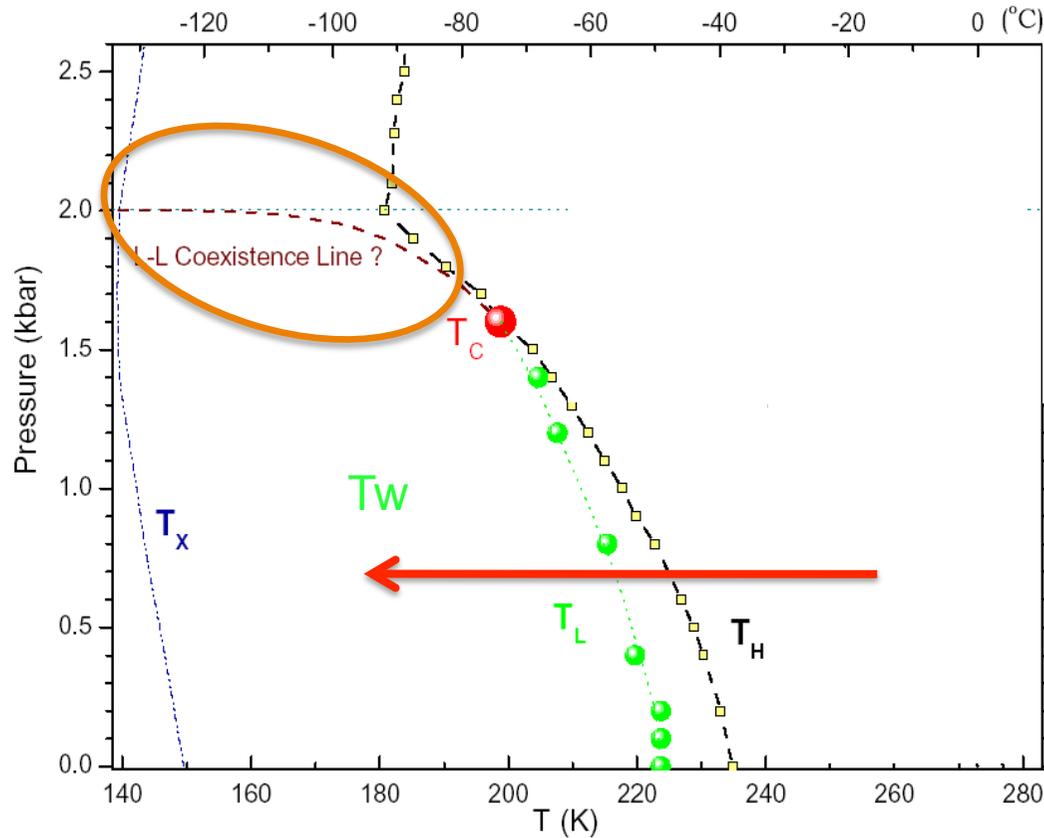


## Specific Heat



Maruyama et. al., AIP conf. Proc. 708, 675 (2004)

# Detection of second critical point in experiment

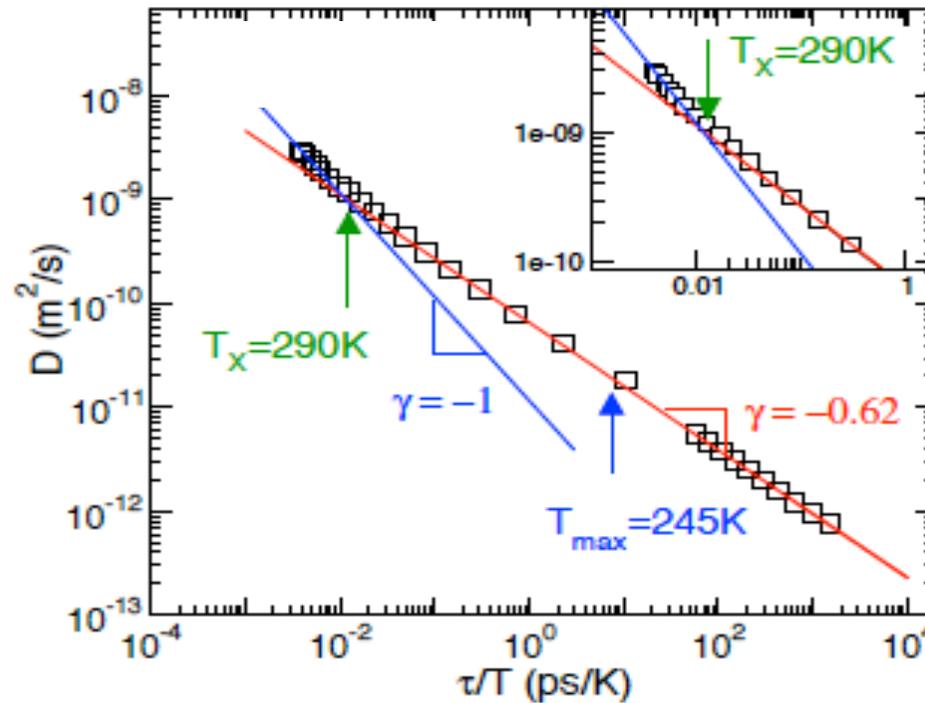


**Region to detect:** high temperature and low pressure  
**Criteria:** response function maxima

L. Xu et. al, Proc. Natl. Acad. Sci. **102**, 16558-16562 (2005)

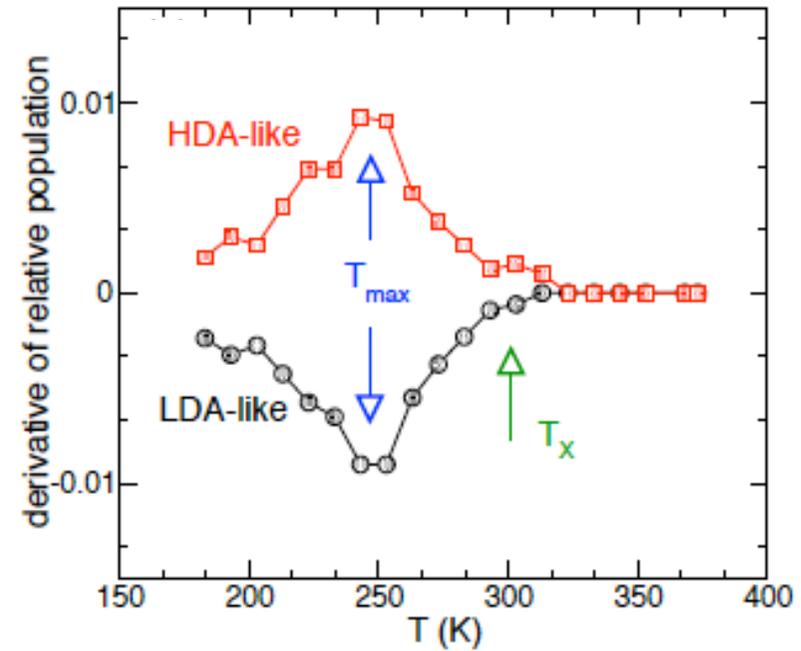
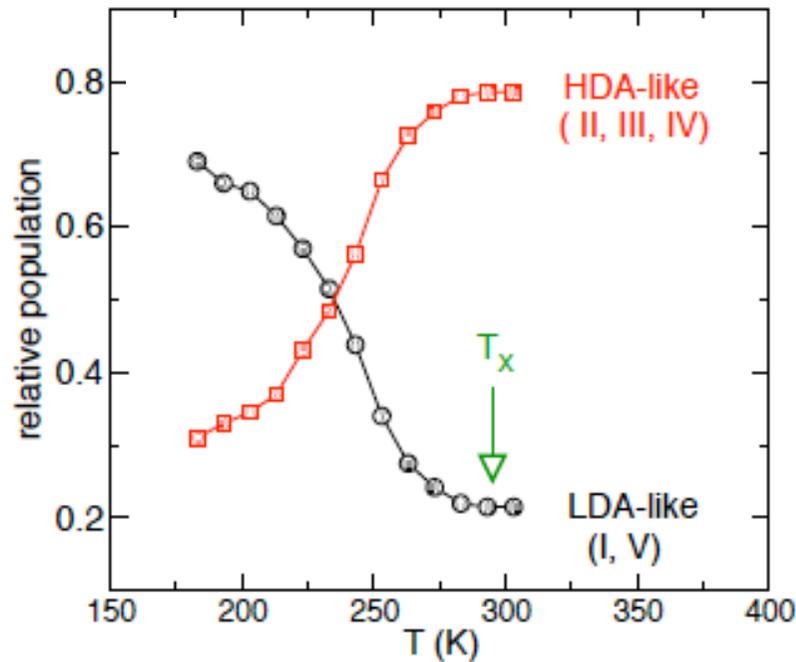
L. Liu et. at. PRL 95, 117802 (2005)

# Fractional Stokes-Einstein relation and its breakdown



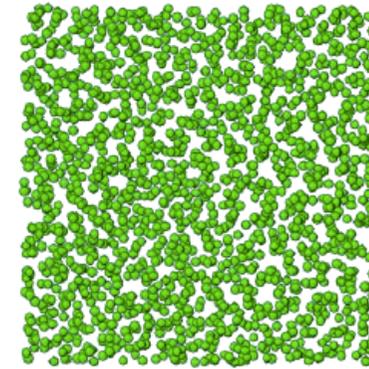
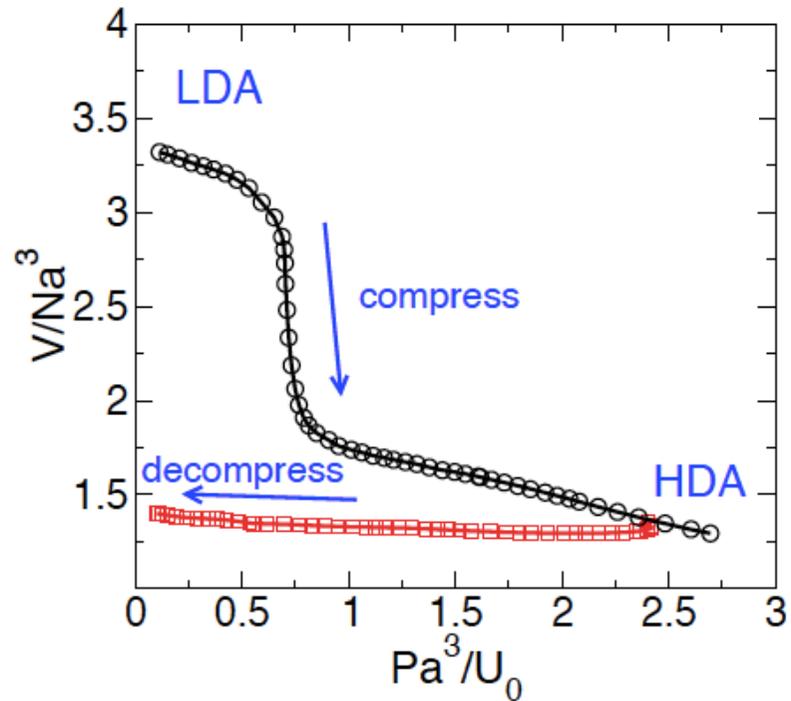
- There exists a fractional Stokes-Einstein relation, the breakdown temperature  $T_x \sim 290\text{K}$
- Breakdown of Stokes-Einstein relation  $T_x \sim 290\text{K} > T_w > T_g$ , not directly associated with the Widom line

# Changes in structure and breakdown of SER (EXP)

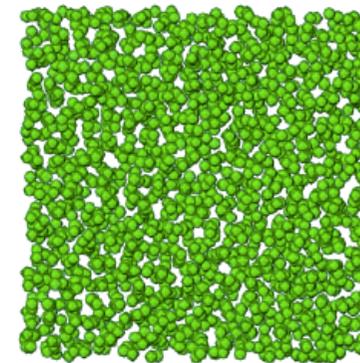


- SER breakdown occurs at temperature where the local structure of water changes
- Near the Widom line temperature, structure change is a maximum

# Low density glass to high density glass transition



LDA

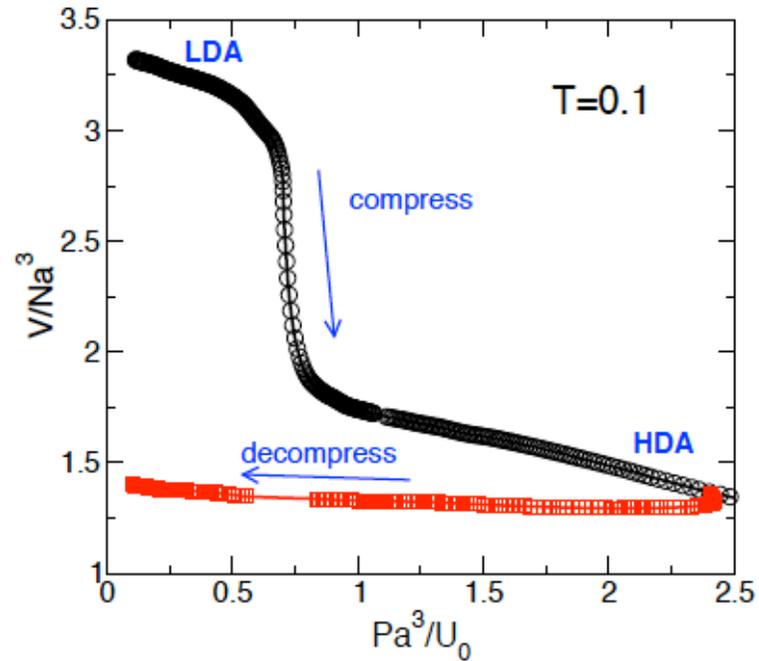


HDA

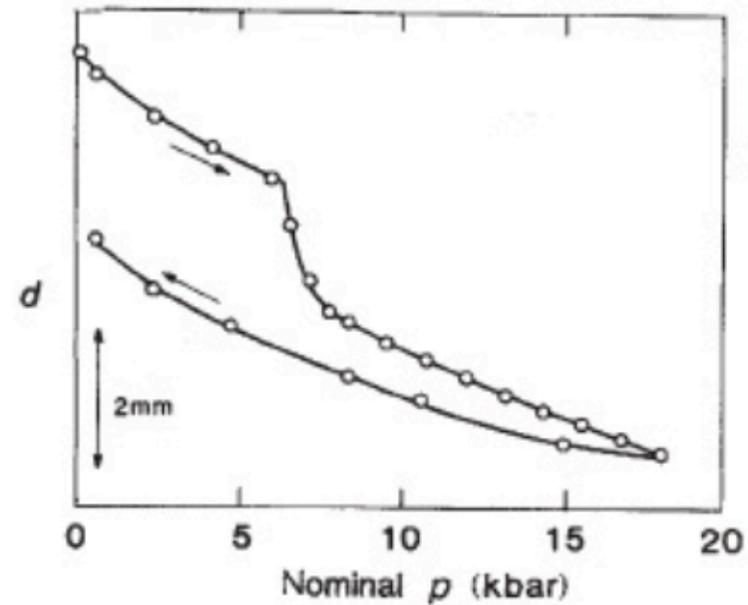
Formation of new high density glasses by compression and decompression along constant pressure

L. Xu et. al, J. Chem. Phys. 134, 064507 (2011)

# Polyamorphism



EXP

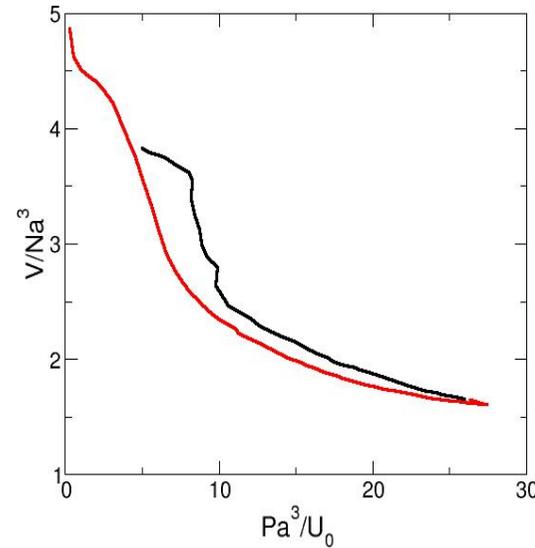
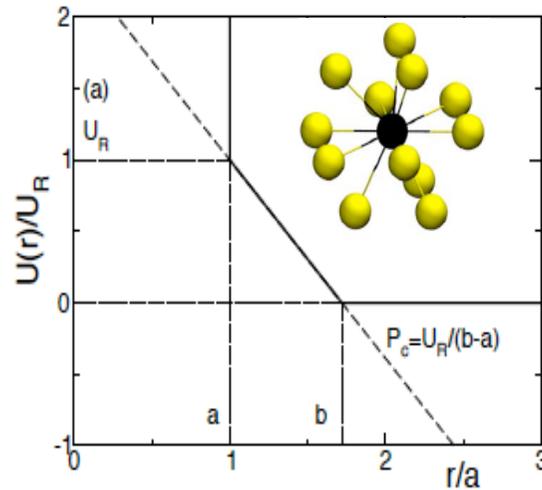


Mishima, L. D. Calvert, and E. Whalley, Nature (London) **310**, 393 (1984)

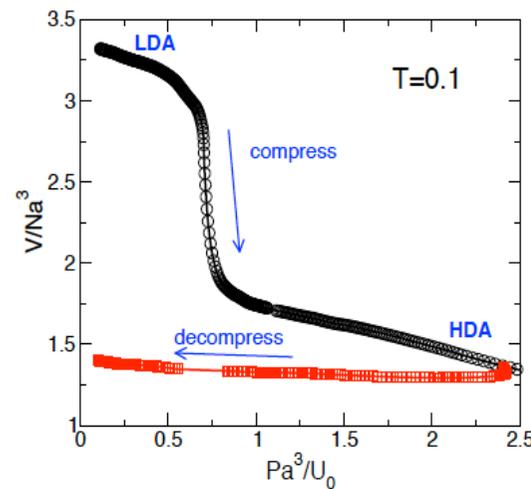
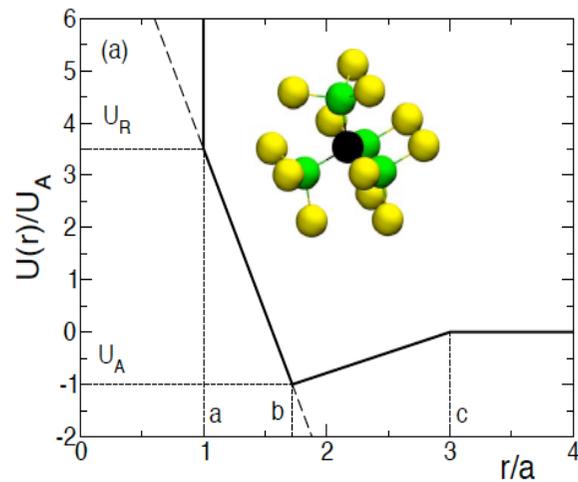
HDA is stable at low pressure upon decompression

L. Xu, S. V. Buldyrev, N. Giovambattista, C. A. Angell, H. E. Stanley, JCP (2009)

# Stability of liquid-liquid critical point and polyamorphism



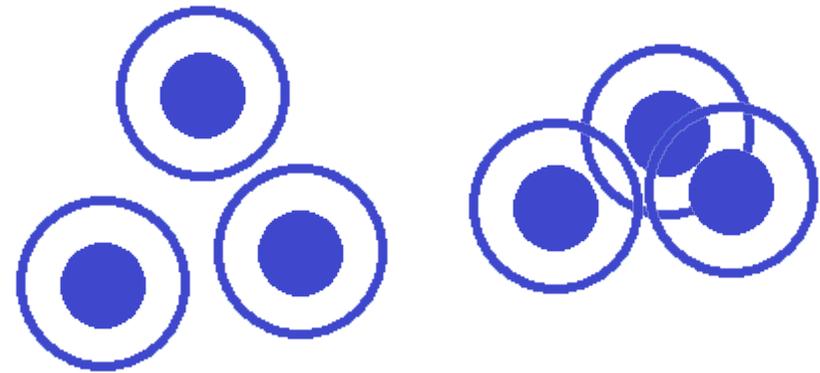
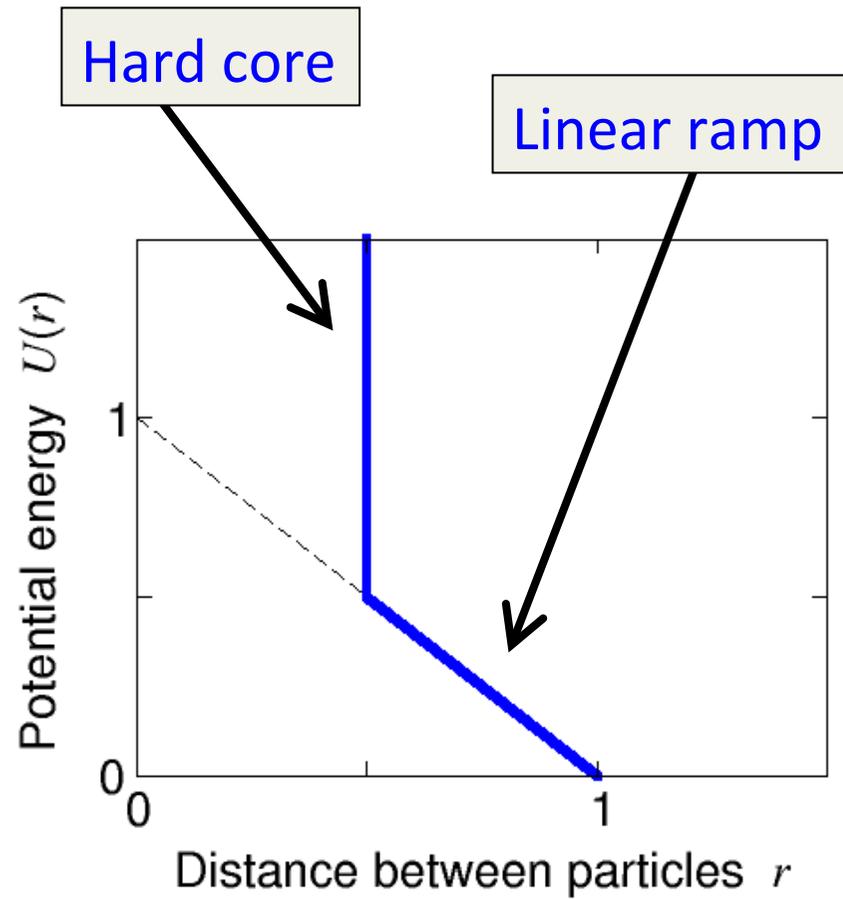
LLCP unaccessible  
TMD, diffusivity, etc



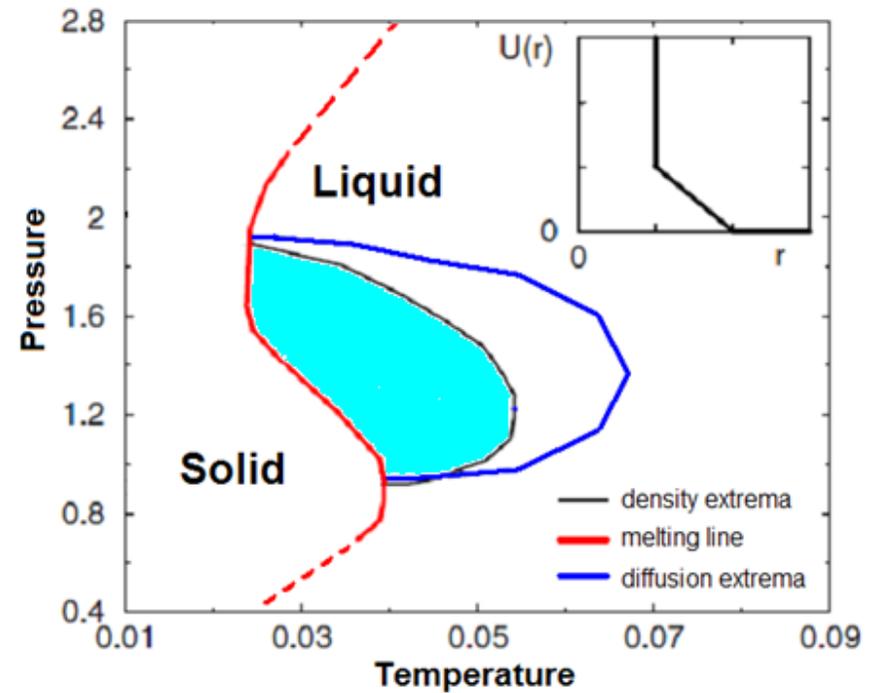
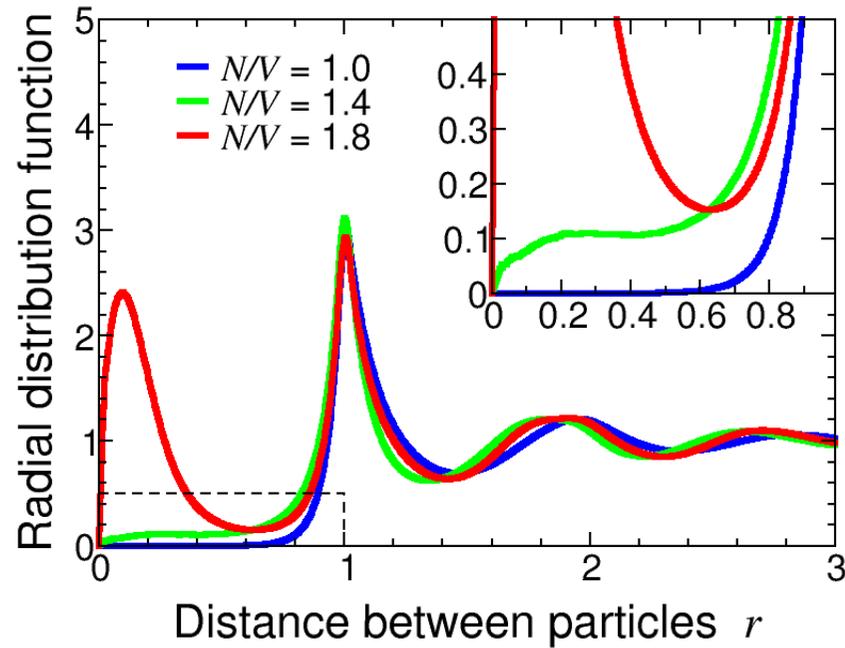
Stable LLCP

# How to understand anomalies with two scales

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## Two "competing" length scales

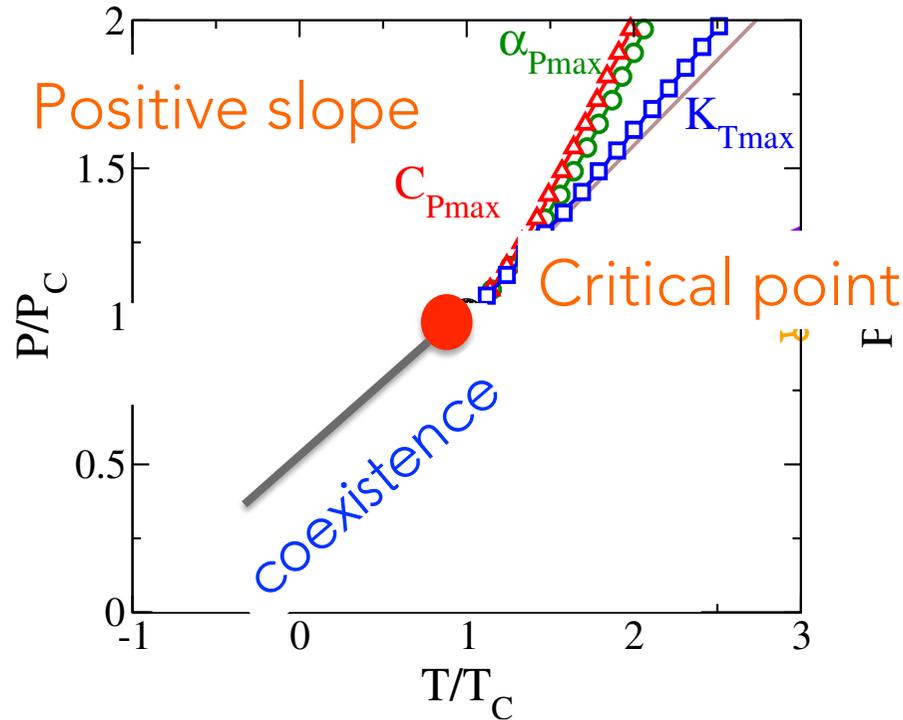


Within anomaly region, some particles are on the ramp

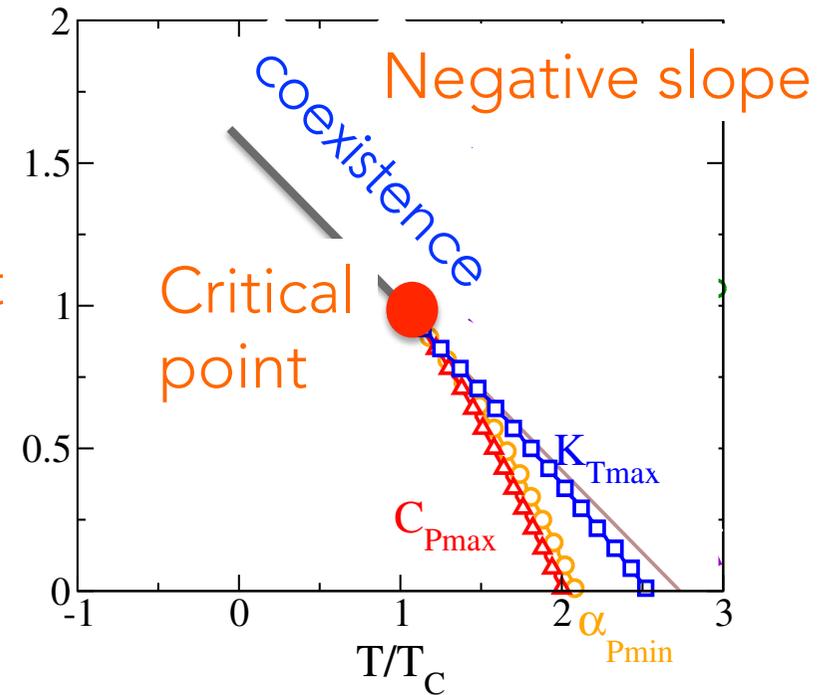


# Supercritical phenomenon in different transitions

## Liquid-gas transition



## Liquid-liquid transition



**Tracing critical point from supercritical region along the Widom can be a rather general approach**

## Conclusion I & II

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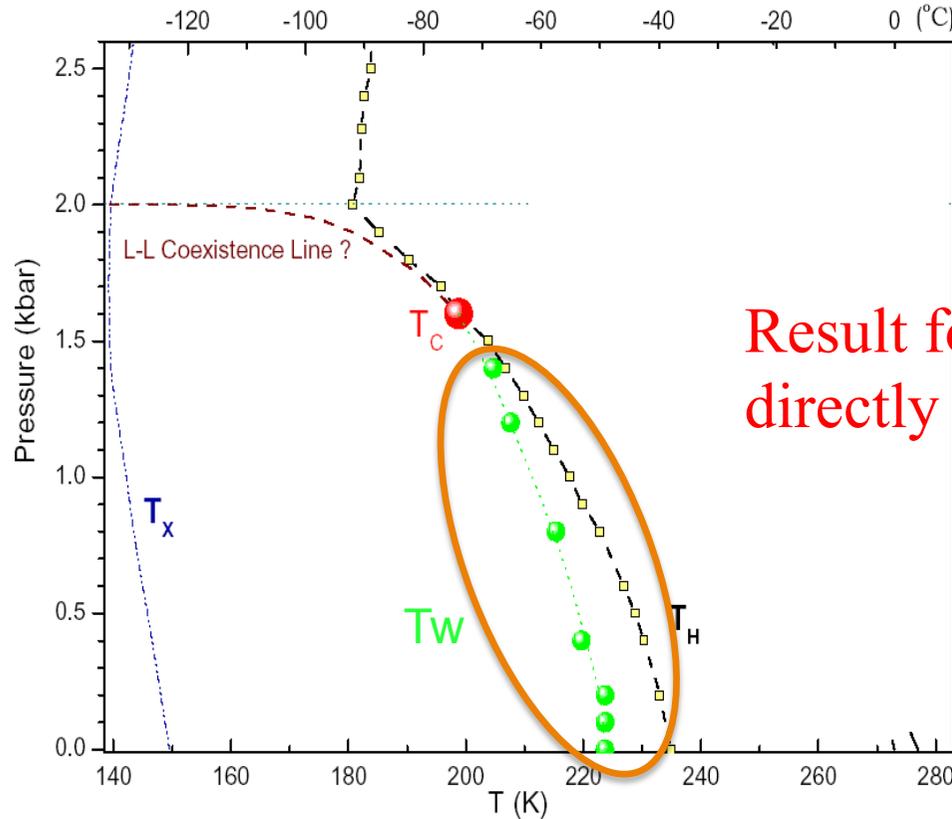
- The two-scale model can reproduce water-like anomalies
- Thermodynamic and dynamic quantities shows changes upon crossing the Widom line, not upon crossing the coexistence line
- Provide a way for experiments to locate the possible existence of liquid-liquid critical point
- Maybe not hydrogen bond, not tetrahedral local structure, but the two-scale matters for some of water-like anomalies?

□ Can we construct a simple model that shows **stable** LLCPC and captures water-like anomalies?

□ How to understand properties of water in terms of isotropic two-scale interactions

□ How confinement and surface chemistry affect the anomalous properties and phase behaviors?

# Bulk and confined water



Result for confined system  
directly map to bulk system

## Question:

Is the phase diagram for confined system good to represent the phase diagram of bulk water?

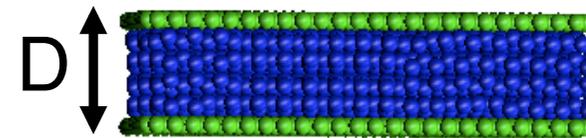
# Difference between bulk and confined system

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**Bulk liquid:** temperature  $T$ , pressure  $P$

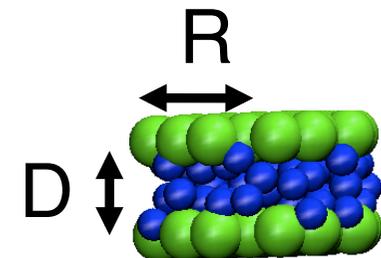
**Confined liquid:**

◆ Infinite plates:  $T, P, D$

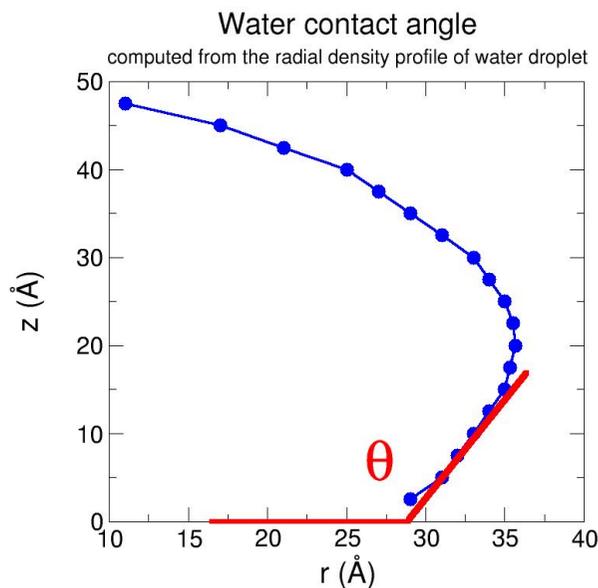
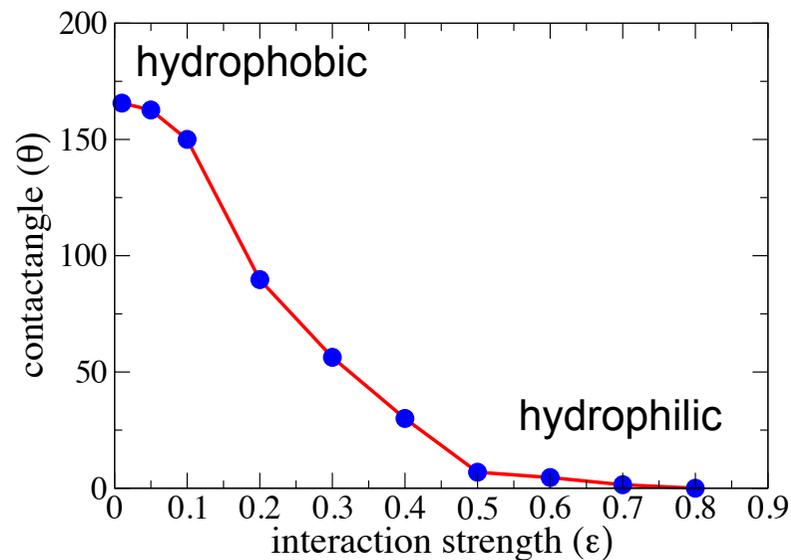
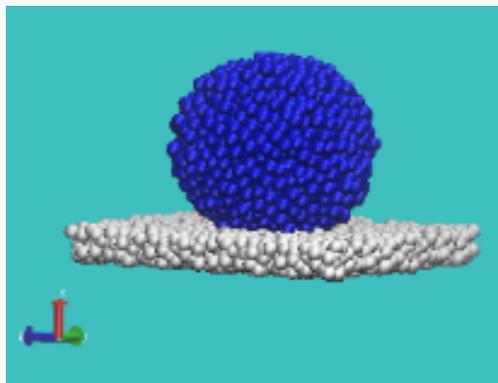


◆ Finite plates:  $T, P, D, R$

◆ Confined effect and surface interaction effect



# Surface chemistry: hydrophobic or hydrophilic

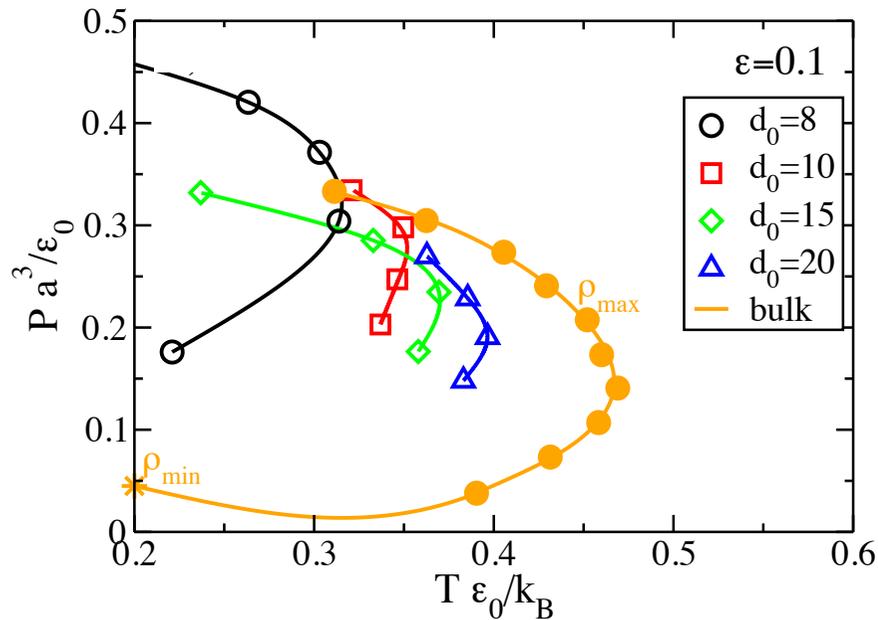


The surface is hydrophobic for interaction length  $\epsilon < 0.2$ , and hydrophilic for  $\epsilon > 0.2$

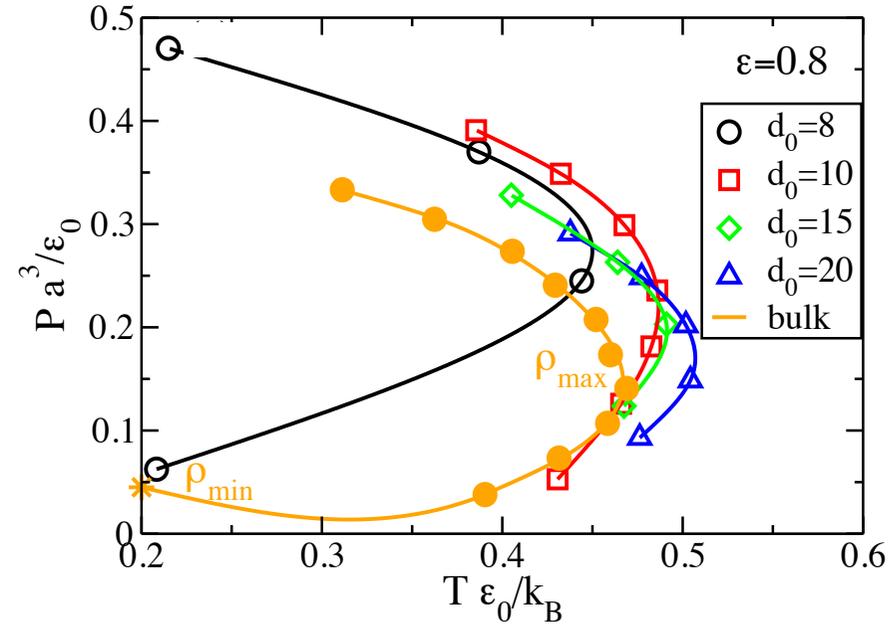
L. Xu, V. Molinero JPCB (2010); JPCB (2011)

# Effect of confinement on density anomaly

## Hydrophobic



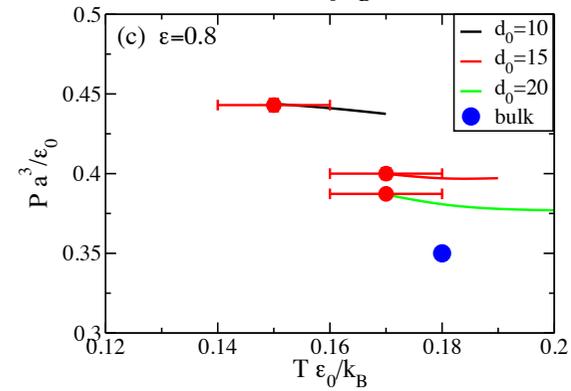
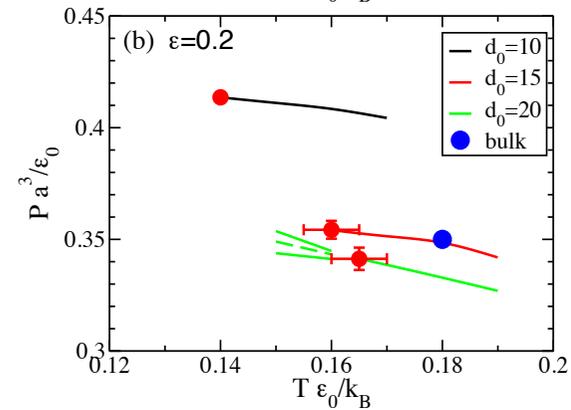
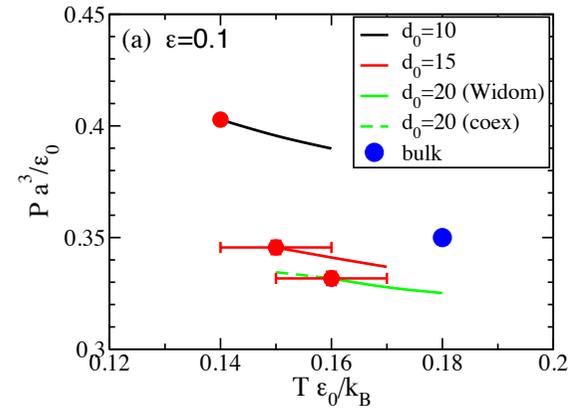
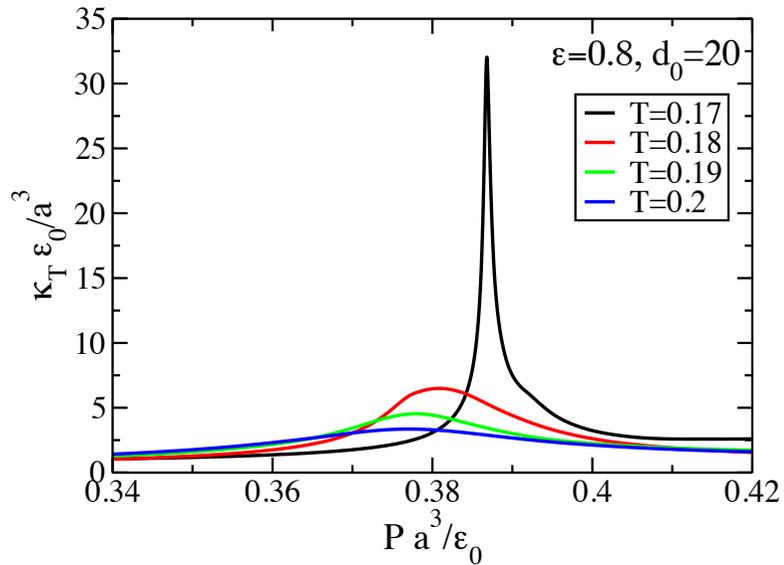
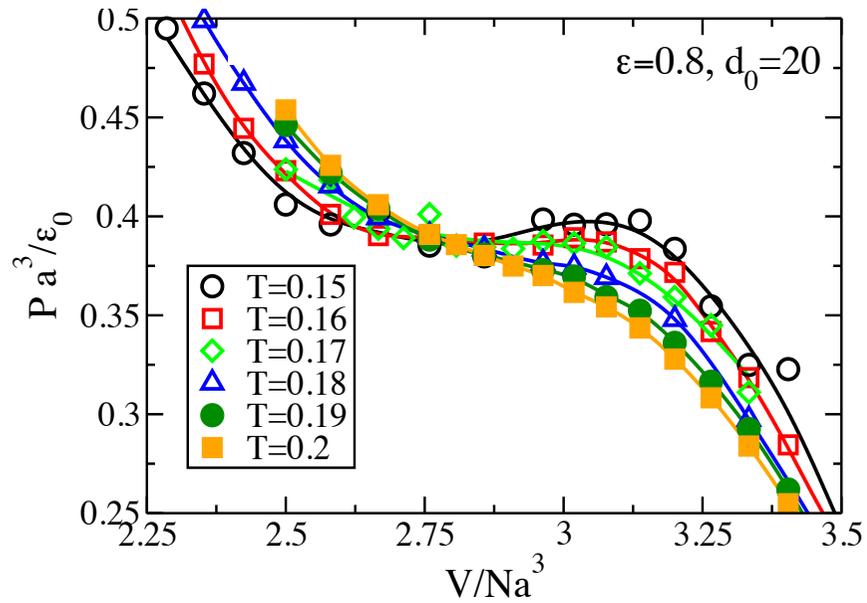
## Hydrophilic



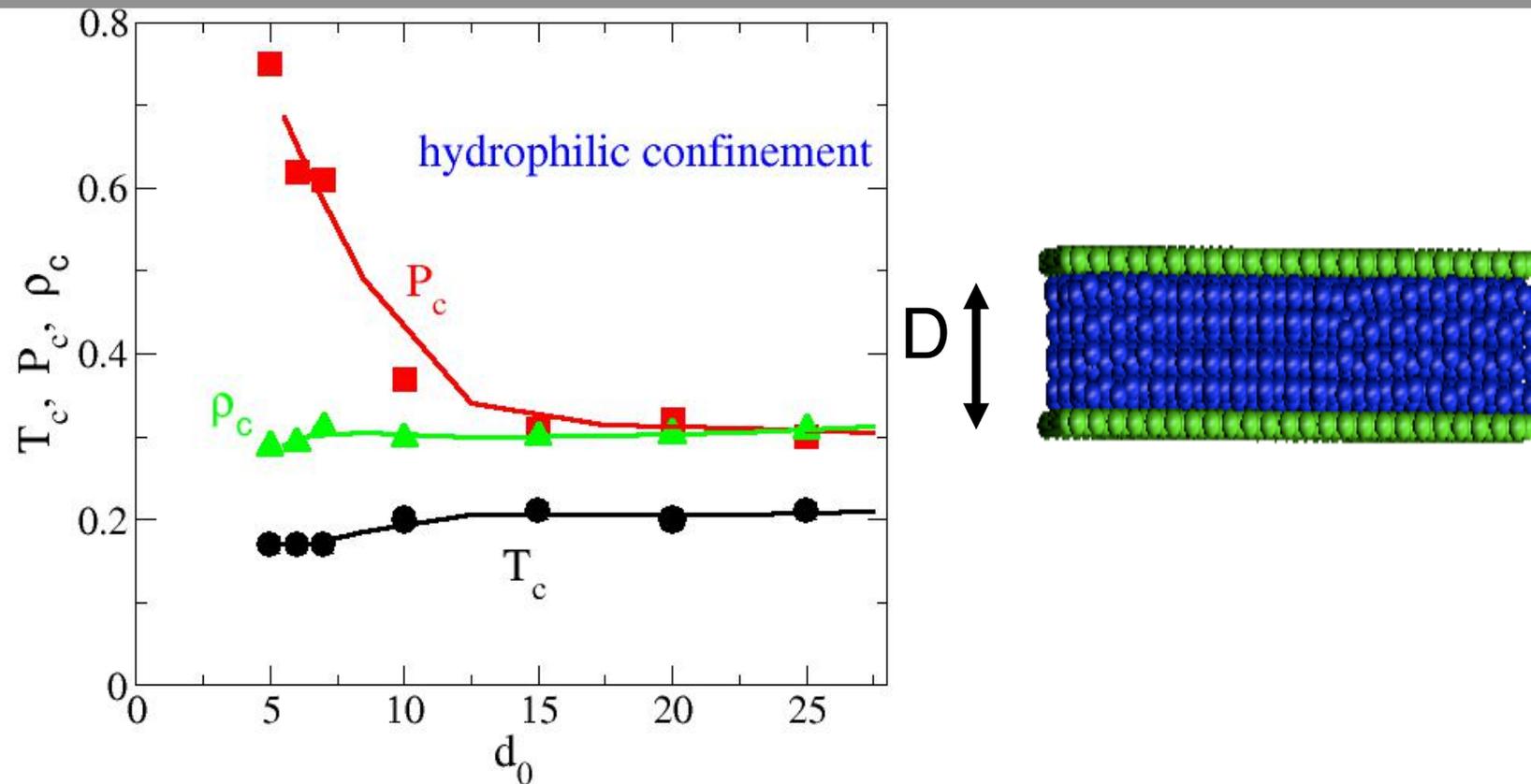
**Hydrophobic surface: phase diagram shifts to low-temperature**

**Hydrophilic surface: phase behavior is not significantly affected**

# Equation of state for hydrophilic confinement



# Confinement effect on liquid-liquid critical point

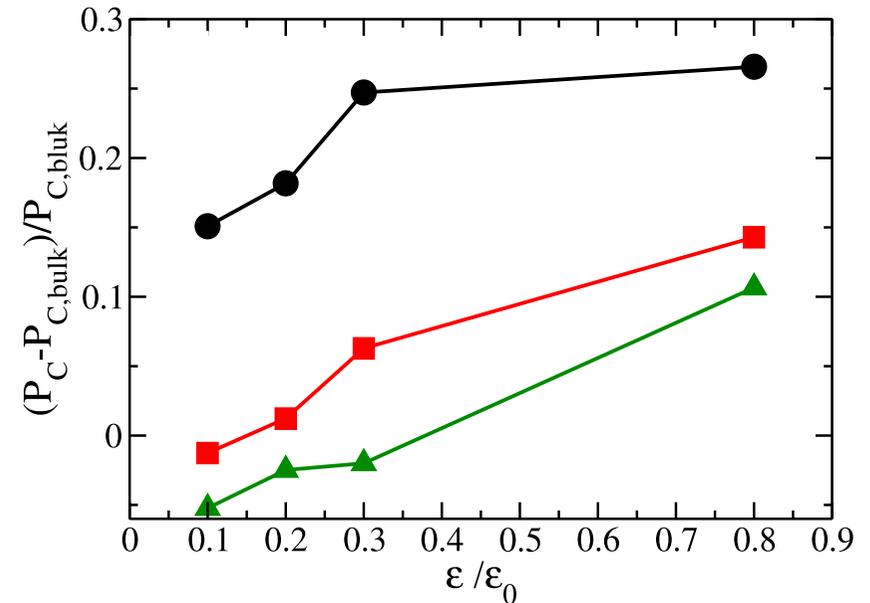
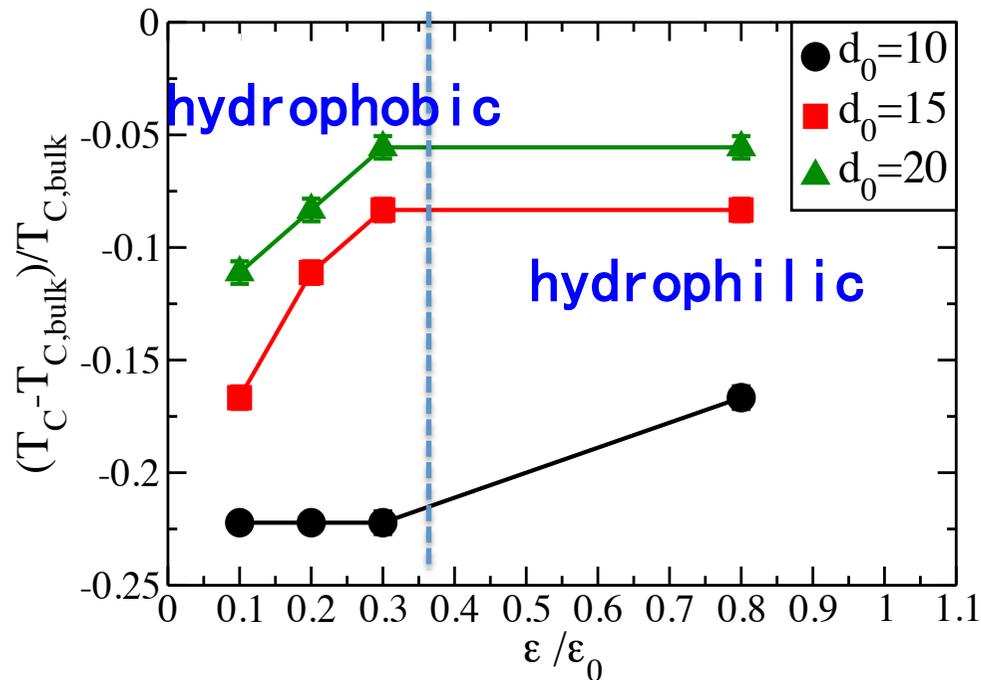


## Confinement effect:

- second critical point shifts to high pressure when strongly confined
- phase behavior approaches to that of bulk liquid at larger separation

G. Sun, N. Giovambattista, L. Xu, J. Chem. Phys. (2015)

# Surface chemistry effect on liquid-liquid critical point



**Hydrophobic:** Critical point move to lower temperatures

**Hydrophilic:**

- Severe confinement,  $T_C$  lower than that of bulk
- Under not severe confinement,  $T_C$  is not much affected

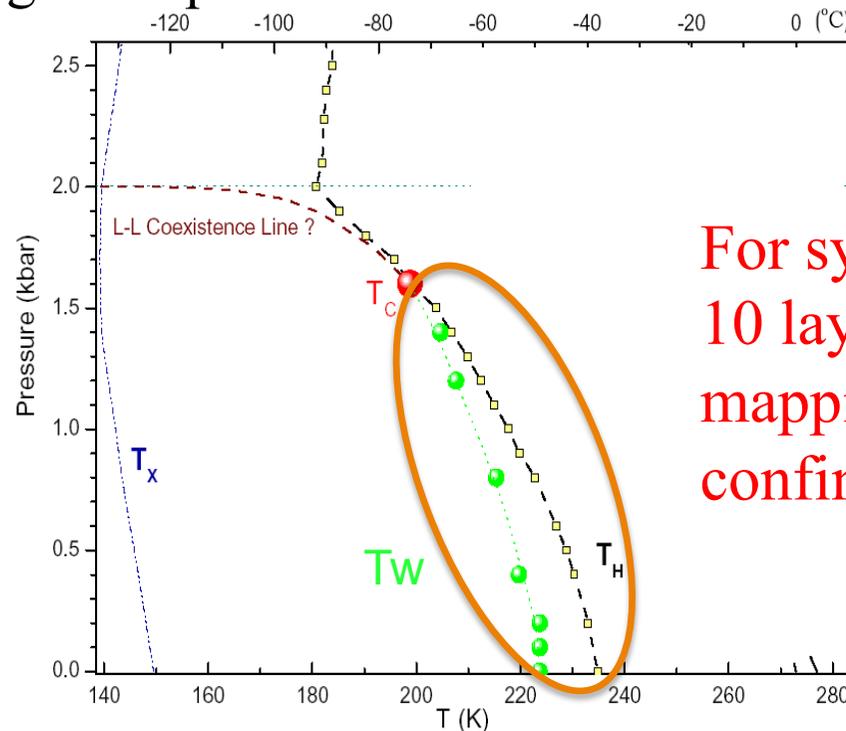
# Summary

✓ Phase behaviors depends on confinement and chemistry of surface

✓ Hydrophilic confinement effect:

◆ No significant effect on the phase diagram for systems with more than 10 layers of water molecules

◆ Drastically changes the phase behavior for severe confined systems



# Acknowledgement

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Francesco Mallamace	University of Messina
H Eugene Stanley	Boston University

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