

# Instrument Modelling Analytical Methods

6<sup>th</sup> April 2016

Ken Andersen

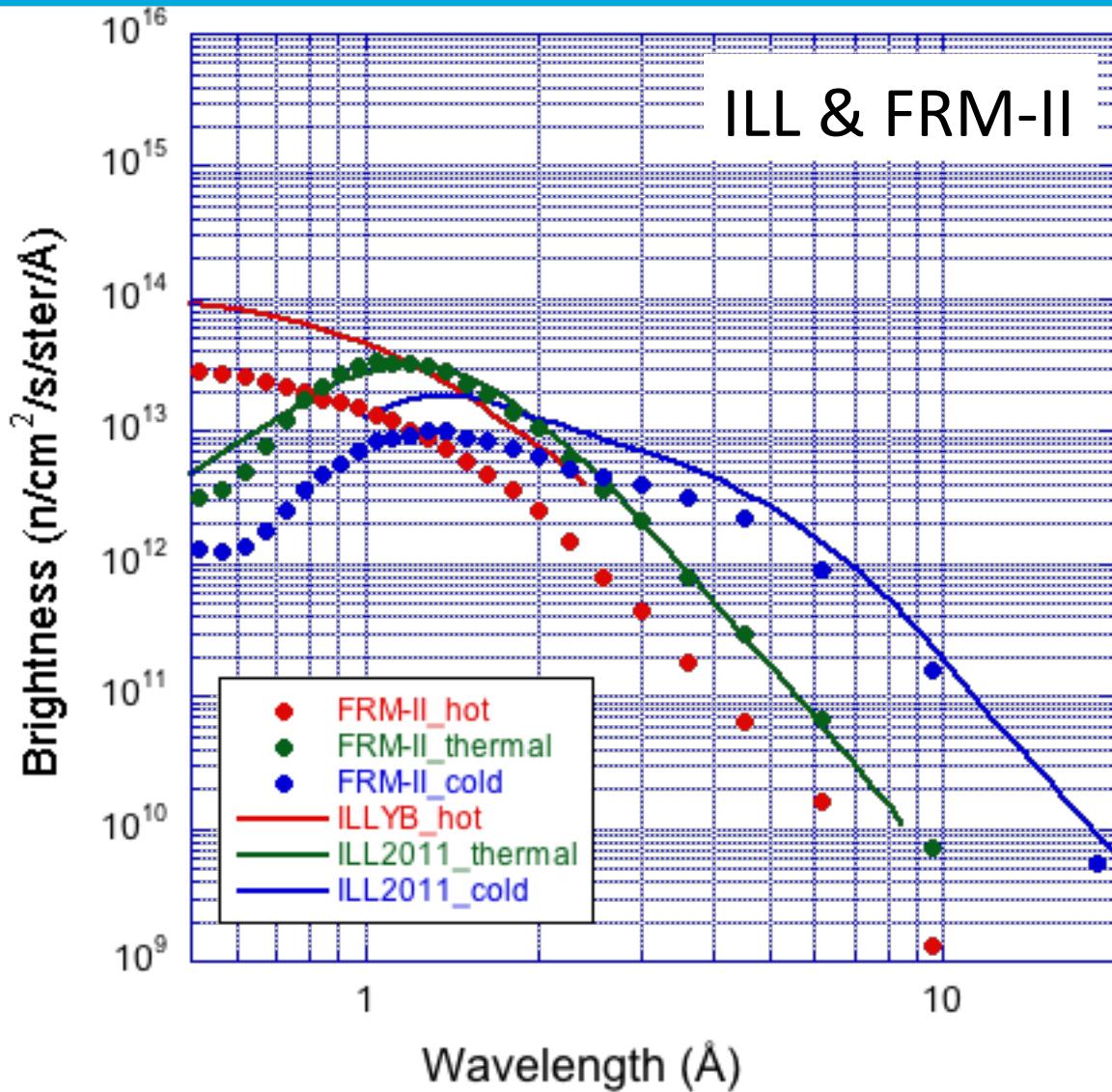
# Slow Neutrons vs Light

	light	neutrons
$\lambda$	$< \mu\text{m}$	$< \text{nm}$
E	$> \text{eV}$	$> \text{meV}$
n	$1 \rightarrow 4$	$0.9997 \rightarrow 1.0001$
$\theta_c$	$90^\circ$	$1^\circ$
B	$10^{18} \text{ p/cm}^2/\text{ster/s}$ (60W lightbulb)	$10^{14} \text{ n/cm}^2/\text{ster/s}$ (60MW reactor)
spin	1	$\frac{1}{2}$
interaction	electromagnetic	strong force, magnetic
charge	0	0

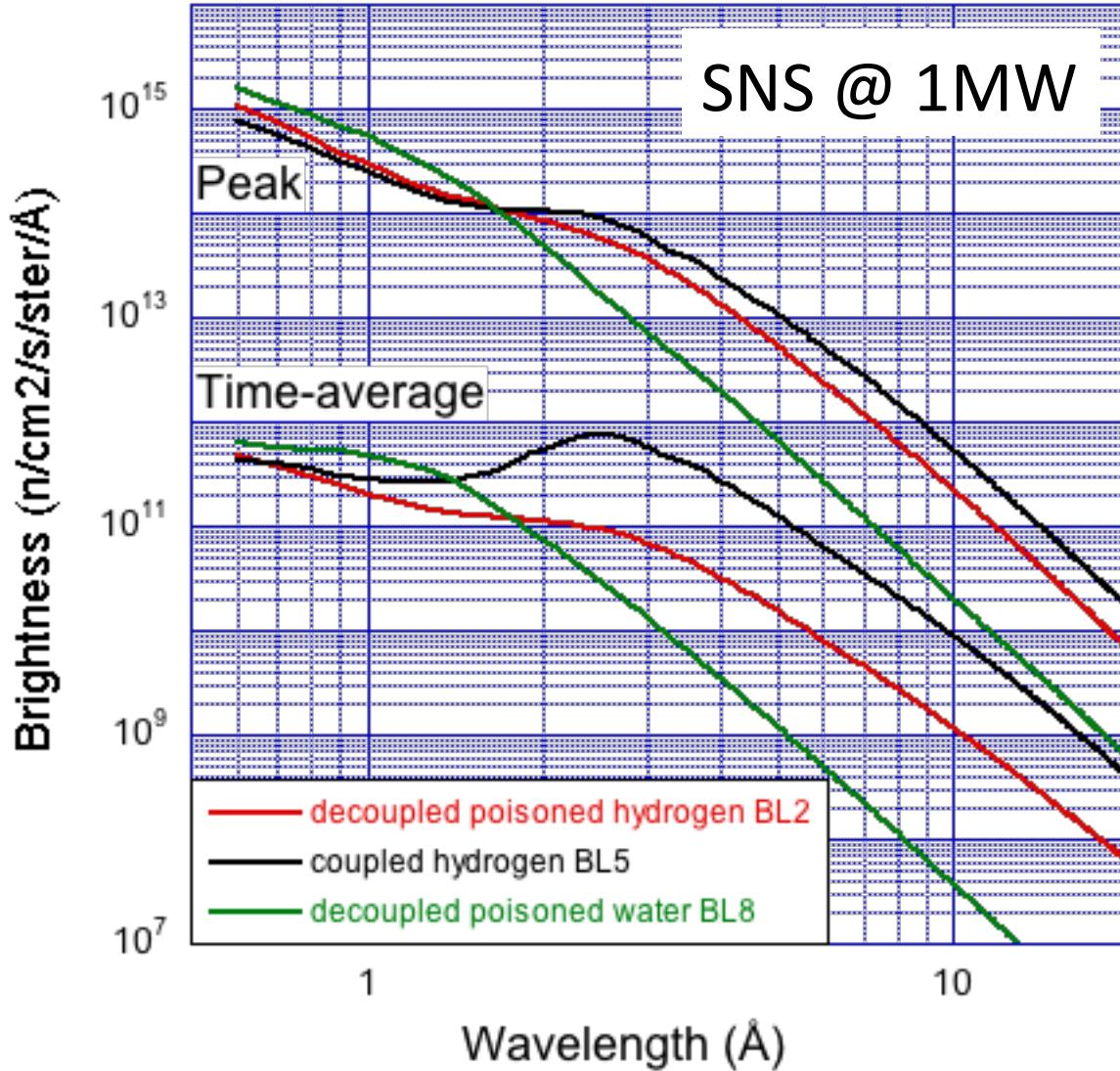
# Topics

- Flux calculations
  - source brightness
  - brilliance transfer
  - calculation of flux given instrument parameters
  - example: ESS powder diffractometer
  - example: SNS STS chopper spectrometer
- Resolution calculations
  - calculation of instrument parameters given resolution requirement
  - formulate the measurement requirement
  - calculate partial differentials
  - match resolution contributions
  - example: ESS powder diffractometer
  - example: SNS STS chopper spectrometer

# Source Brightness

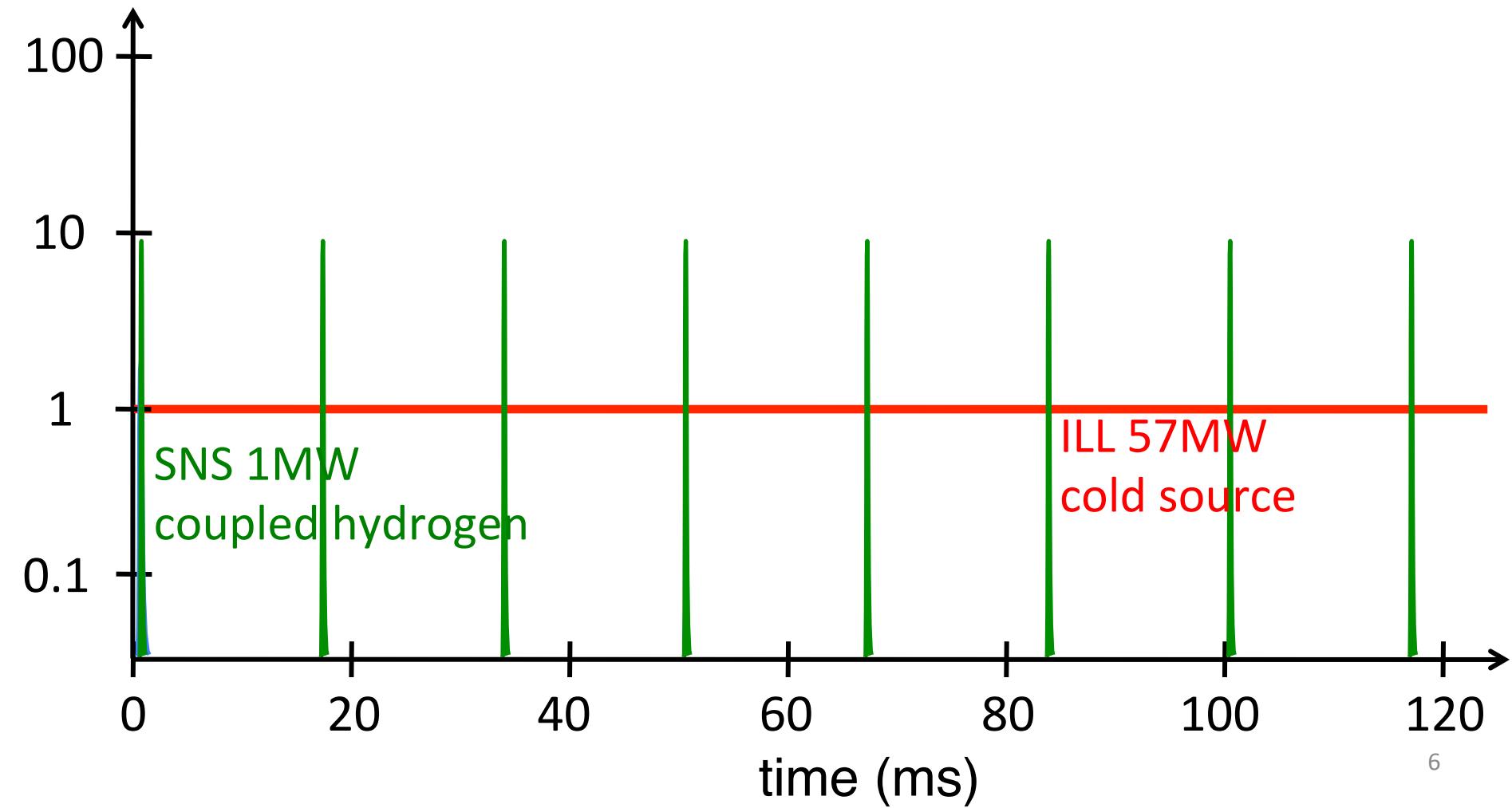


# Source Brightness

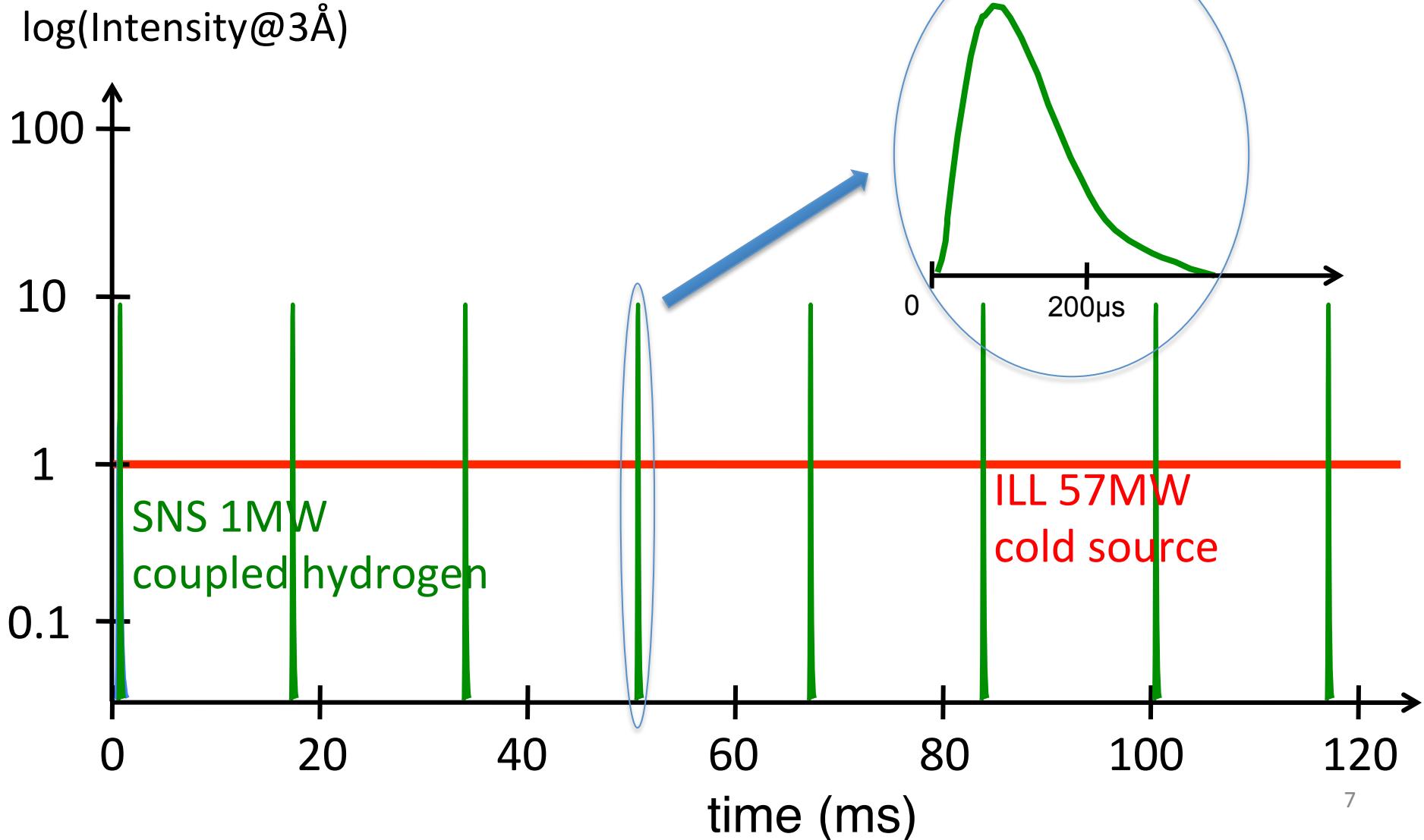


# Source Brightness

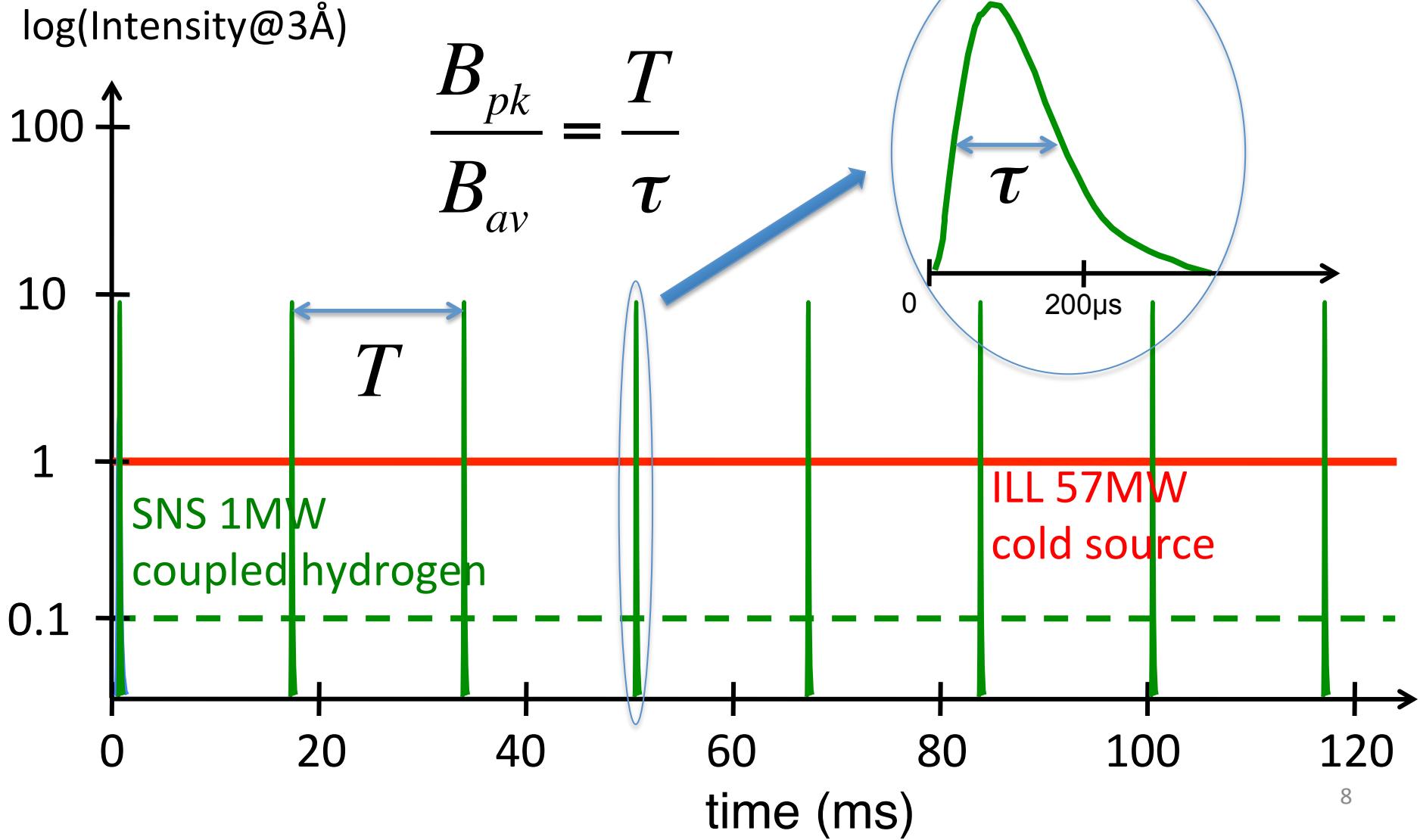
$\log(\text{Intensity}@3\text{\AA})$



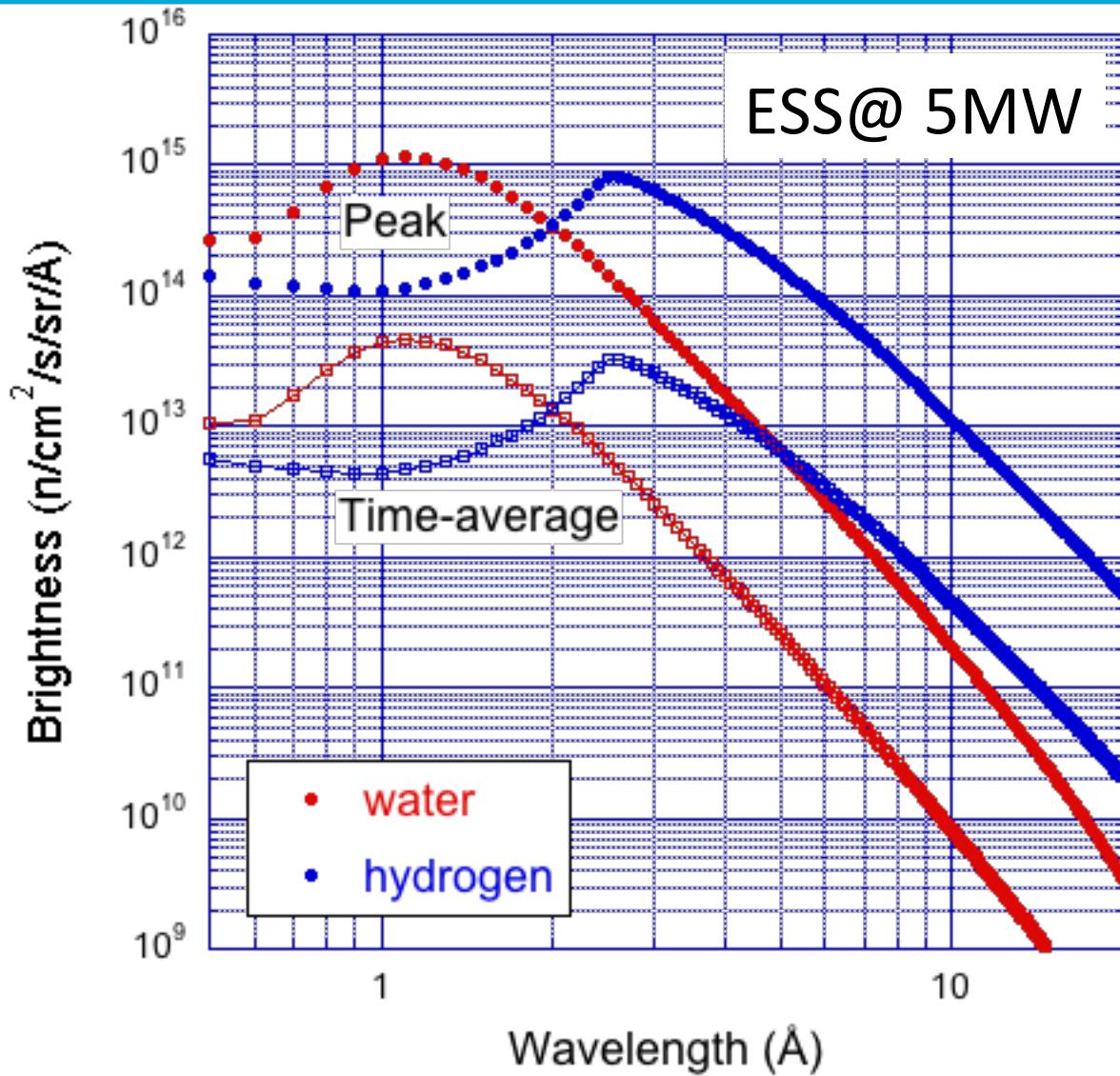
# Source Brightness



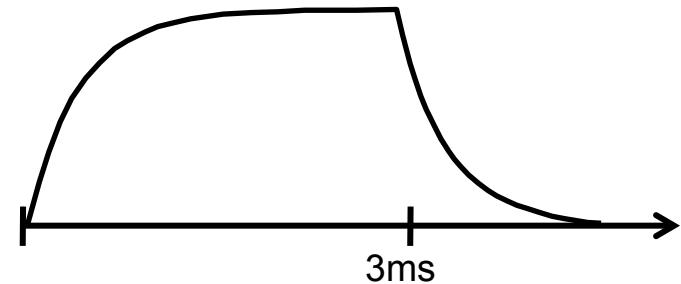
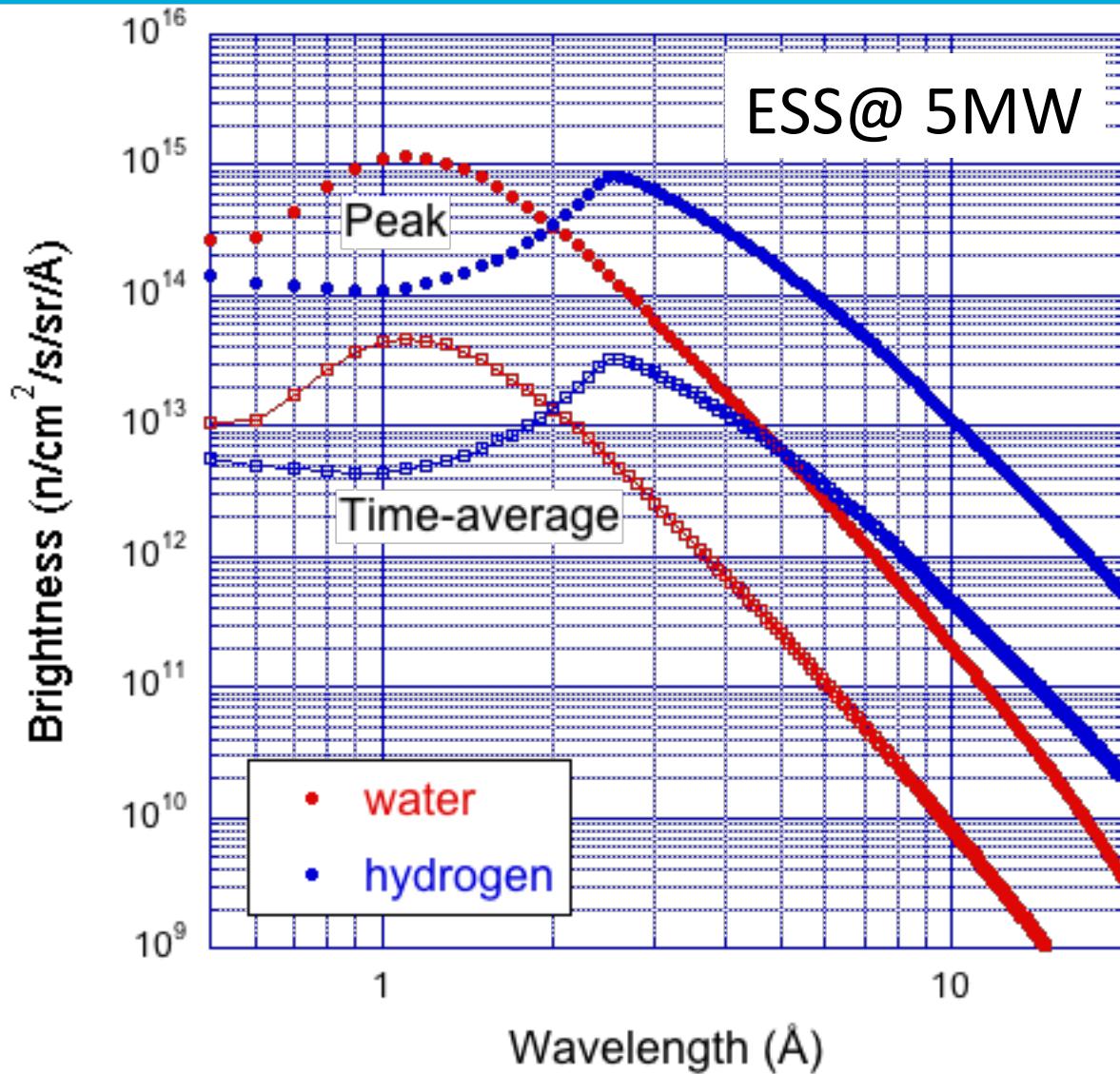
# Source Brightness



# Source Brightness



# Source Brightness



$$\frac{B_{pk}}{B_{av}} = \frac{T}{\tau} = 25$$

# Flux Calculations

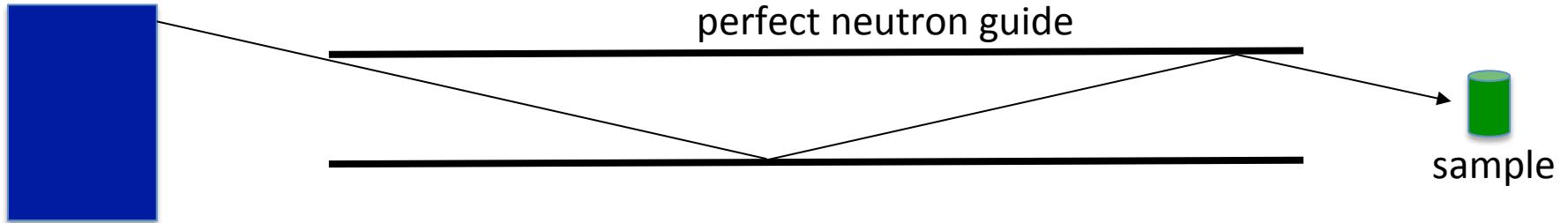
- Basic approach:

$$\Phi = \int_{\lambda_{\min}}^{\lambda_{\max}} B_{av} d\lambda \times \Delta\Omega$$

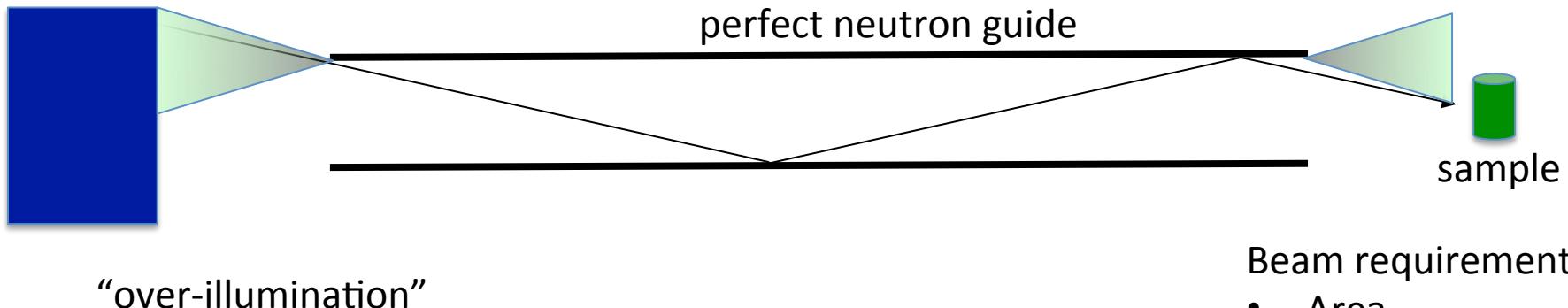
# Flux Calculations

- Basic approach: 
$$\Phi = \int_{\lambda_{\min}}^{\lambda_{\max}} B_{av} d\lambda \times \Delta\Omega$$
- Transported solid angle:  $\Delta\Omega = \Delta\theta_H \times \Delta\theta_V \times BT$
- $BT$  = Brilliance Transfer: between 0 and 1
  - how effective the guide system is
  - if you don't know, set it to 0.5

# Guide Illumination



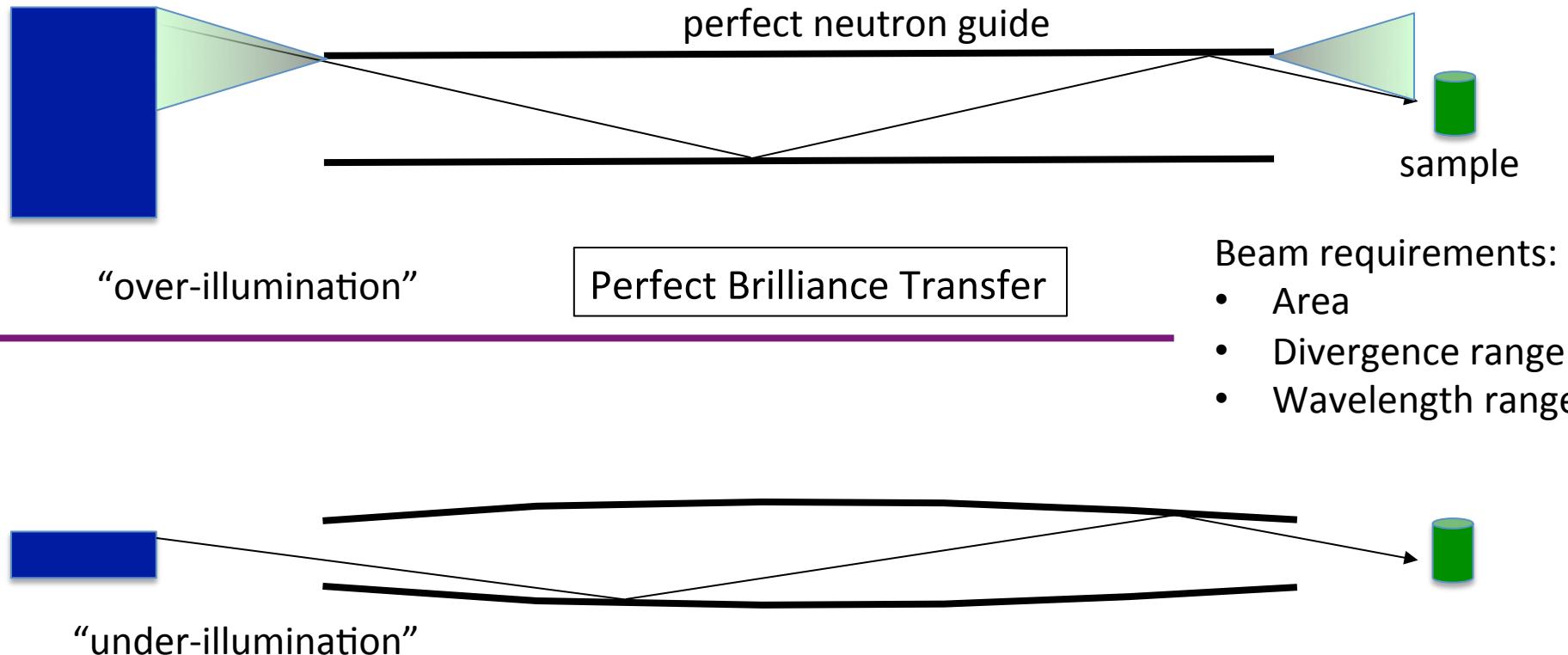
# Guide Illumination



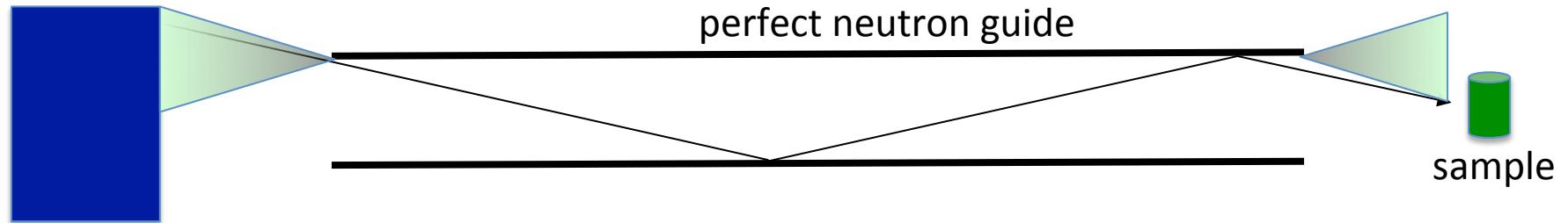
Beam requirements:

- Area
- Divergence range
- Wavelength range

# Guide Illumination



# Guide Illumination



"over-illumination"

Perfect Brilliance Transfer

sample

Beam requirements:

- Area
- Divergence range
- Wavelength range

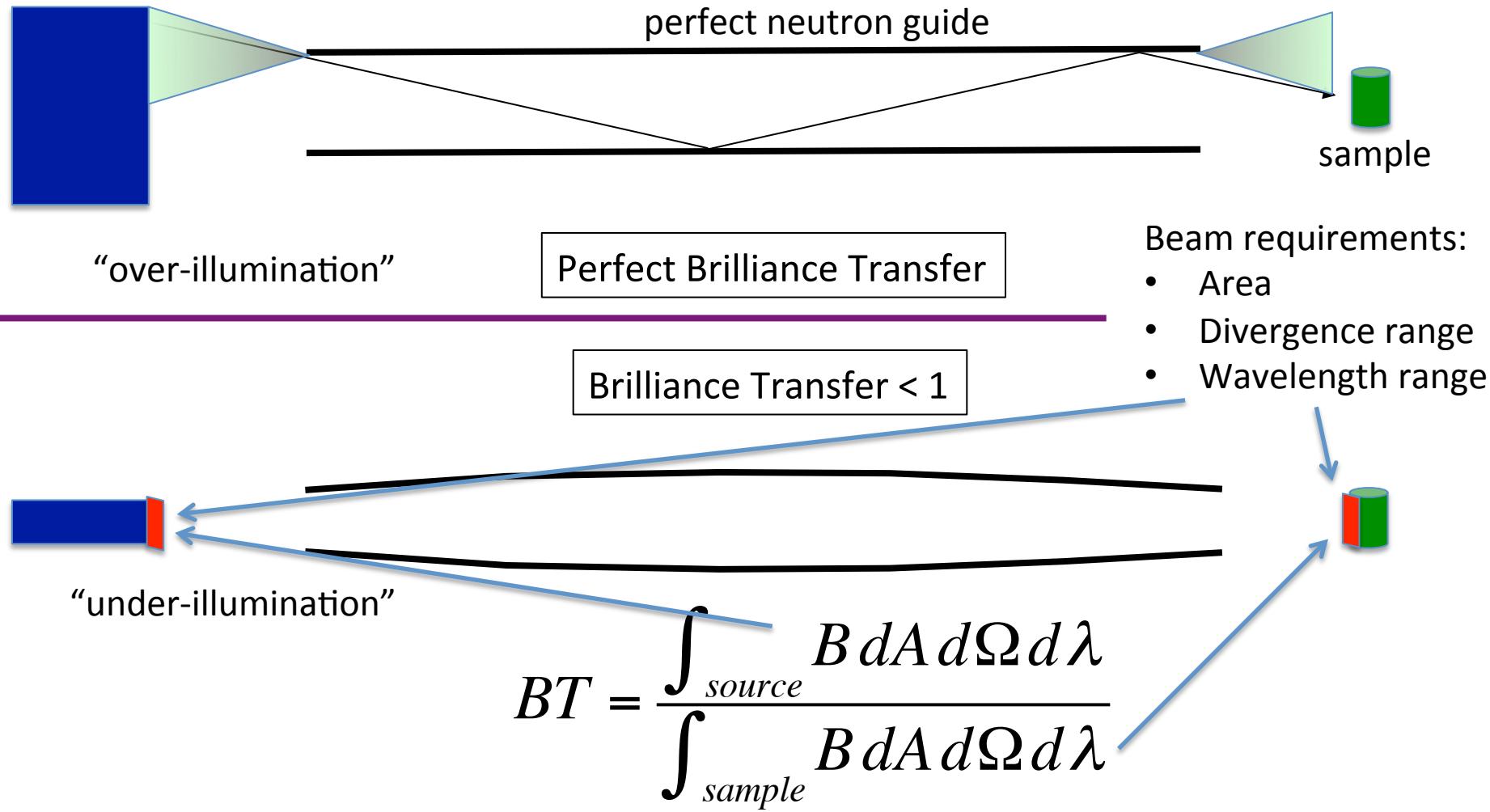
Brilliance Transfer < 1



"under-illumination"

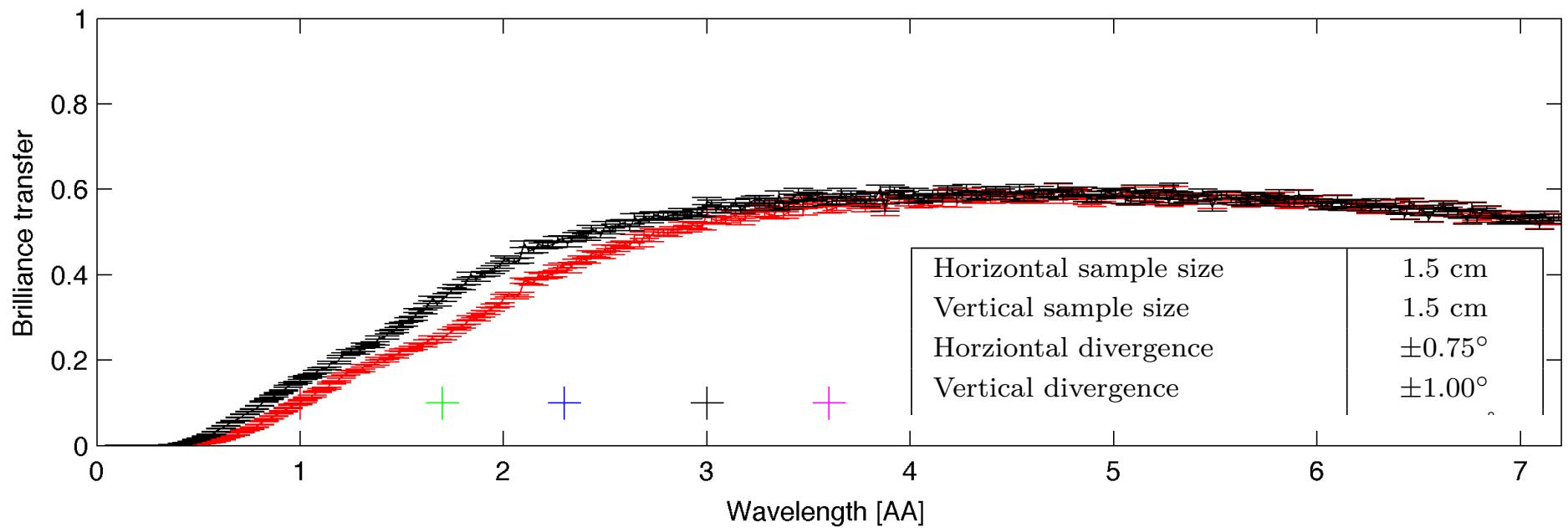
=> less efficient guides

# Guide Illumination



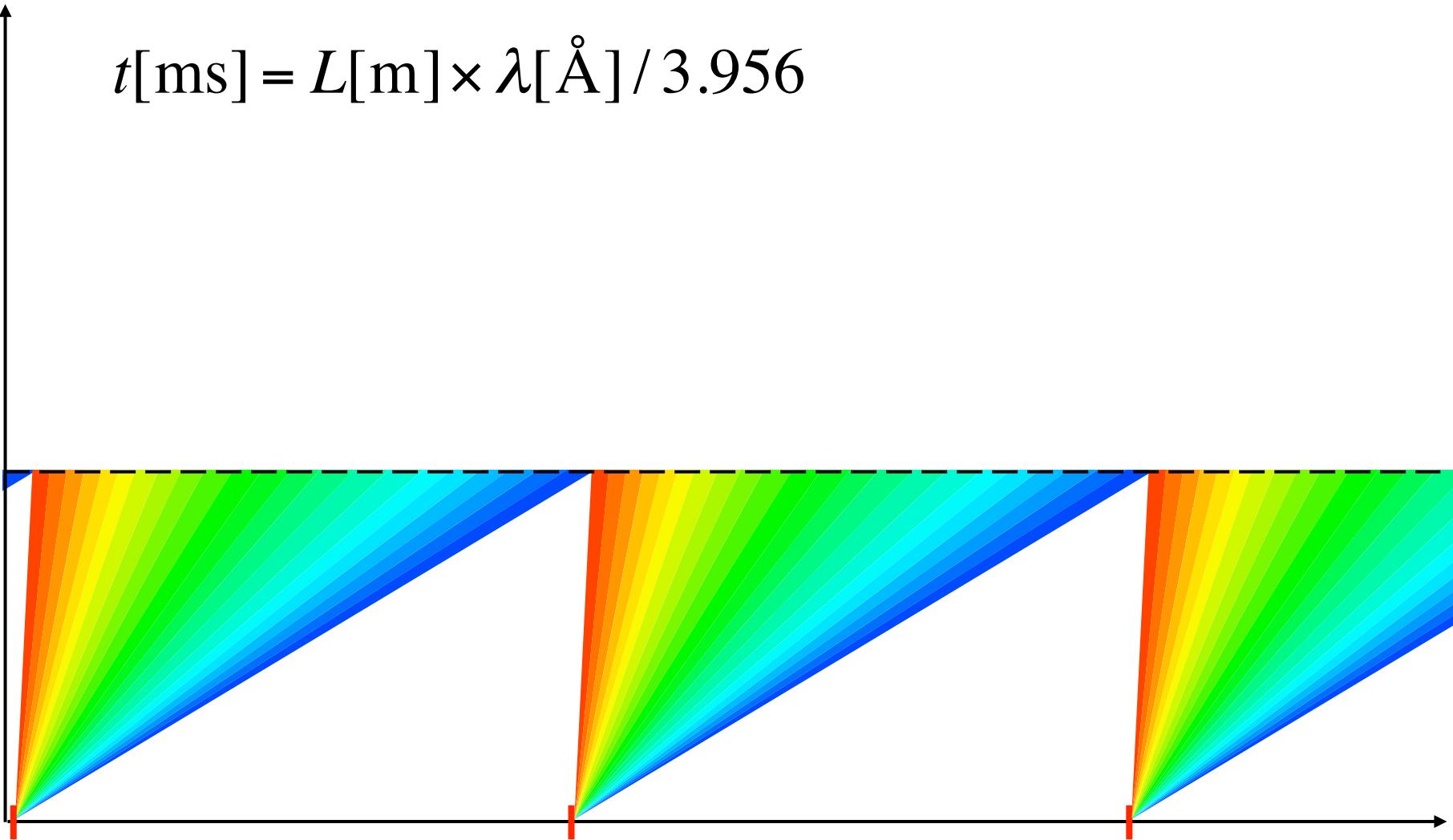
# Brilliance Transfer as a Guide Design Diagnostic

McStas simulations for BIFROST spectrometer at ESS viewing 3 cm tall cold moderator



# Example: ESS SANS

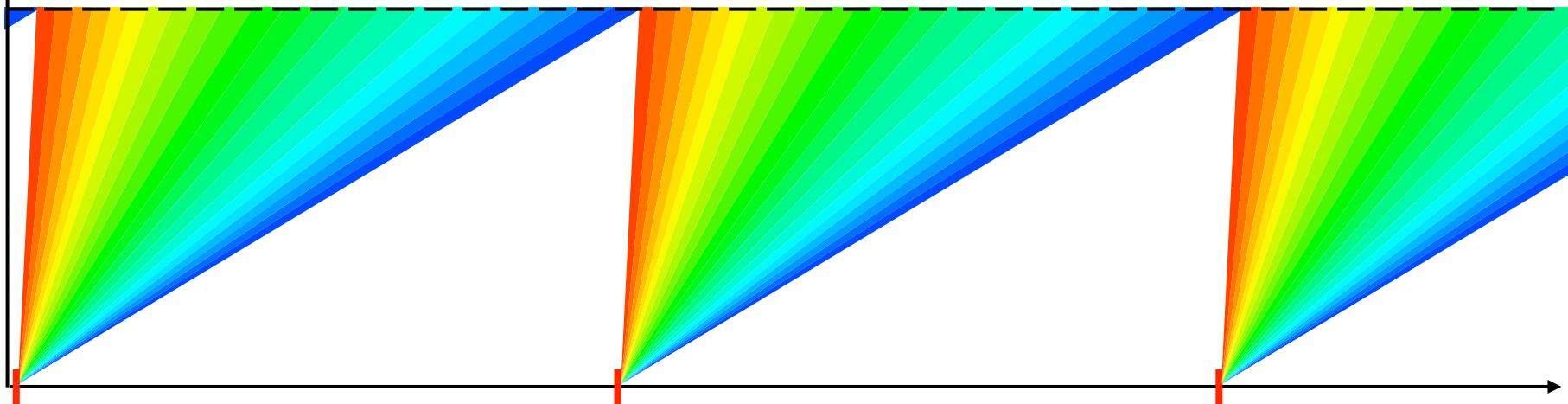
$$t[\text{ms}] = L[\text{m}] \times \lambda[\text{\AA}] / 3.956$$



# Example: ESS SANS

$$t[\text{ms}] = L[\text{m}] \times \lambda[\text{\AA}] / 3.956$$

$$L[\text{m}] = T[\text{ms}] \times 3.956 / \Delta\lambda[\text{\AA}]$$



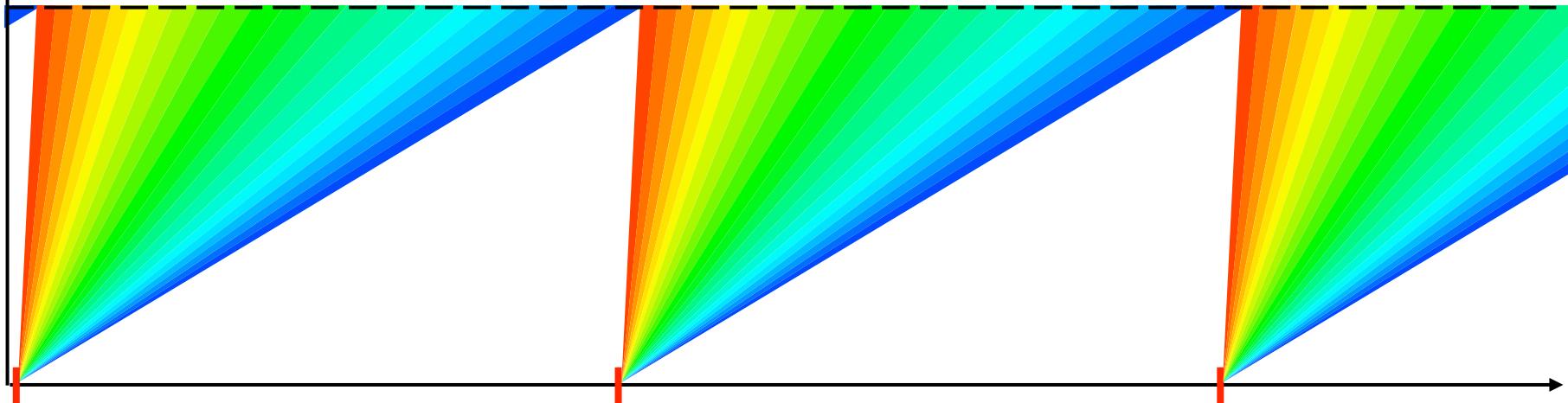
# Example: ESS SANS

$$t[\text{ms}] = L[\text{m}] \times \lambda[\text{\AA}] / 3.956$$

$$L[\text{m}] = T[\text{ms}] \times 3.956 / \Delta\lambda[\text{\AA}]$$

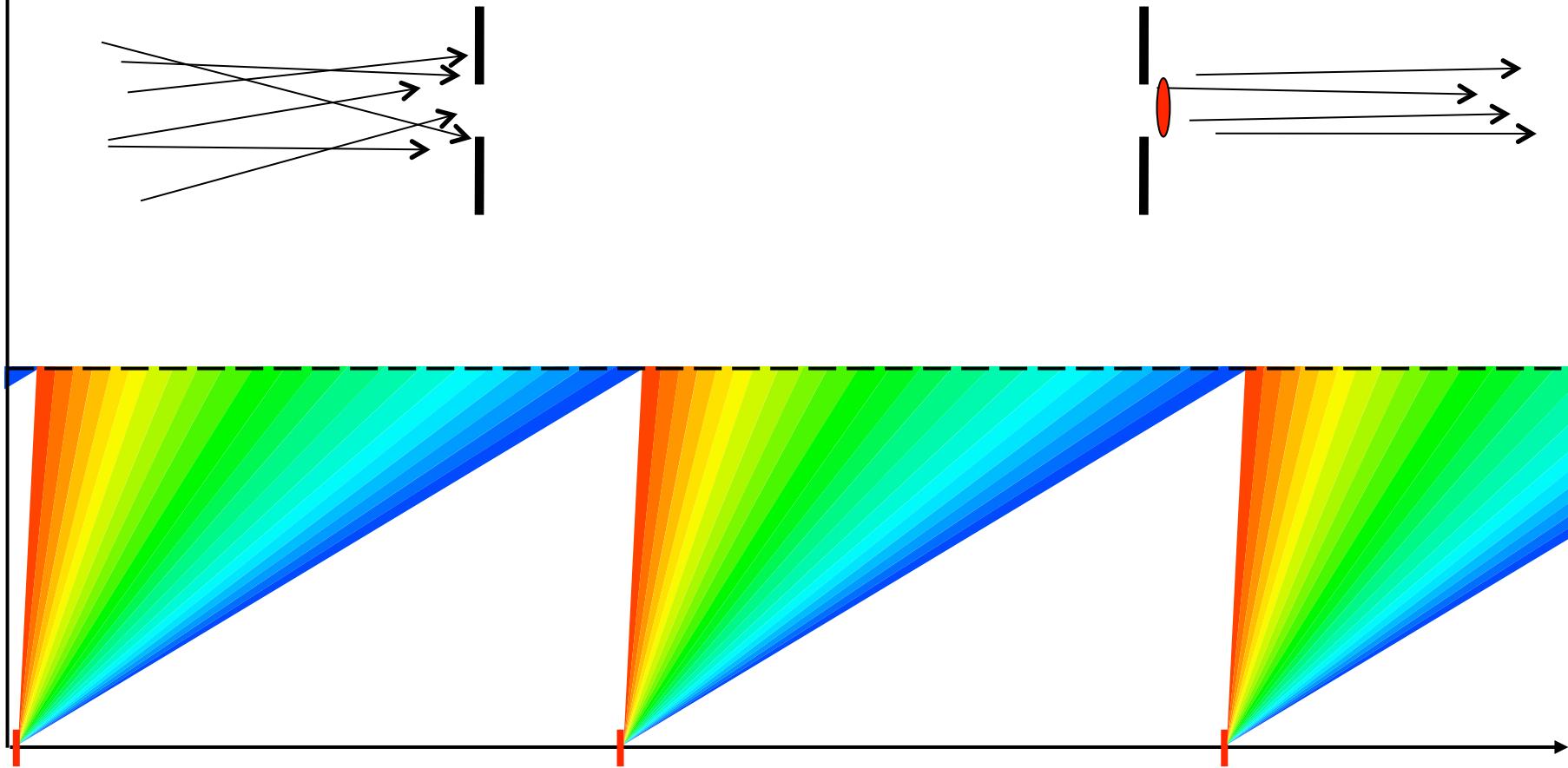
where  $\Delta\lambda$  is determined by the performance requirements:

e.g.  $\lambda_{\min} = 4\text{\AA}$   $\lambda_{\max} = 12\text{\AA} \Rightarrow L = 35\text{m}$



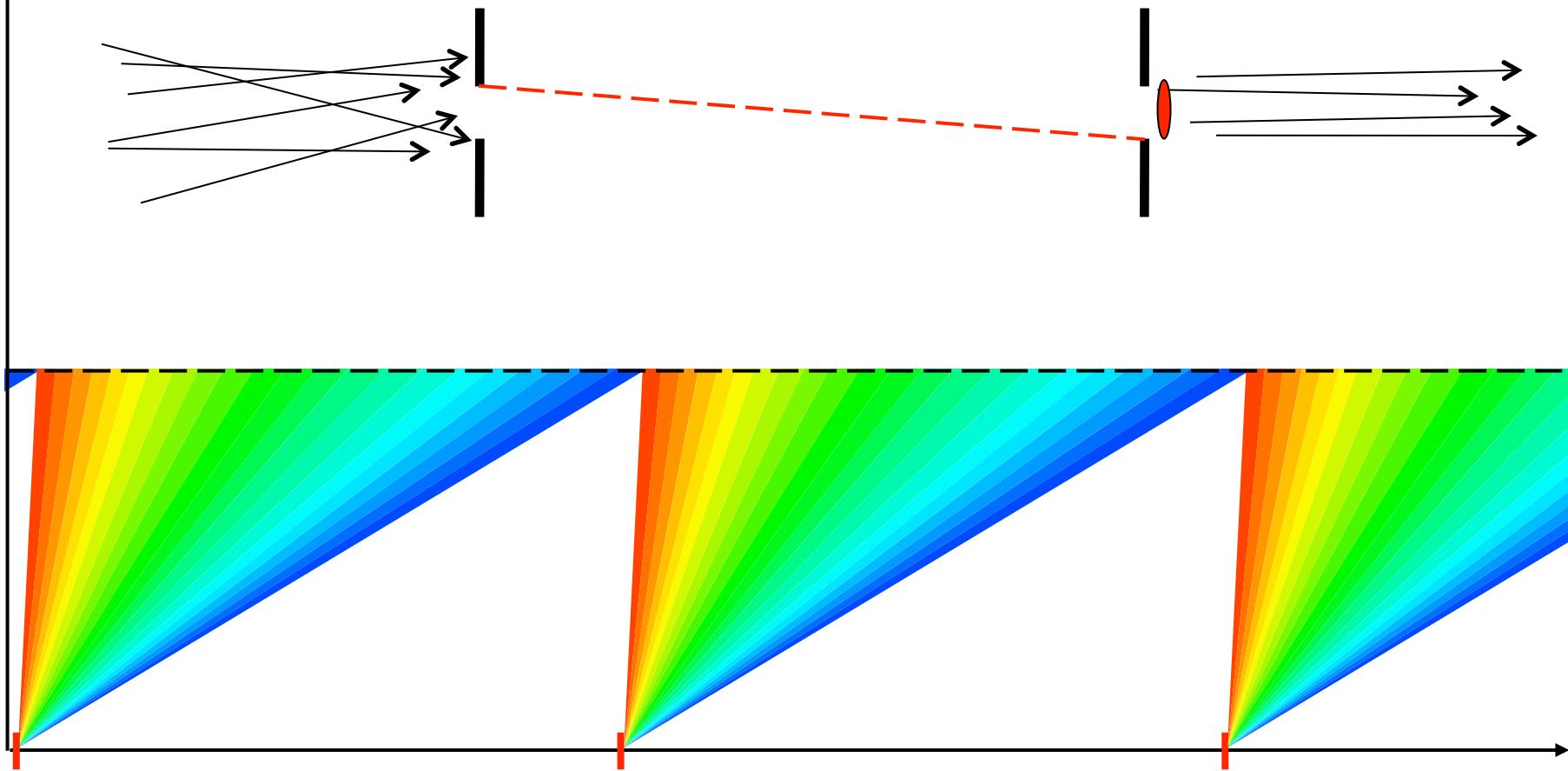
# Example: ESS SANS

Evaluate flux for a 3 cm sample and a 10 m collimation distance:



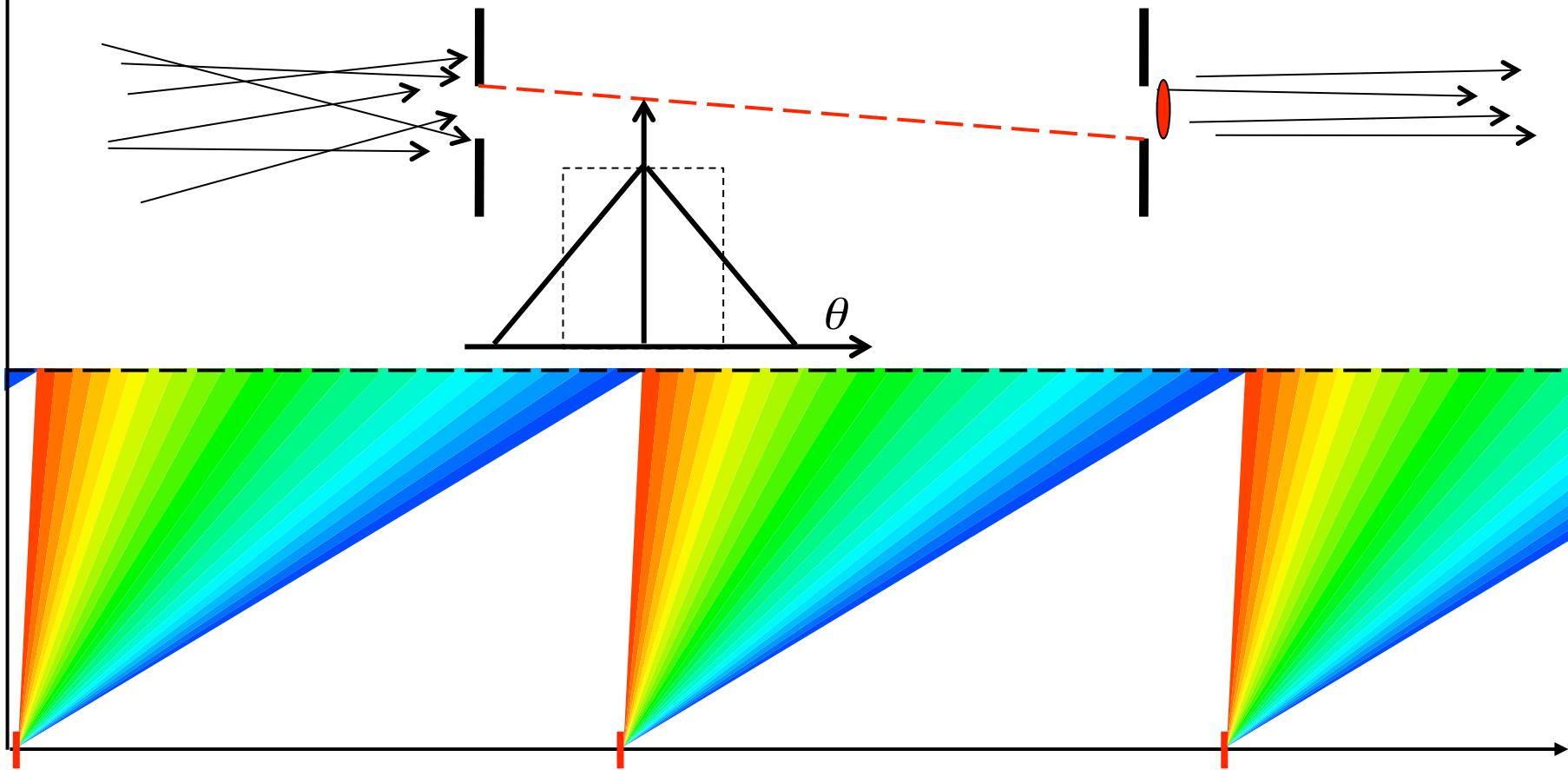
# Example: ESS SANS

Evaluate flux for a 3 cm sample and a 10 m collimation distance:



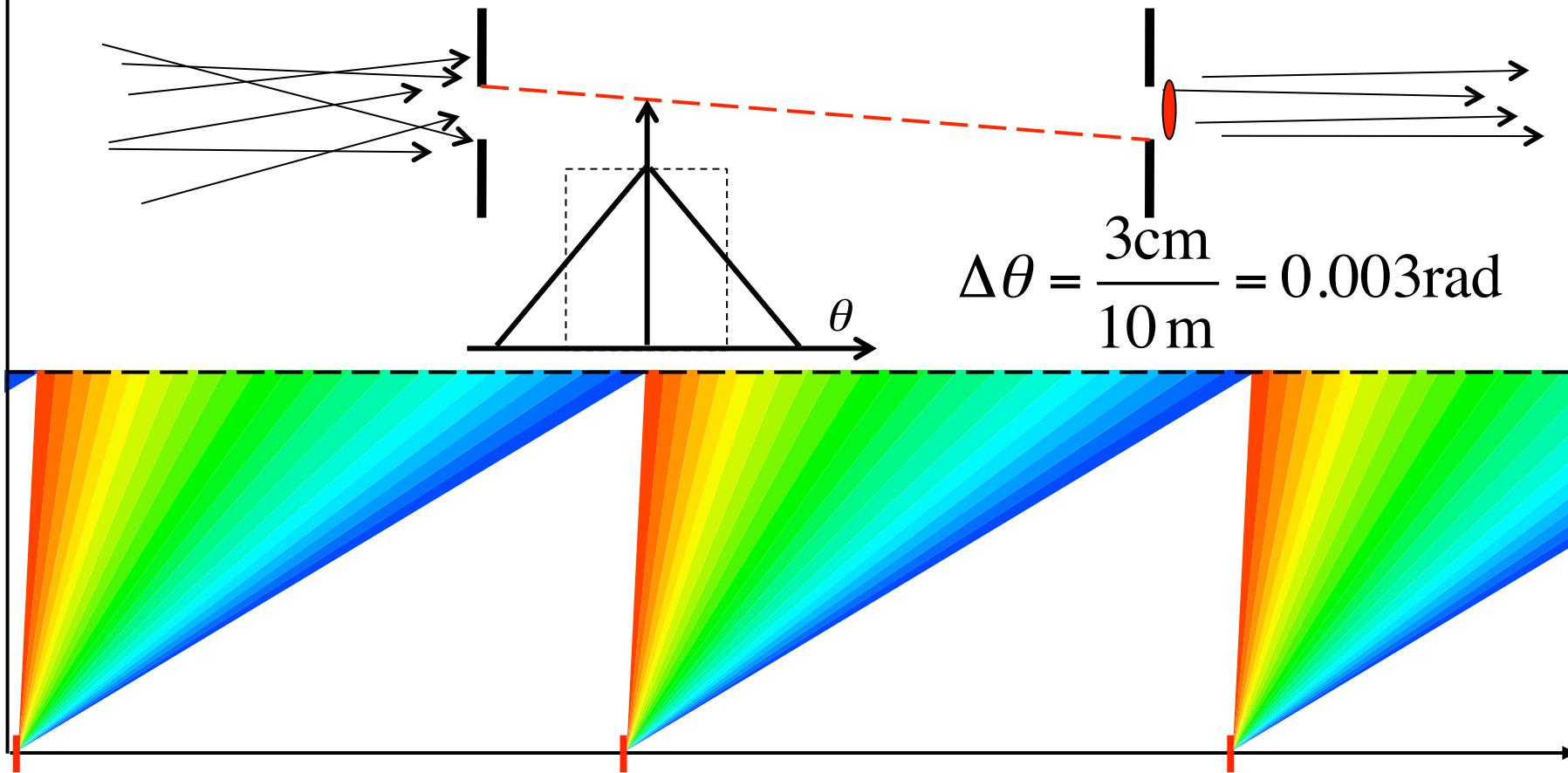
# Example: ESS SANS

Evaluate flux for a 3 cm sample and a 10 m collimation distance:

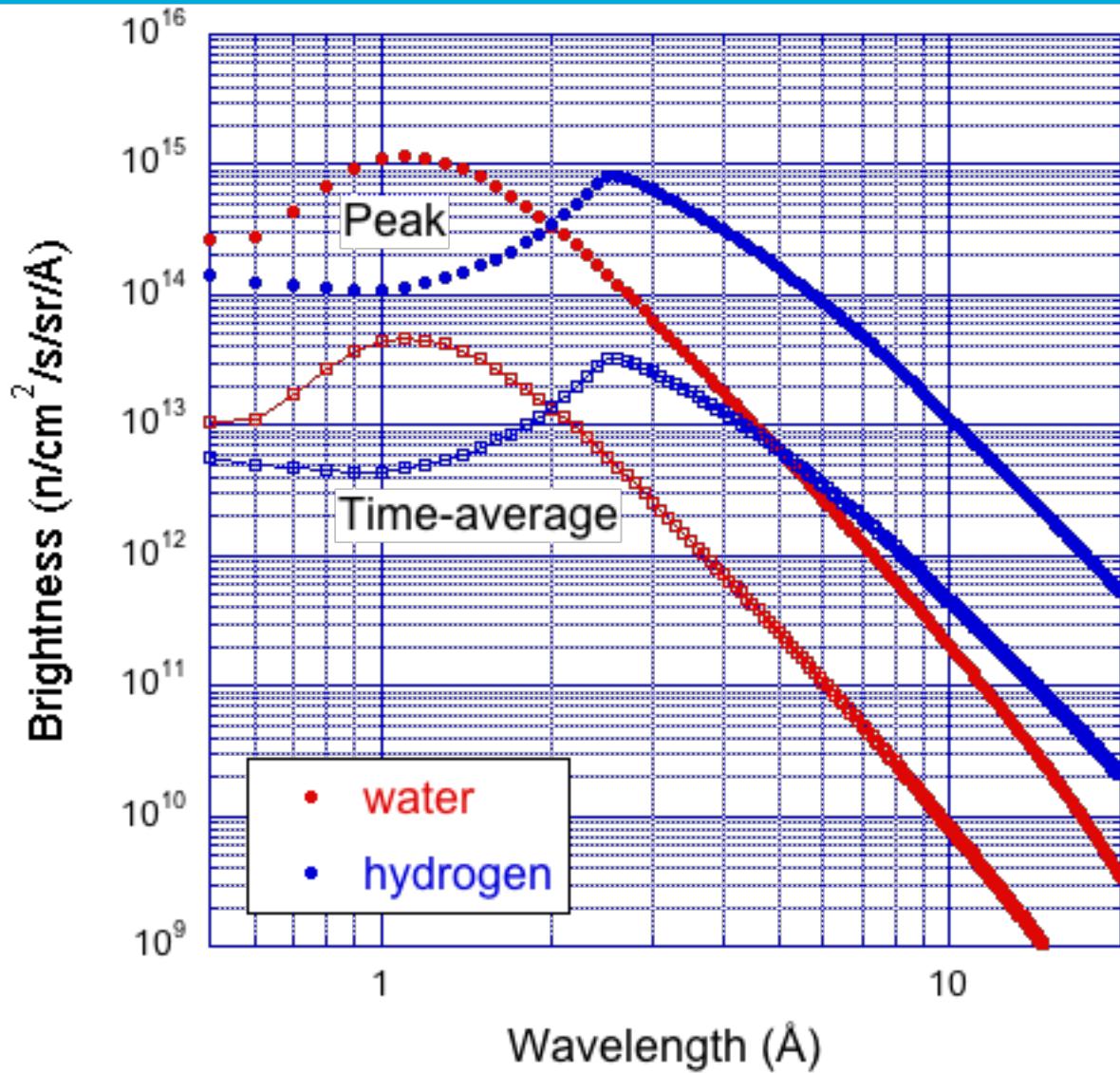


# Example: ESS SANS

Evaluate flux for a 3 cm sample and a 10 m collimation distance:

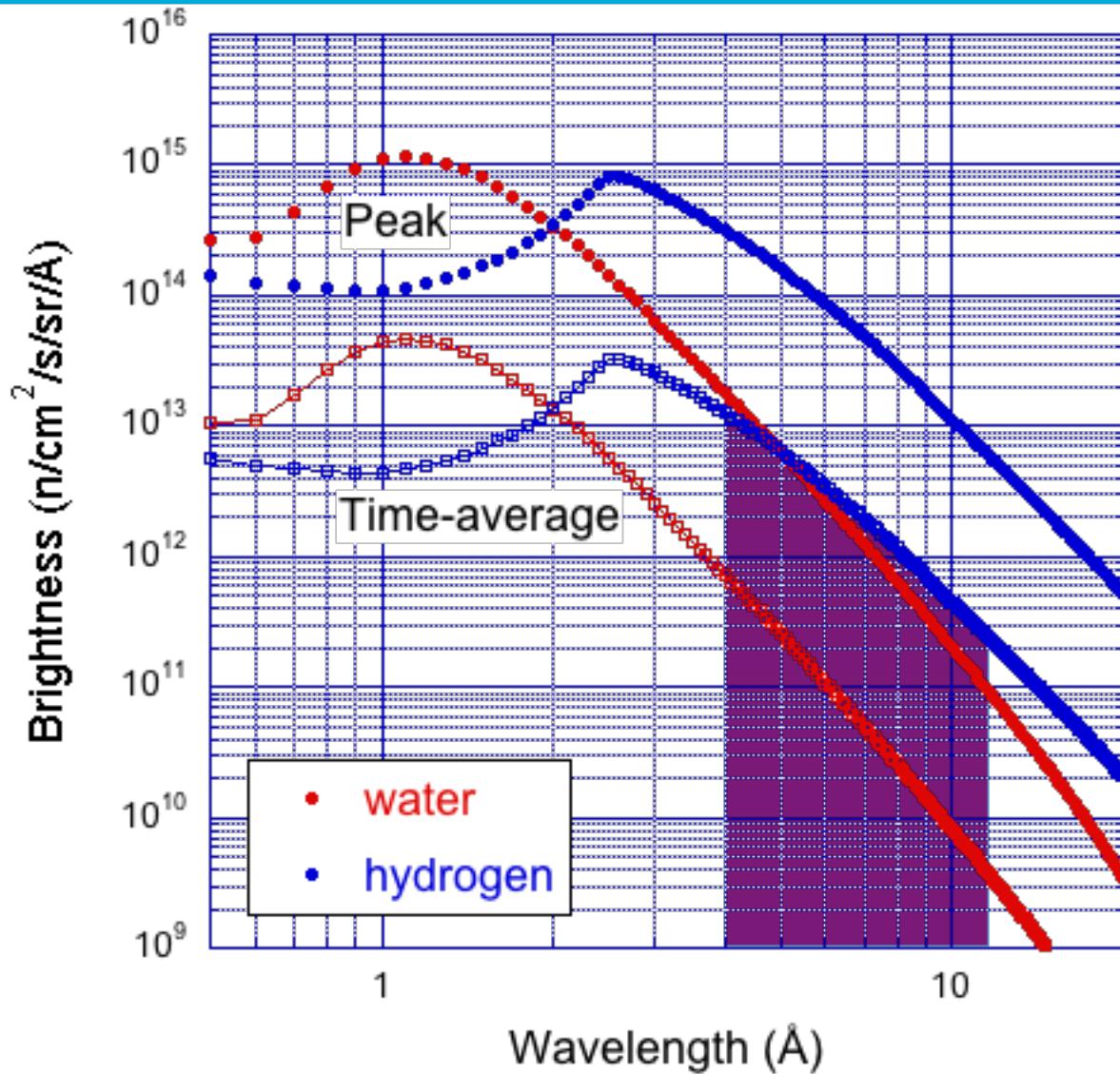


# Example: ESS SANS



$$\Phi = \int_{\lambda_{\min}}^{\lambda_{\max}} B_{av} d\lambda \times \Delta\Omega$$

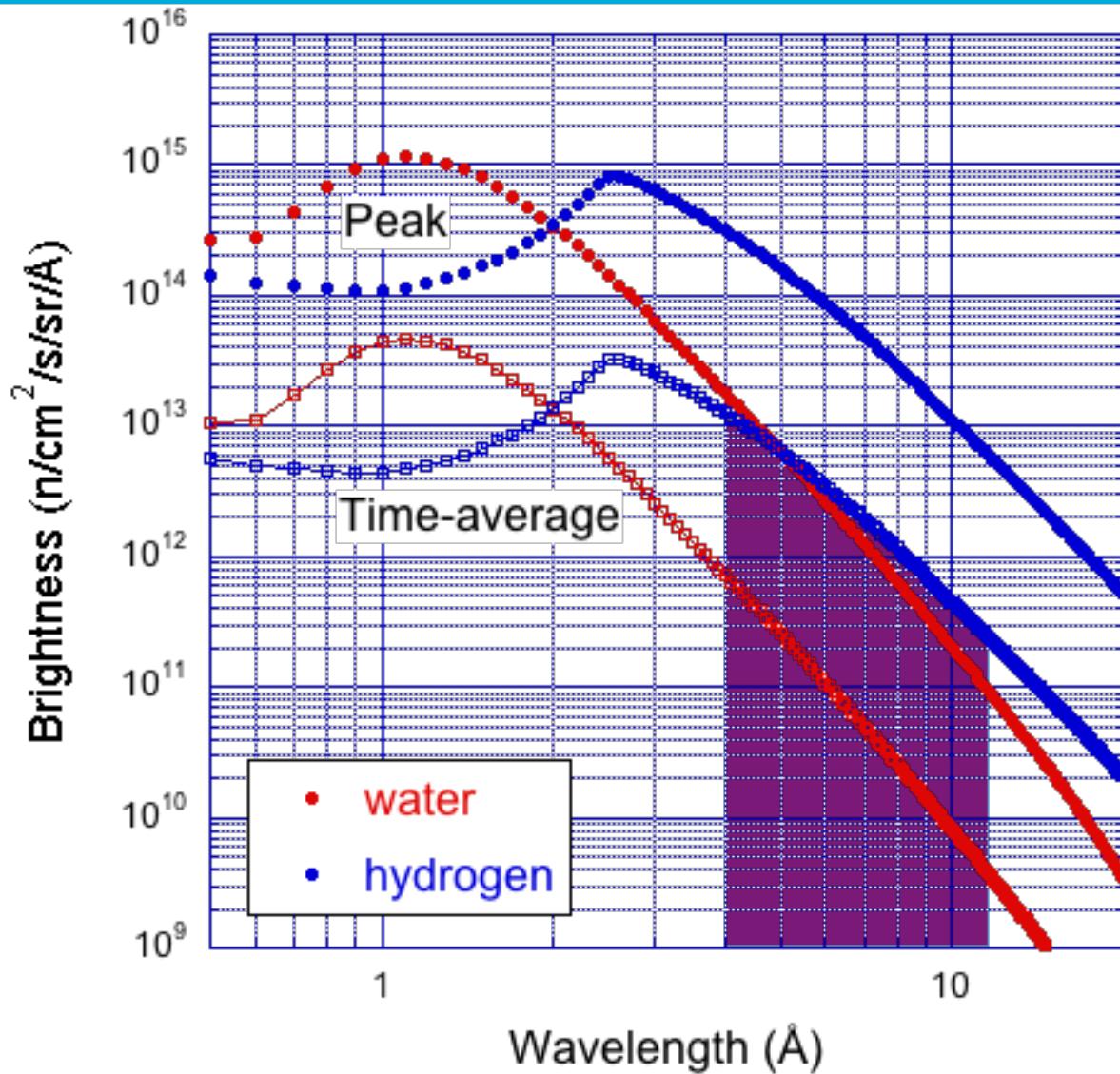
# Example: ESS SANS



$$\Phi = \int_{\lambda_{\min}}^{\lambda_{\max}} B_{av} d\lambda \times \Delta\Omega$$

$$4 \text{\AA} < \lambda < 12 \text{\AA}$$

# Example: ESS SANS



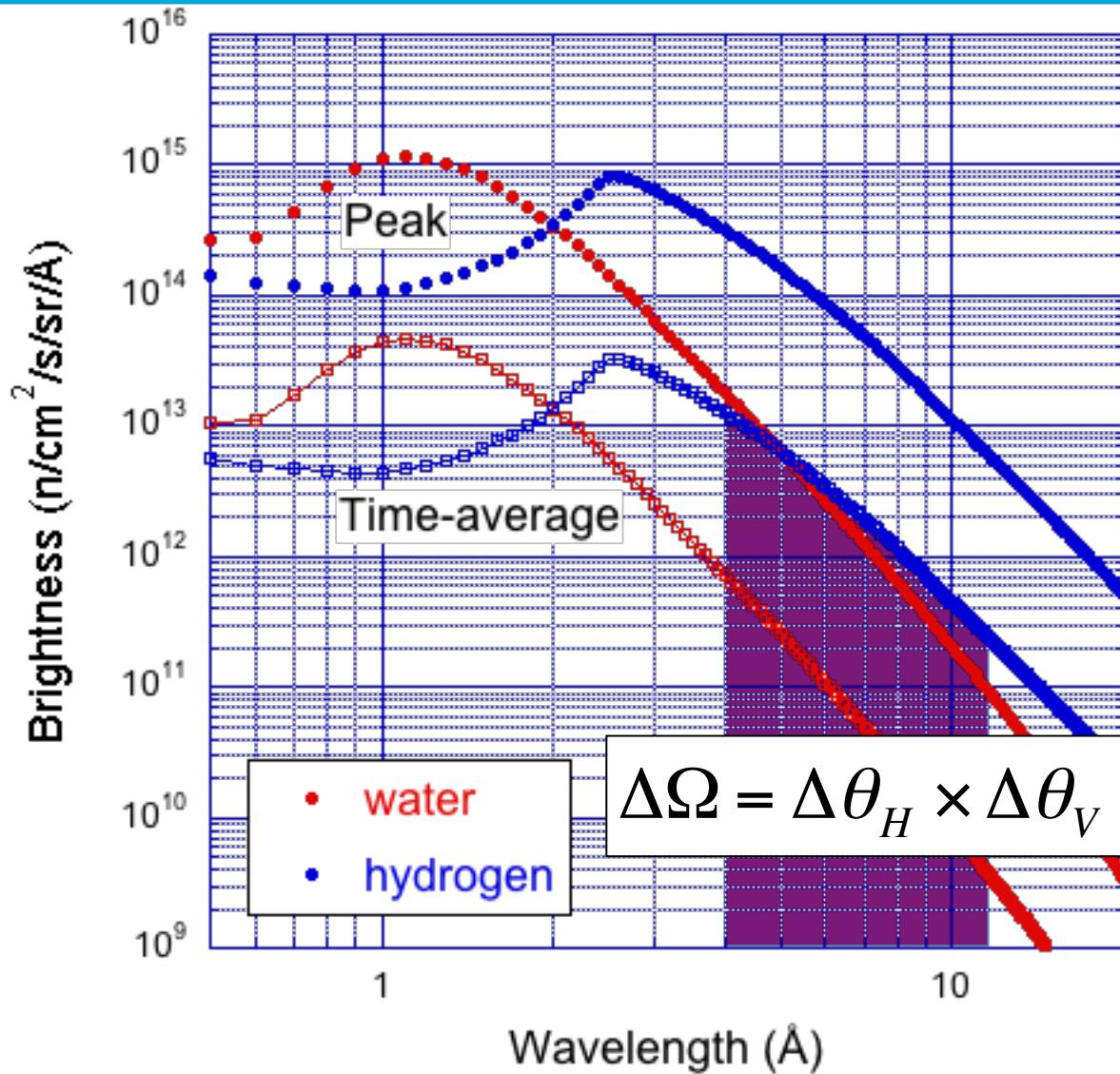
$$\Phi = \int_{\lambda_{\min}}^{\lambda_{\max}} B_{av} d\lambda \times \Delta\Omega$$

$$4\text{\AA} < \lambda < 12\text{\AA}$$

$$\int_{\lambda_{\min}}^{\lambda_{\max}} B_{av} d\lambda$$

$$\approx 2\text{E}13 \text{ n/cm}^2/\text{s/sr}$$

# Example: ESS SANS



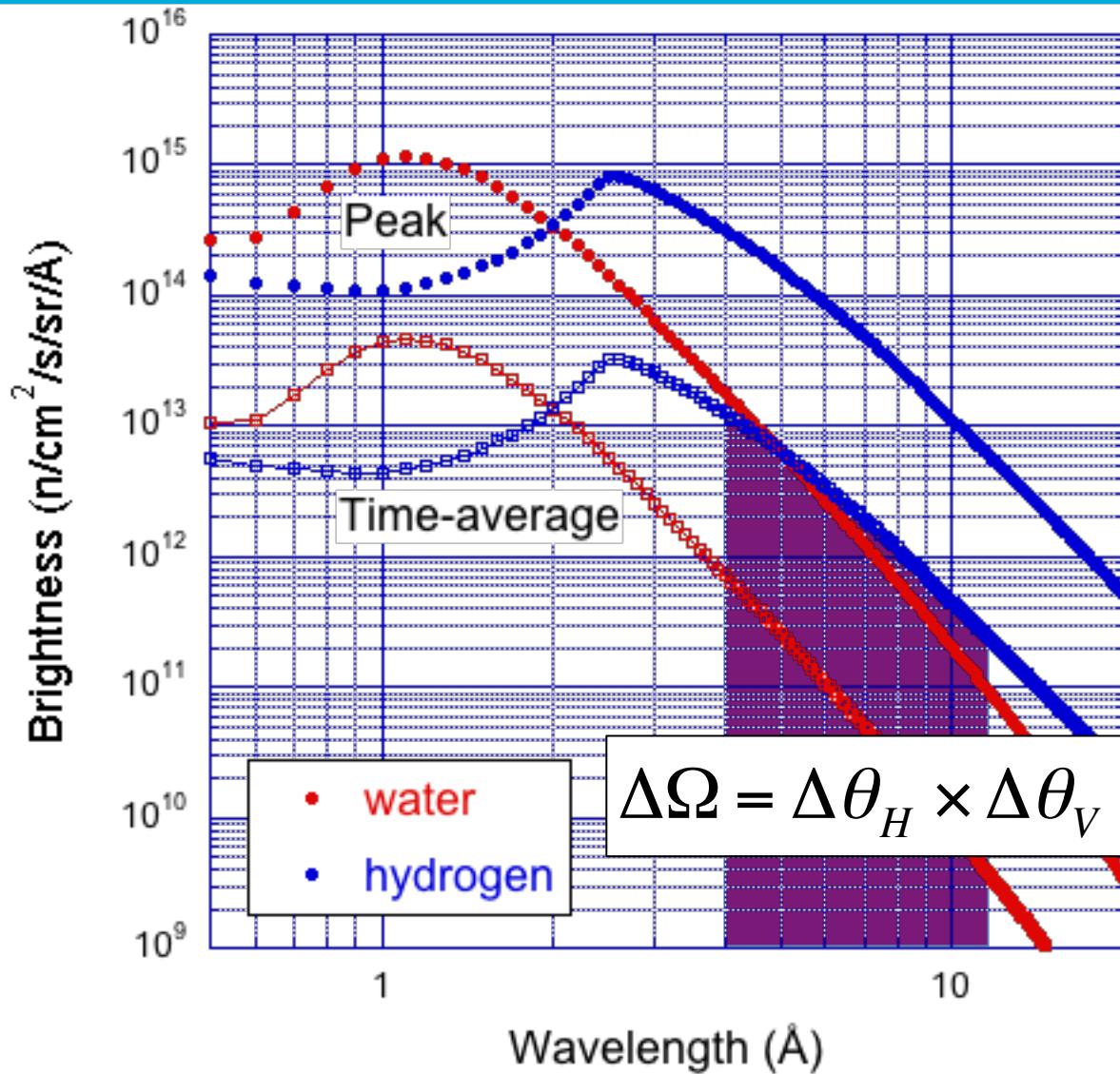
$$\Phi = \int_{\lambda_{\min}}^{\lambda_{\max}} B_{av} d\lambda \times \Delta\Omega$$

$$4\text{\AA} < \lambda < 12\text{\AA}$$

$$\int_{\lambda_{\min}}^{\lambda_{\max}} B_{av} d\lambda$$

$$\approx 2E13 \text{ n/cm}^2/\text{s/sr}$$

# Example: ESS SANS



$$\Phi = \int_{\lambda_{\min}}^{\lambda_{\max}} B_{av} d\lambda \times \Delta\Omega$$

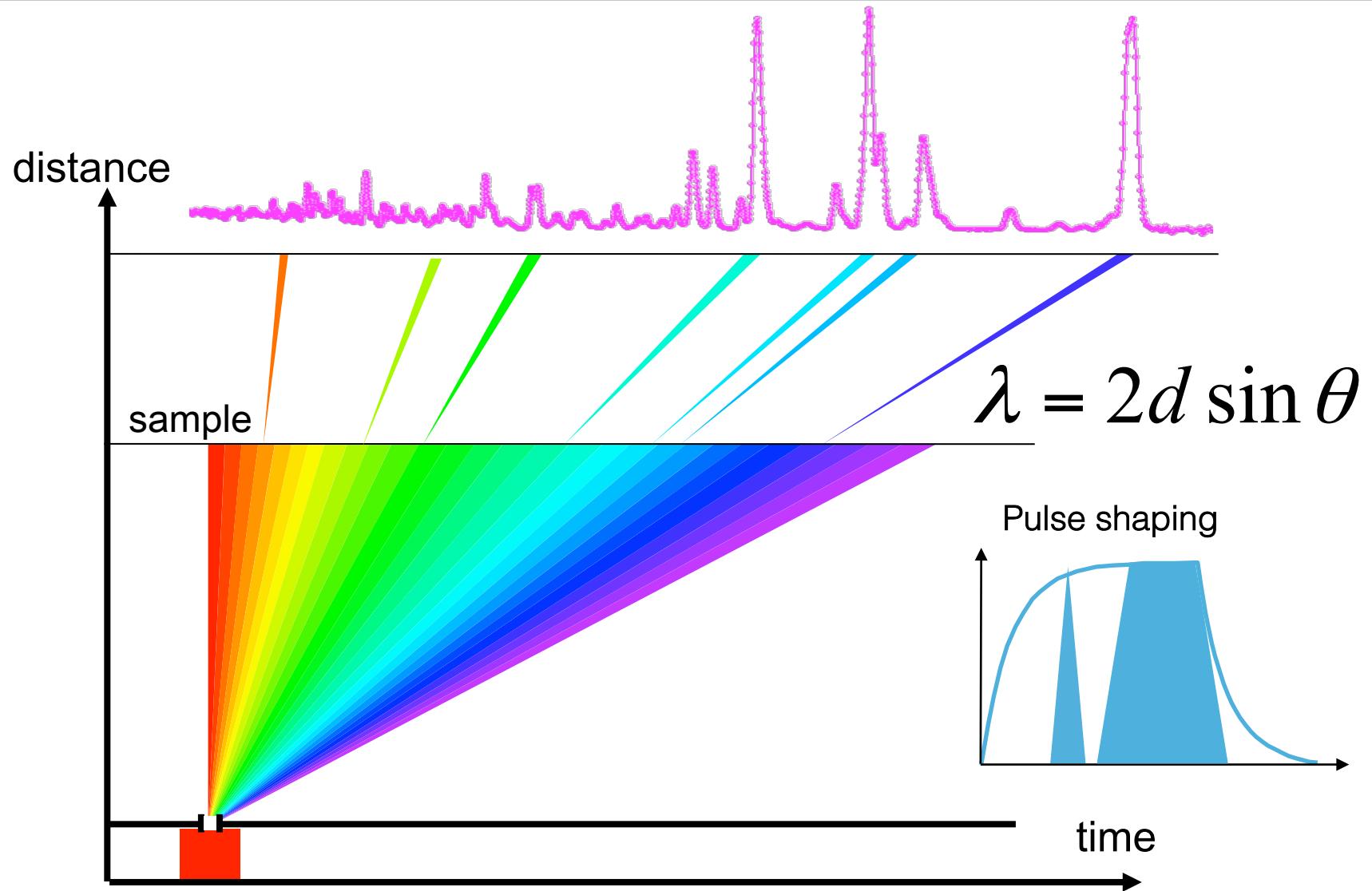
$$4\text{\AA} < \lambda < 12\text{\AA}$$

$$\int_{\lambda_{\min}}^{\lambda_{\max}} B_{av} d\lambda$$

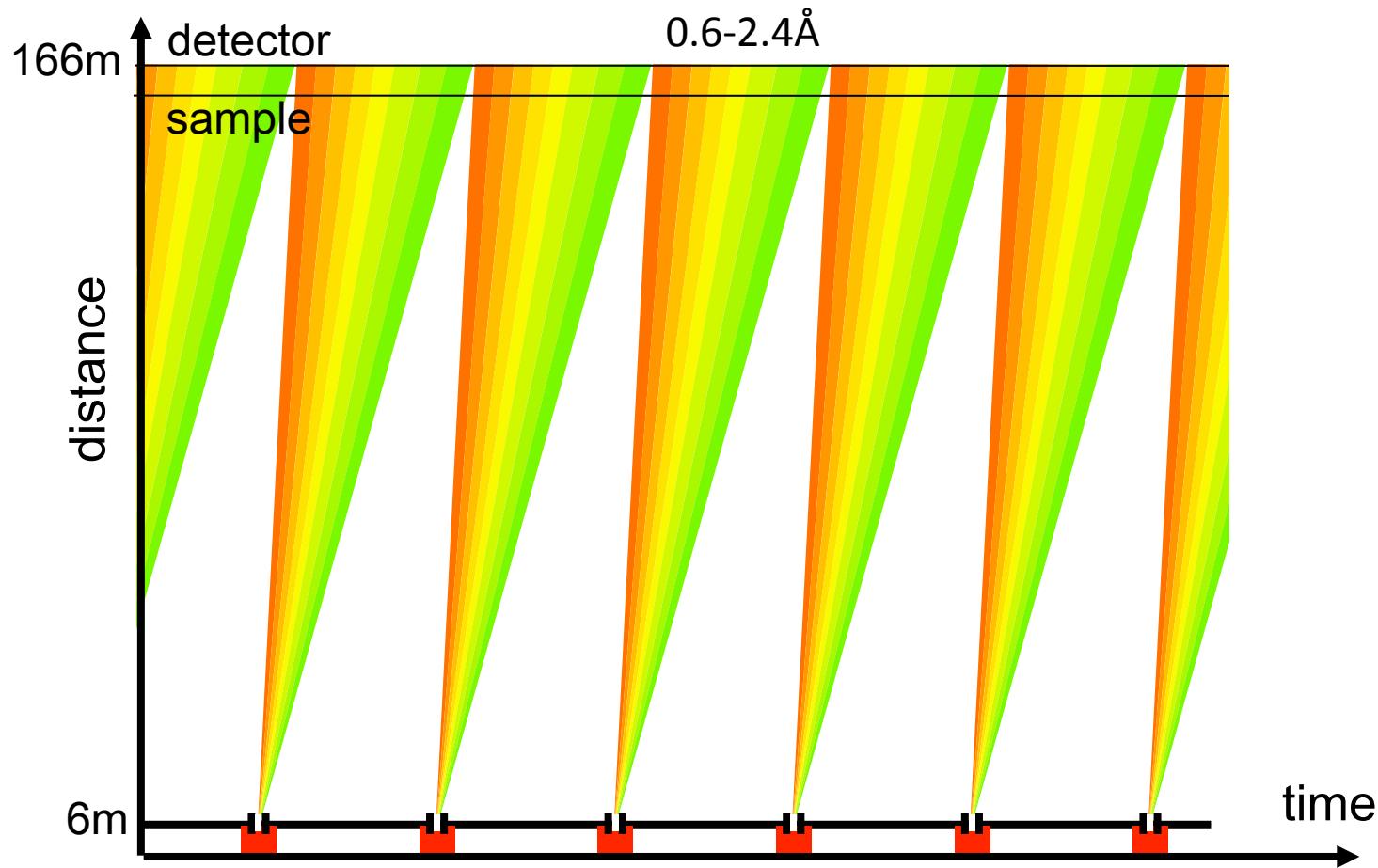
$$\approx 2\text{E}13 \text{ n/cm}^2/\text{s/sr}$$

$$\Phi \approx 9\text{E}7 \text{ n/cm}^2/\text{s}$$

# Example: ESS Powder Diffractometer

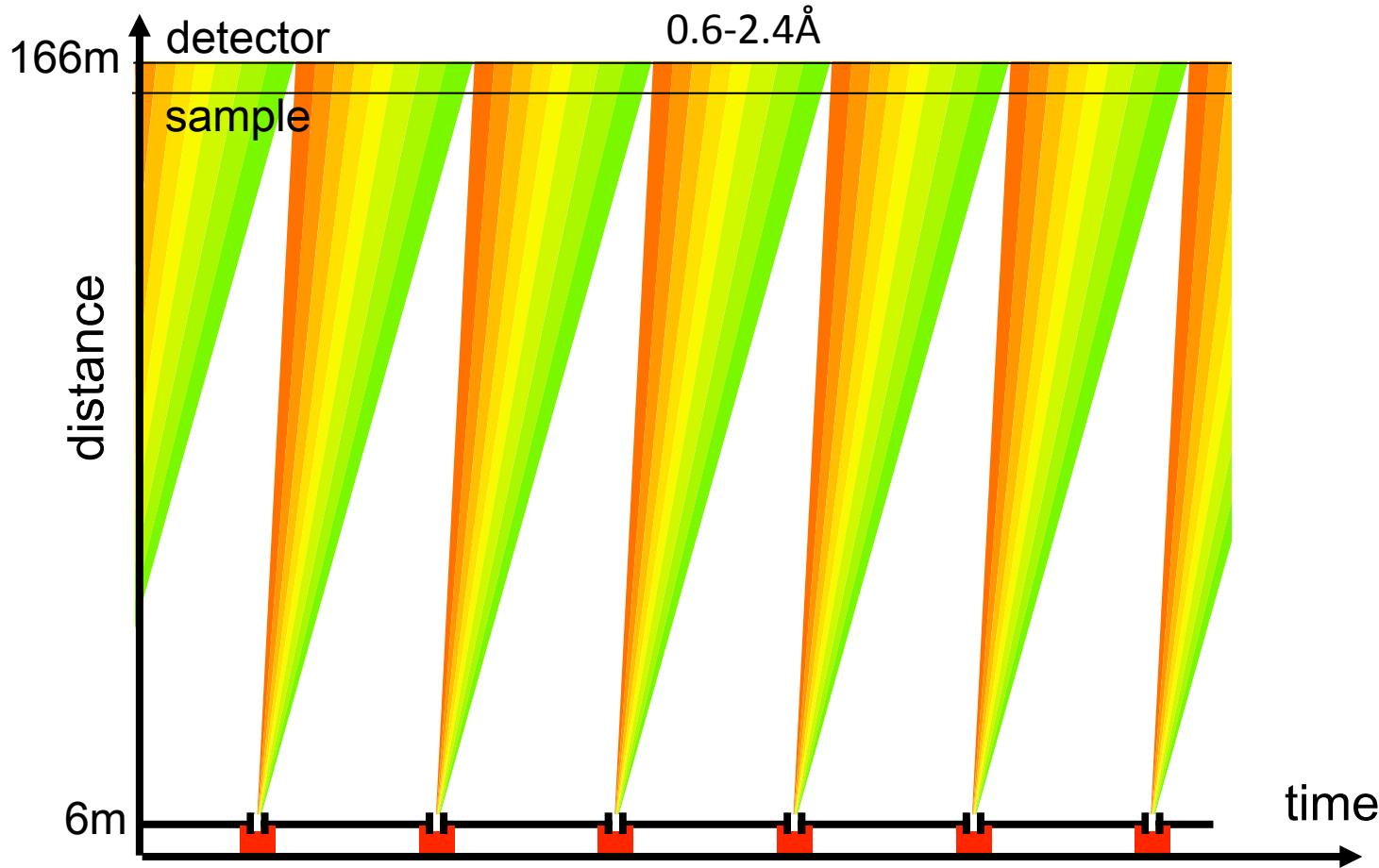


# Example: ESS Powder Diffractometer



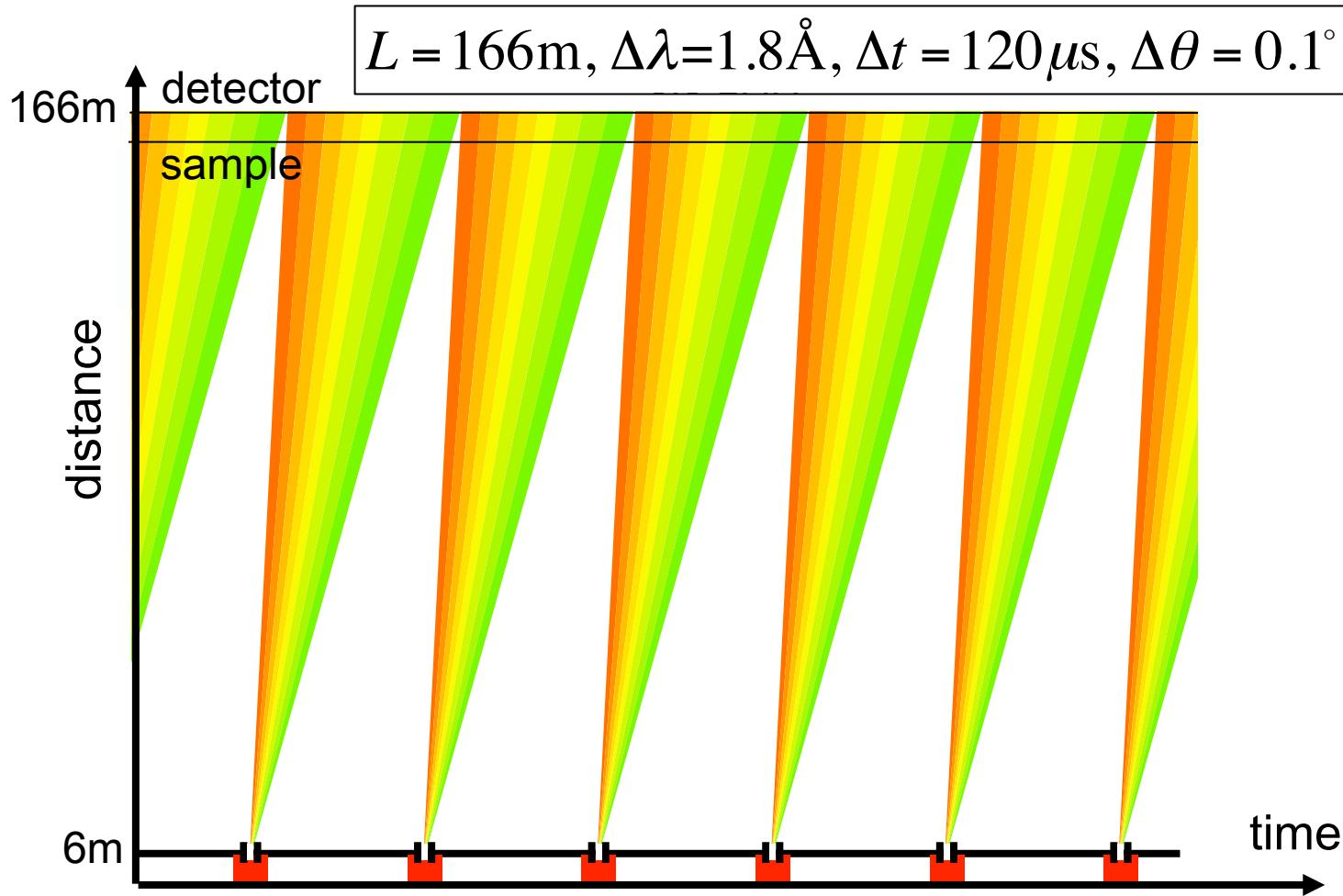
# Example: ESS Powder Diffractometer

$$t[\text{ms}] = L[\text{m}] \times \lambda[\text{\AA}] / 3.956$$

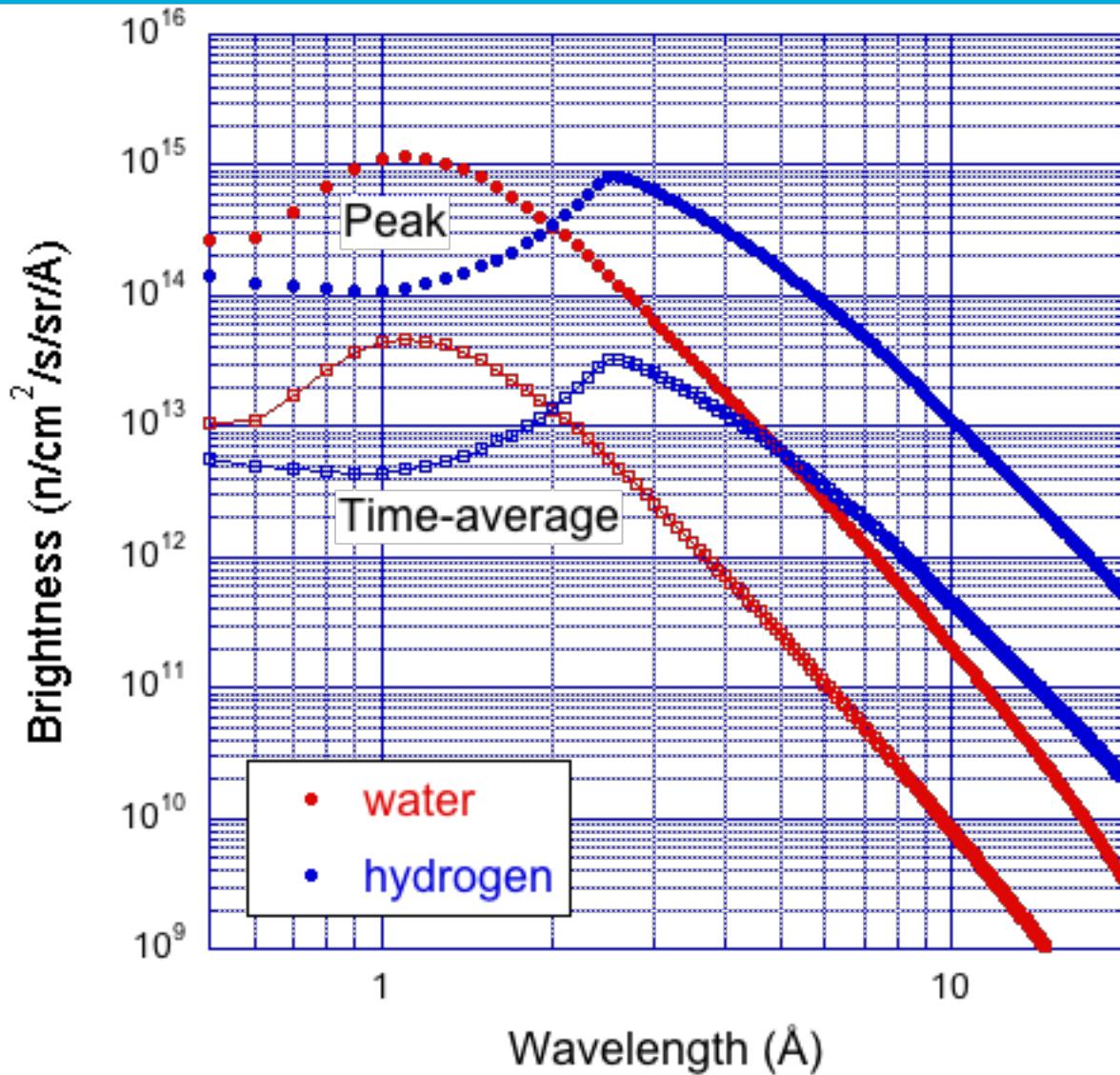


# Example: ESS Powder Diffractometer

$$t[\text{ms}] = L[\text{m}] \times \lambda[\text{\AA}] / 3.956$$

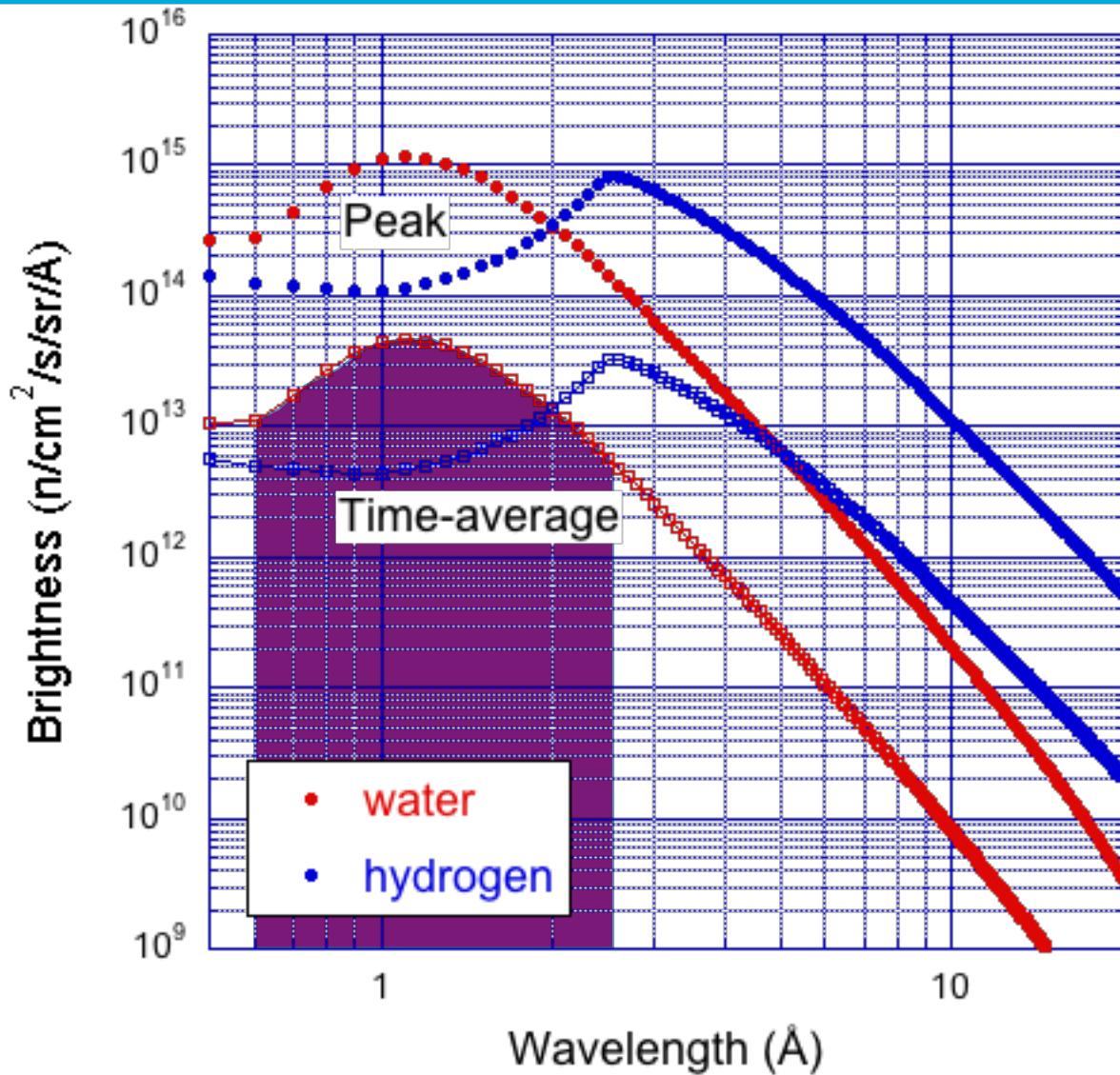


# Example: ESS Powder Diffractometer



$$\Phi = \int_{\lambda_{\min}}^{\lambda_{\max}} B_{av} d\lambda \times \Delta\Omega$$

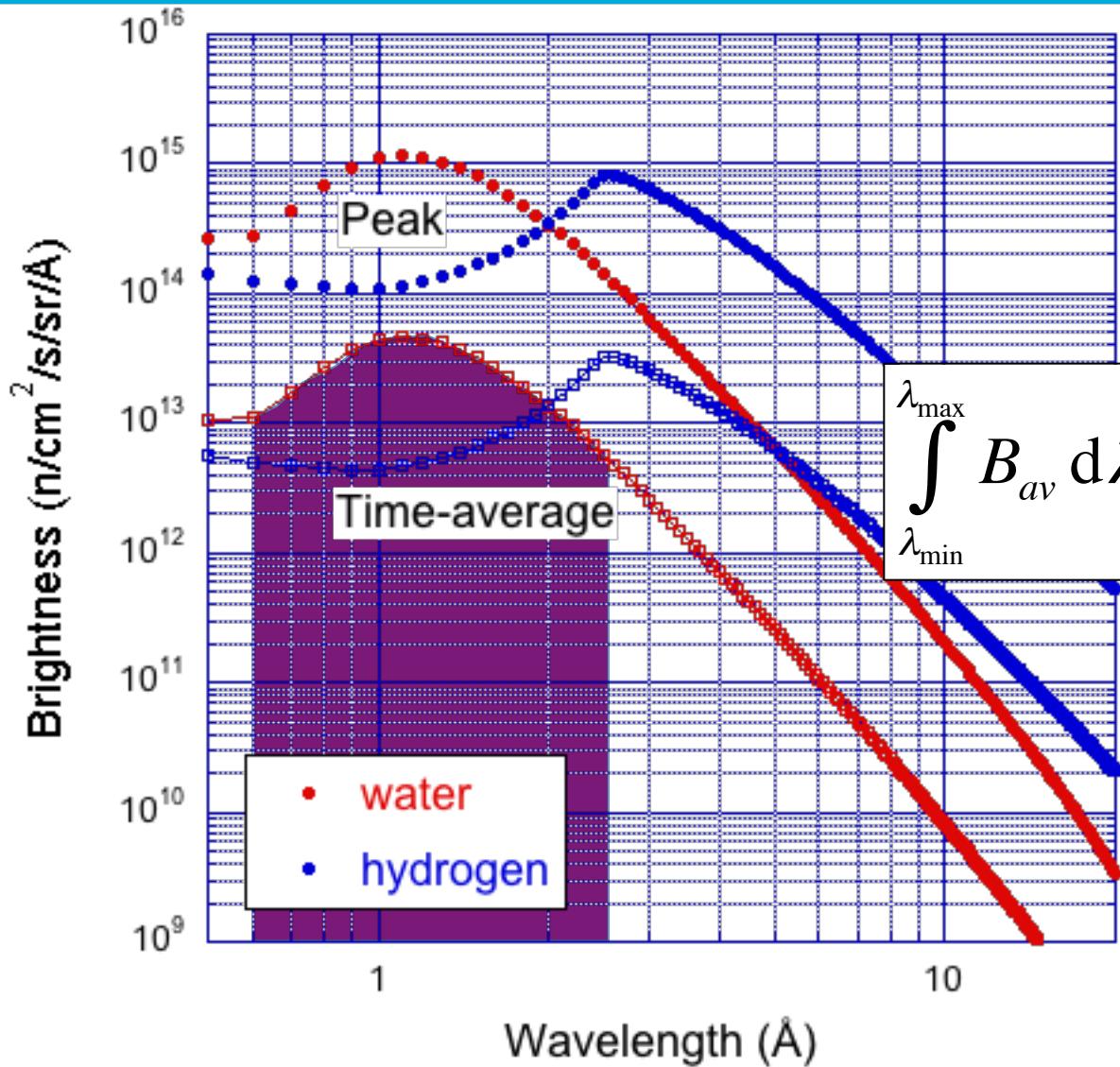
# Example: ESS Powder Diffractometer



$$\Phi = \int_{\lambda_{\min}}^{\lambda_{\max}} B_{av} d\lambda \times \Delta\Omega$$

$$0.6\text{\AA} < \lambda < 2.4\text{\AA}$$

# Example: ESS Powder Diffractometer

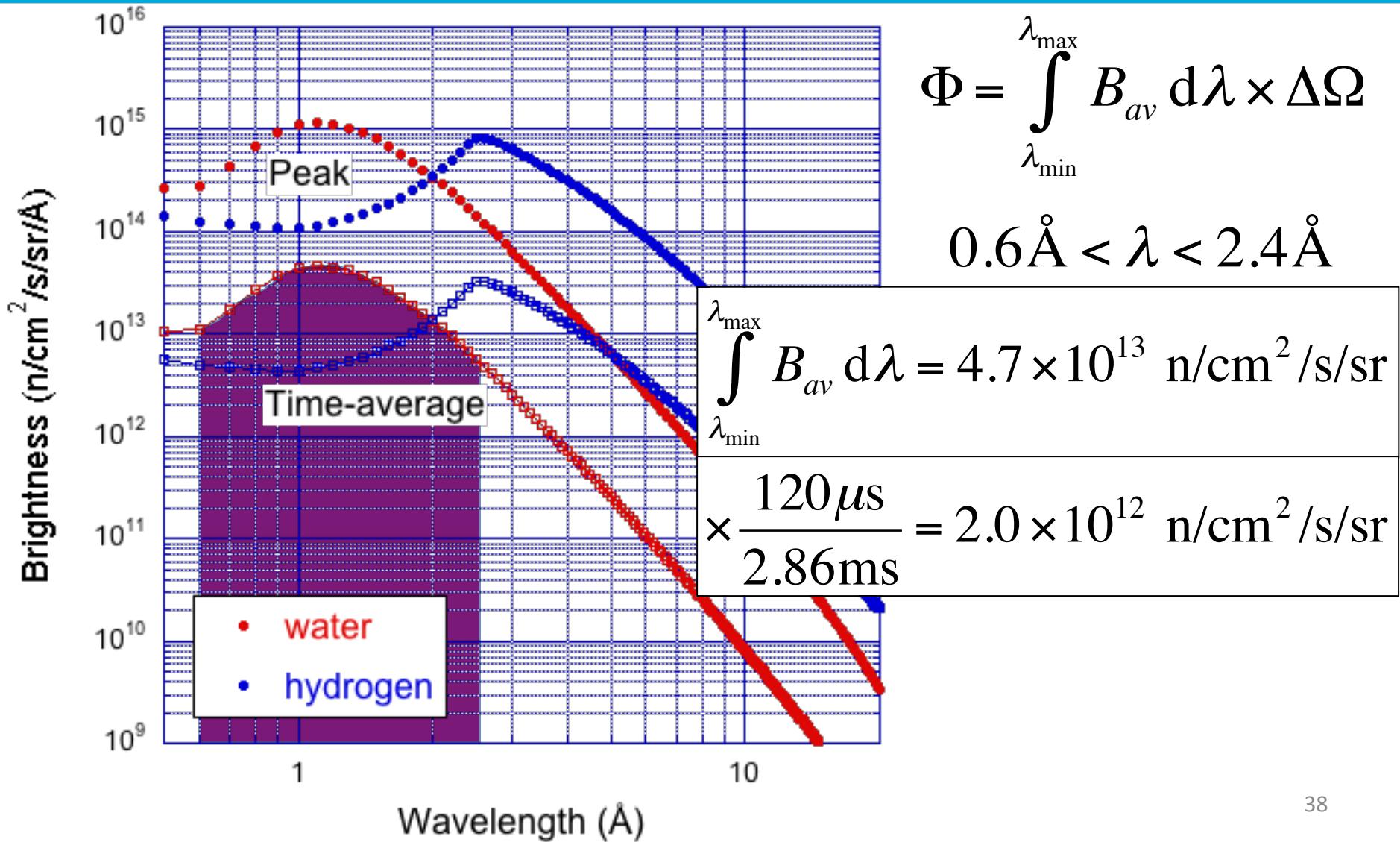


$$\Phi = \int_{\lambda_{\min}}^{\lambda_{\max}} B_{av} d\lambda \times \Delta\Omega$$

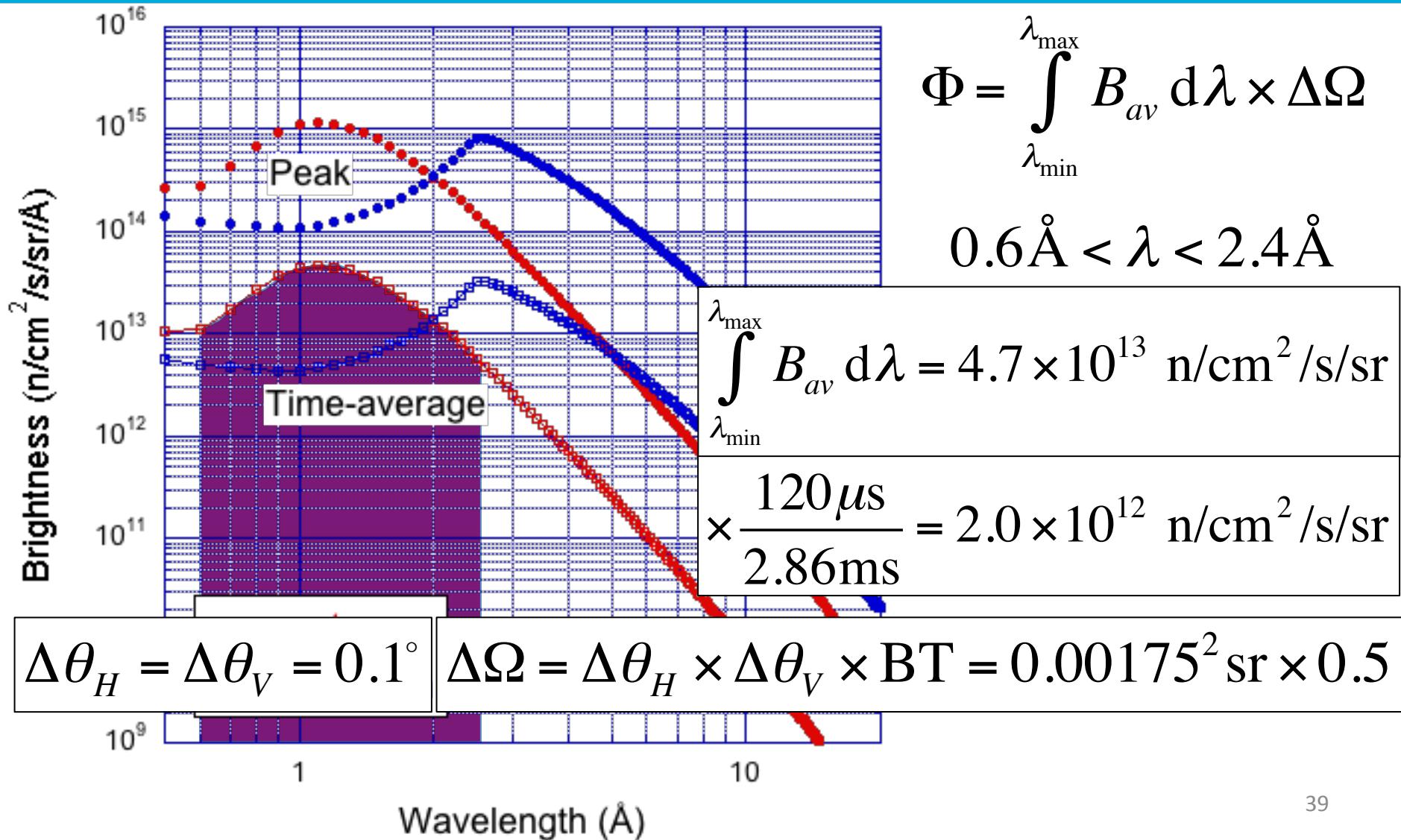
$$0.6\text{\AA} < \lambda < 2.4\text{\AA}$$

$$\int_{\lambda_{\min}}^{\lambda_{\max}} B_{av} d\lambda = 4.7 \times 10^{13} \text{ n/cm}^2/\text{s/sr}$$

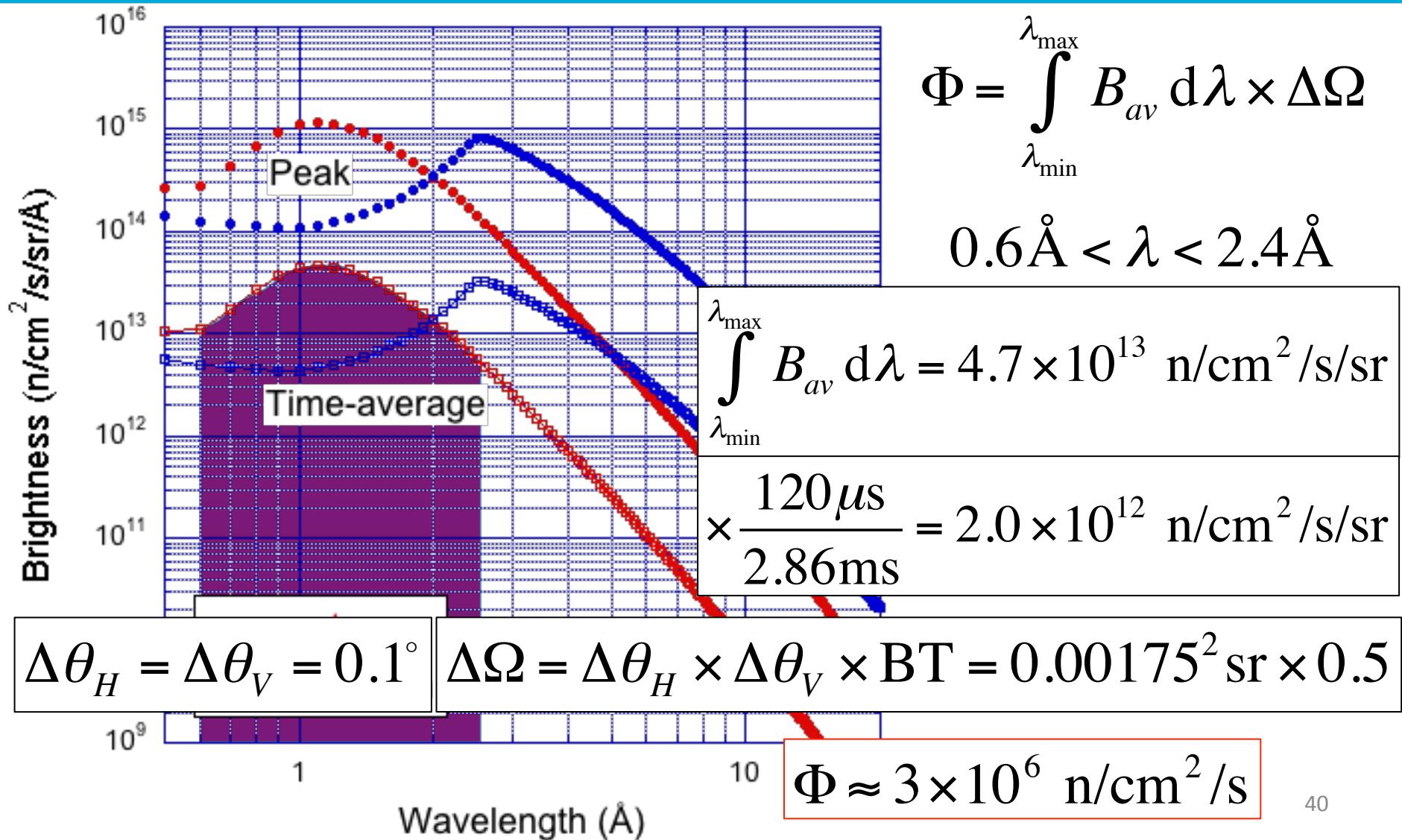
# Example: ESS Powder Diffractometer



# Example: ESS Powder Diffractometer

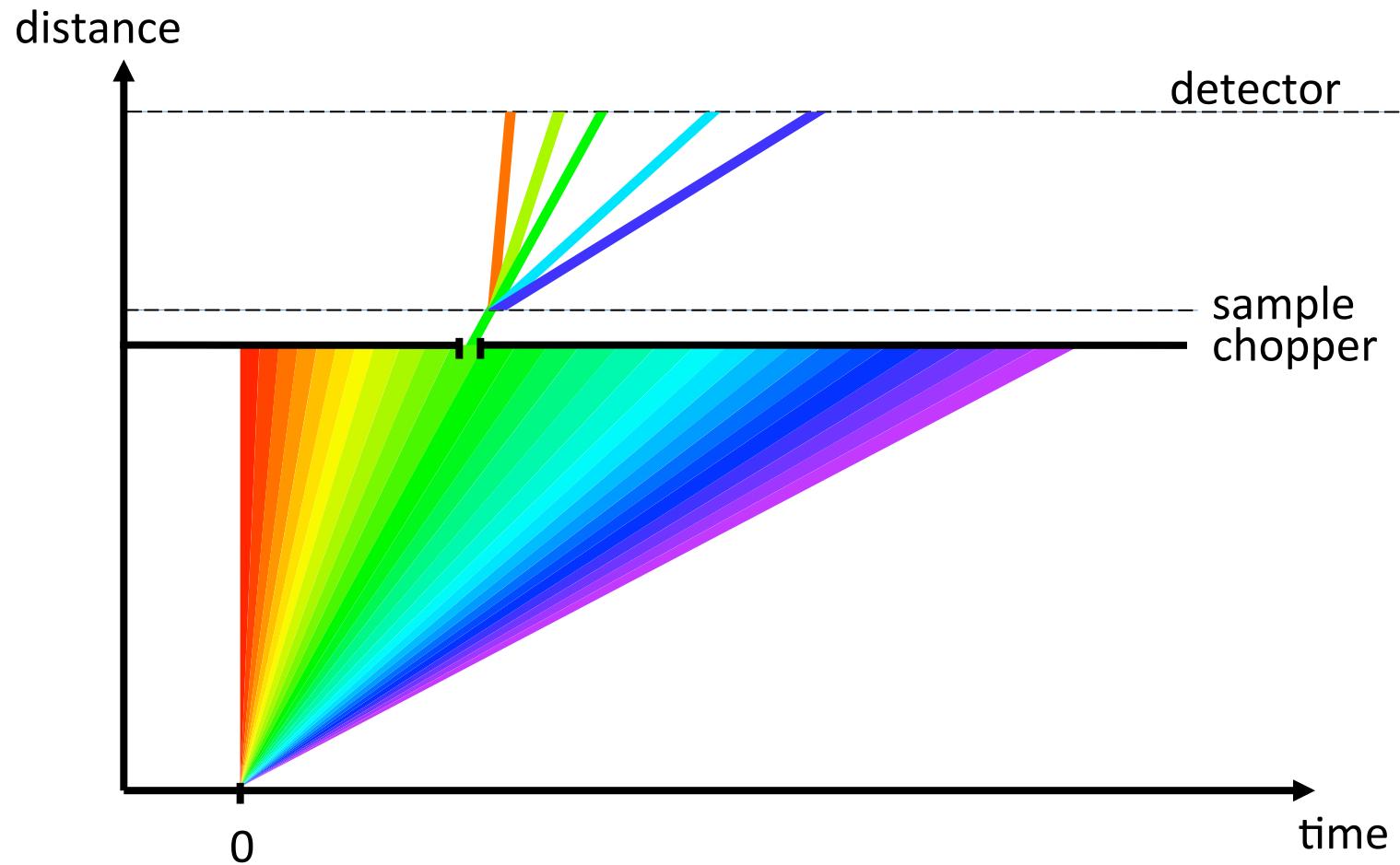


# Example: ESS Powder Diffractometer



# Example: SNS STS Cold Chopper Spectrometer

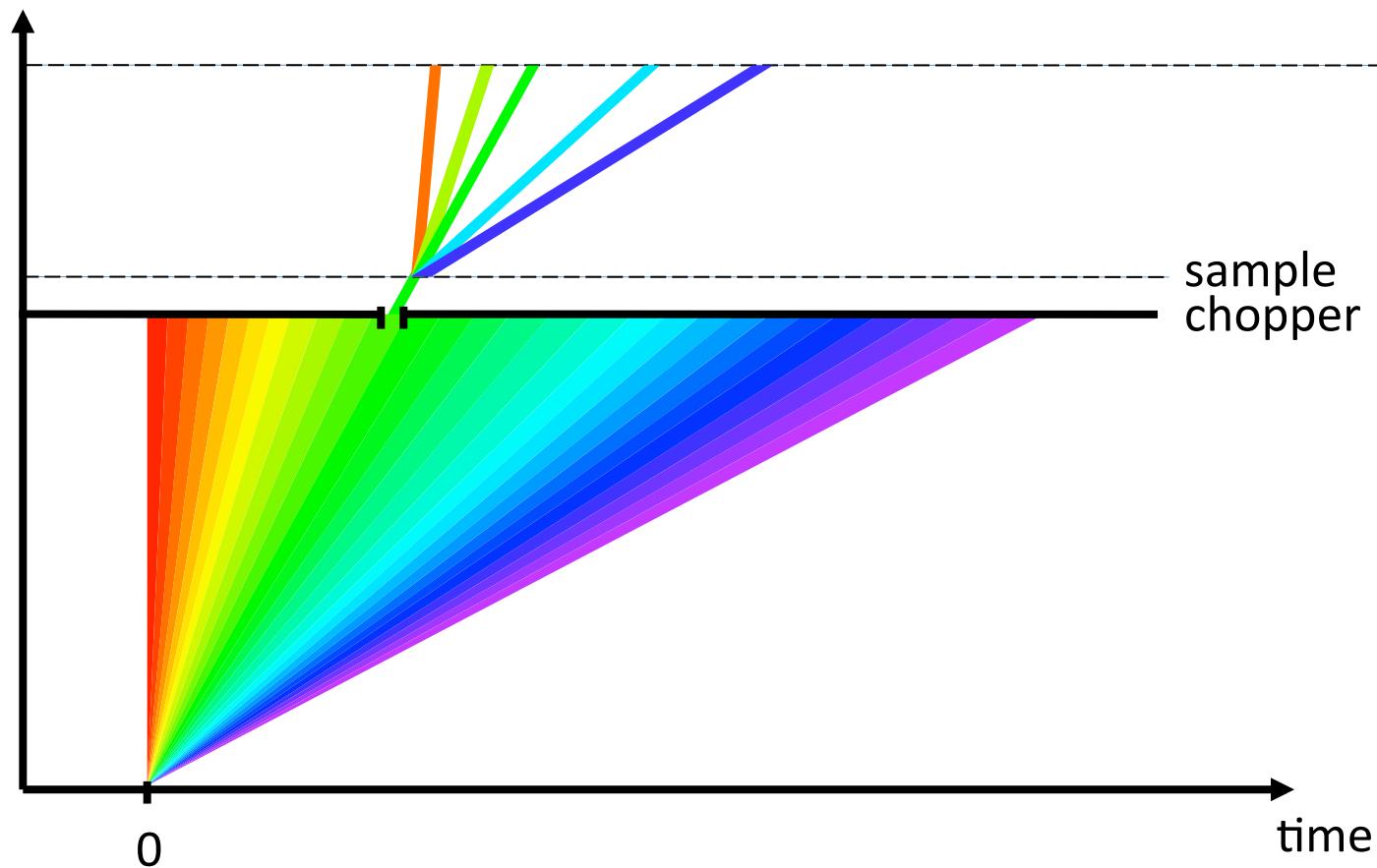
$$t[\text{ms}] = L[\text{m}] \times \lambda[\text{\AA}] / 3.956$$



# Example: SNS STS Cold Chopper Spectrometer

$$t[\text{ms}] = L[\text{m}] \times \lambda[\text{\AA}] / 3.956$$

$$\Delta\lambda[\text{\AA}] = \Delta t[\text{ms}] \times 3.956 / L[\text{m}]$$

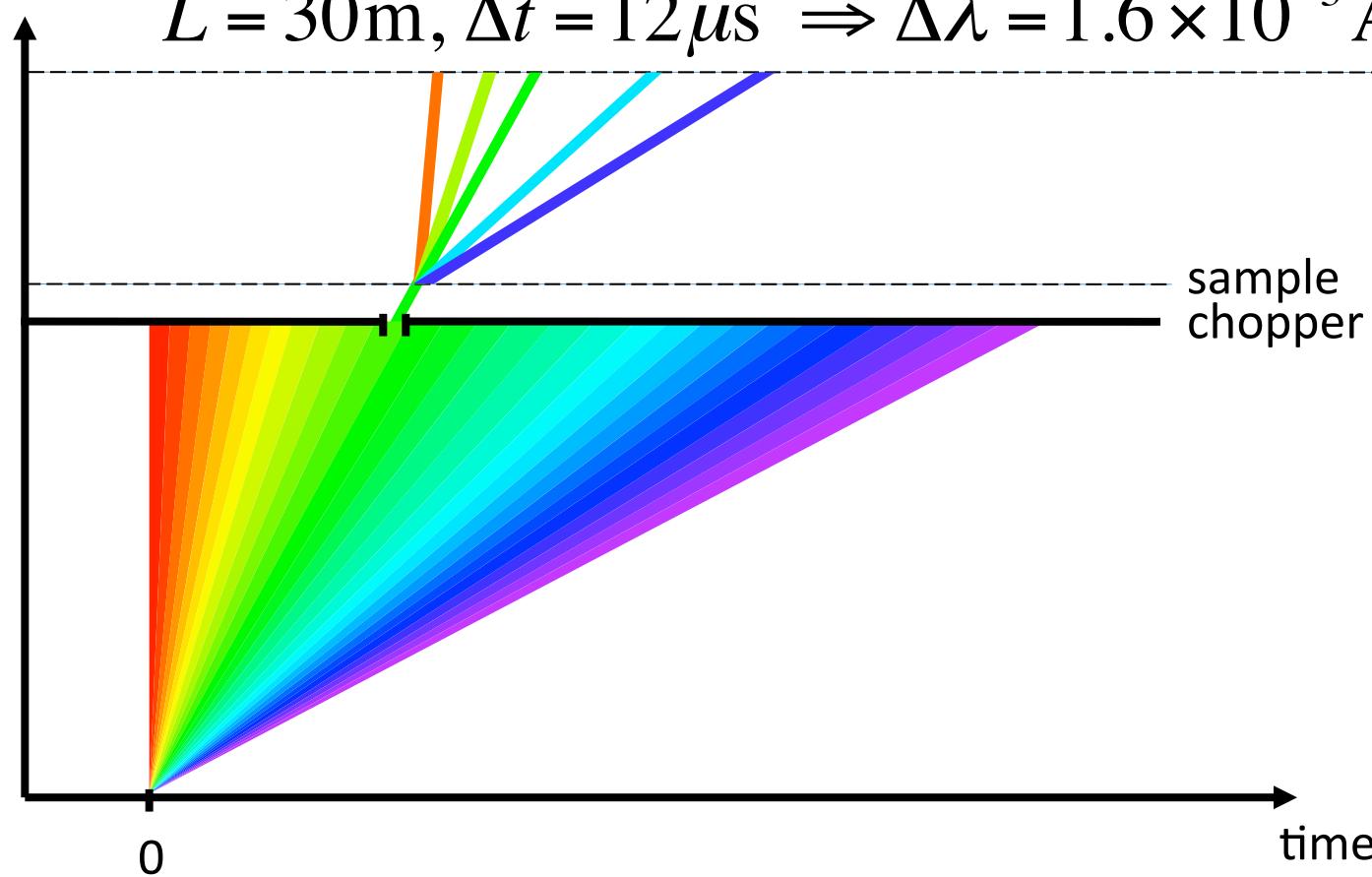


# Example: SNS STS Cold Chopper Spectrometer

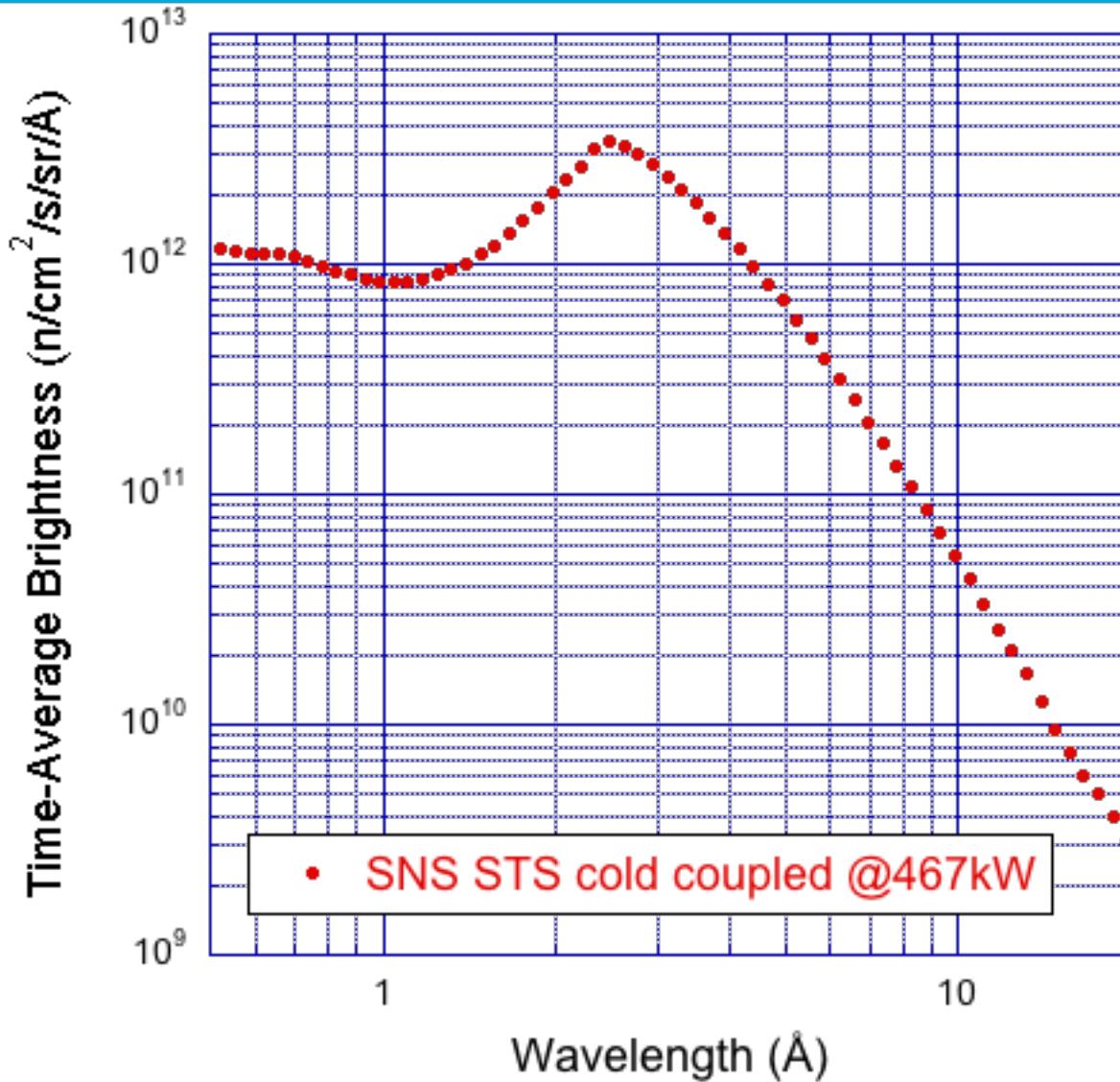
$$t[\text{ms}] = L[\text{m}] \times \lambda[\text{\AA}] / 3.956$$

$$\Delta\lambda[\text{\AA}] = \Delta t[\text{ms}] \times 3.956 / L[\text{m}]$$

$$L = 30\text{m}, \Delta t = 12\mu\text{s} \Rightarrow \Delta\lambda = 1.6 \times 10^{-3} \text{\AA}$$

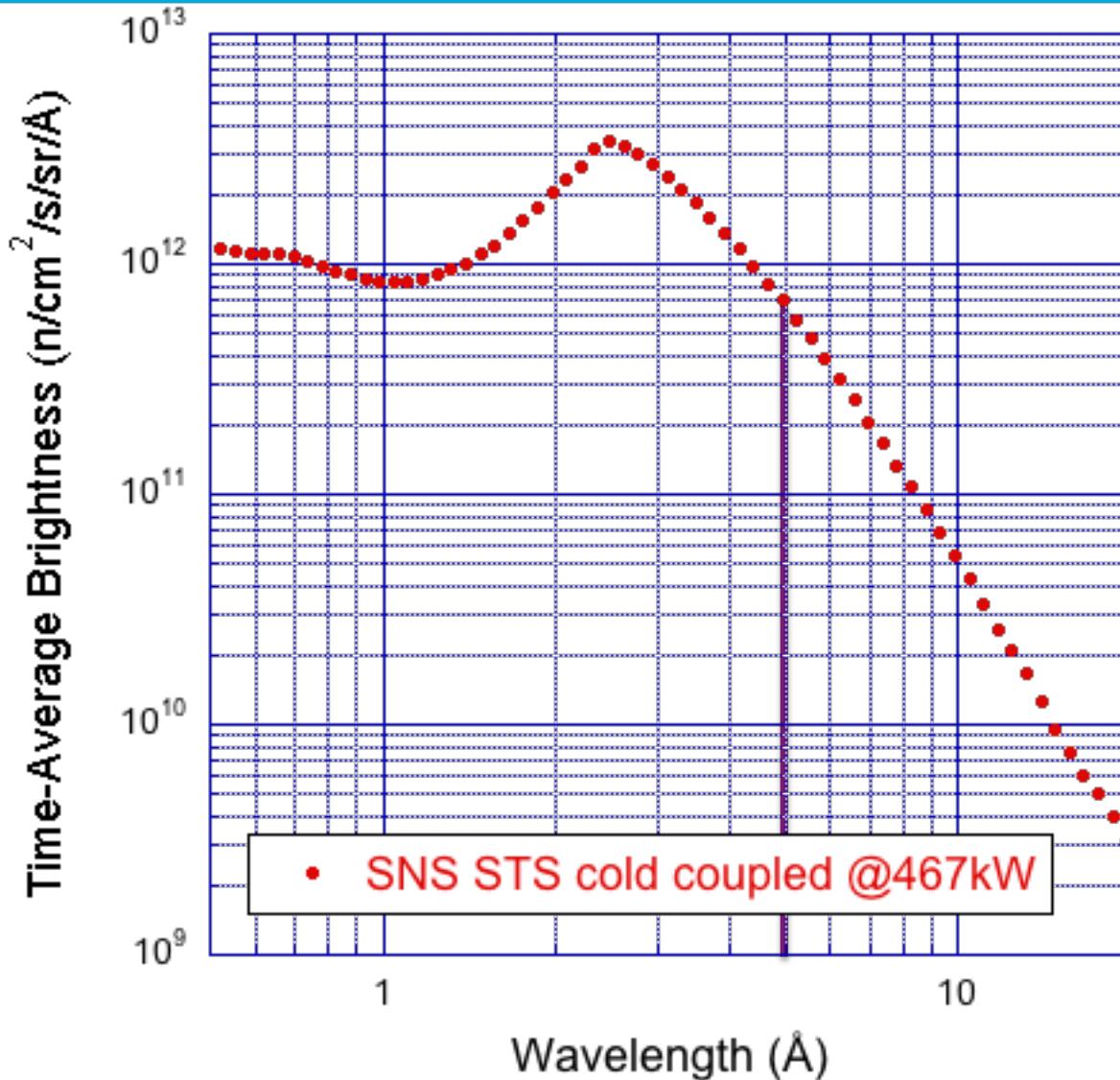


# Example: SNS STS Cold Chopper Spectrometer



$$\Phi = \int_{\lambda_{\min}}^{\lambda_{\max}} B_{av} d\lambda \times \Delta\Omega$$

# Example: SNS STS Cold Chopper Spectrometer

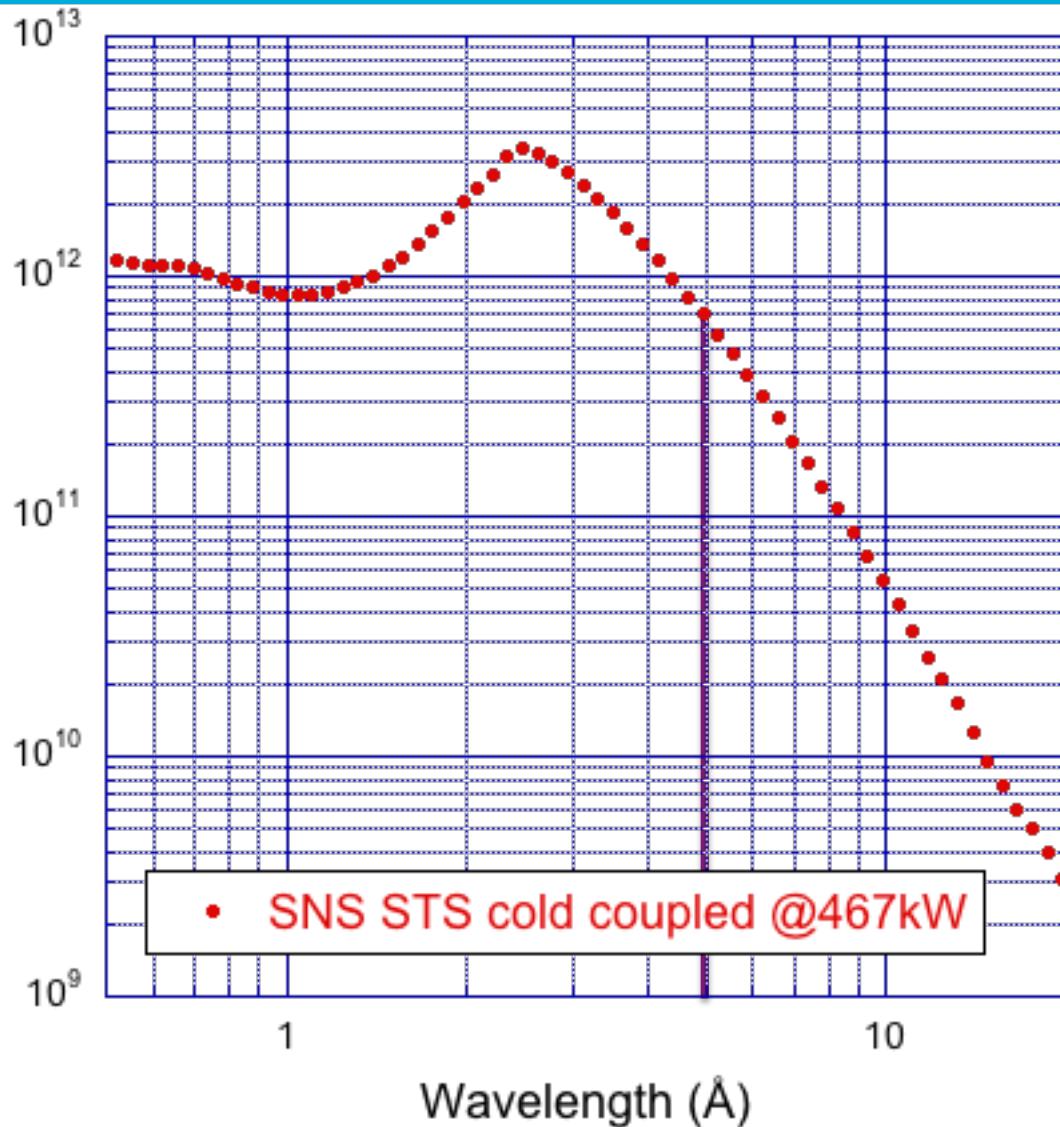


$$\Phi = \int_{\lambda_{\min}}^{\lambda_{\max}} B_{av} d\lambda \times \Delta\Omega$$

$$\Delta\lambda = 1.6 \times 10^{-3} \text{ Å}$$

# Example: SNS STS Cold Chopper Spectrometer

Time-Average Brightness ( $\text{n/cm}^2/\text{s/sr}/\text{\AA}$ )



$$\Phi = \int_{\lambda_{\min}}^{\lambda_{\max}} B_{av} d\lambda \times \Delta\Omega$$

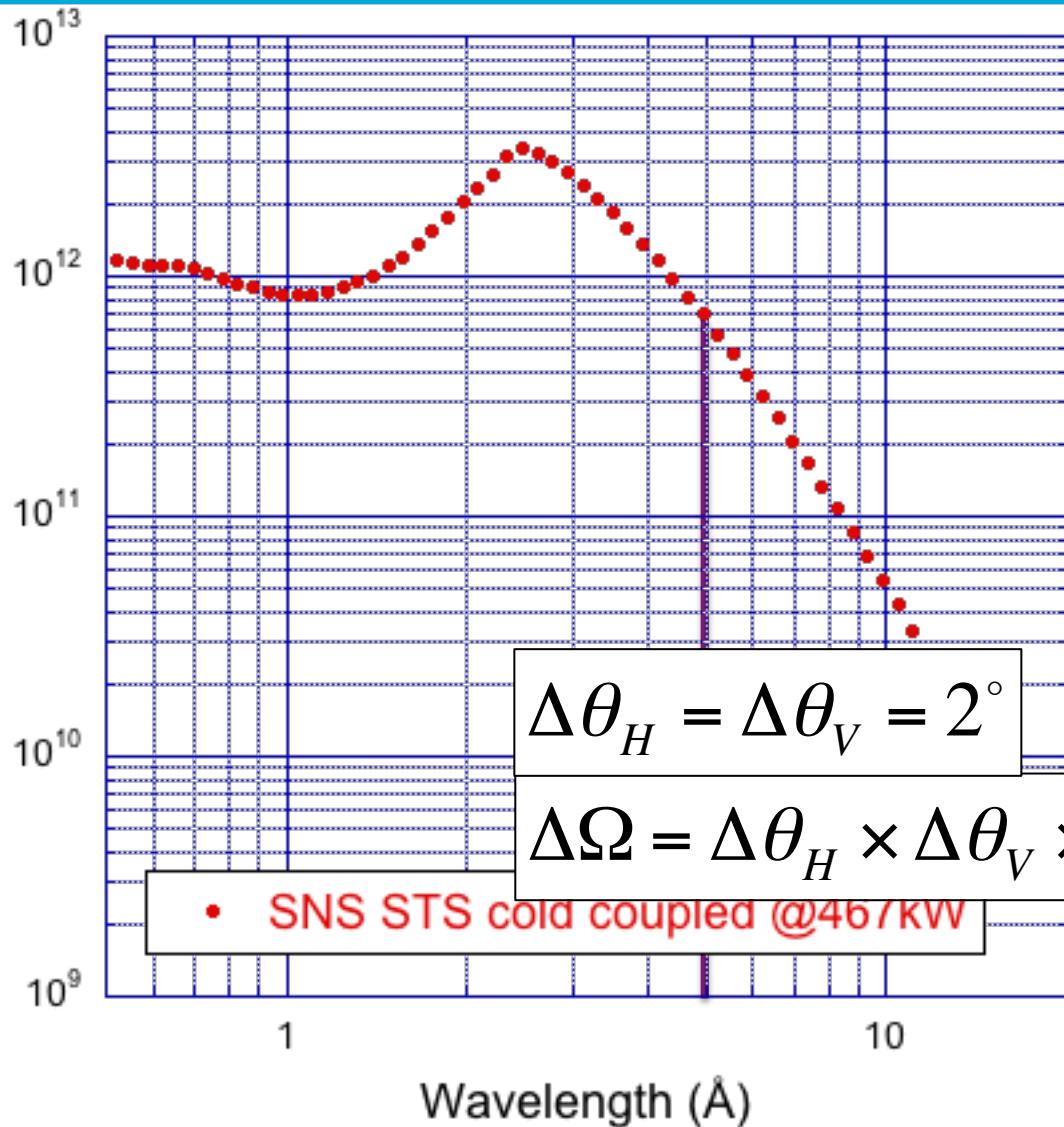
$$\Delta\lambda = 1.6 \times 10^{-3} \text{\AA}$$

$$\int_{\lambda_{\min}}^{\lambda_{\max}} B_{av} d\lambda \text{ at } \lambda = 5 \text{\AA}$$

$$\approx 1.1 \times 10^9 \text{ n/cm}^2/\text{s/sr}$$

# Example: SNS STS Cold Chopper Spectrometer

Time-Average Brightness ( $\text{n/cm}^2/\text{s/sr}/\text{\AA}$ )



$$\Phi = \int_{\lambda_{\min}}^{\lambda_{\max}} B_{av} d\lambda \times \Delta\Omega$$

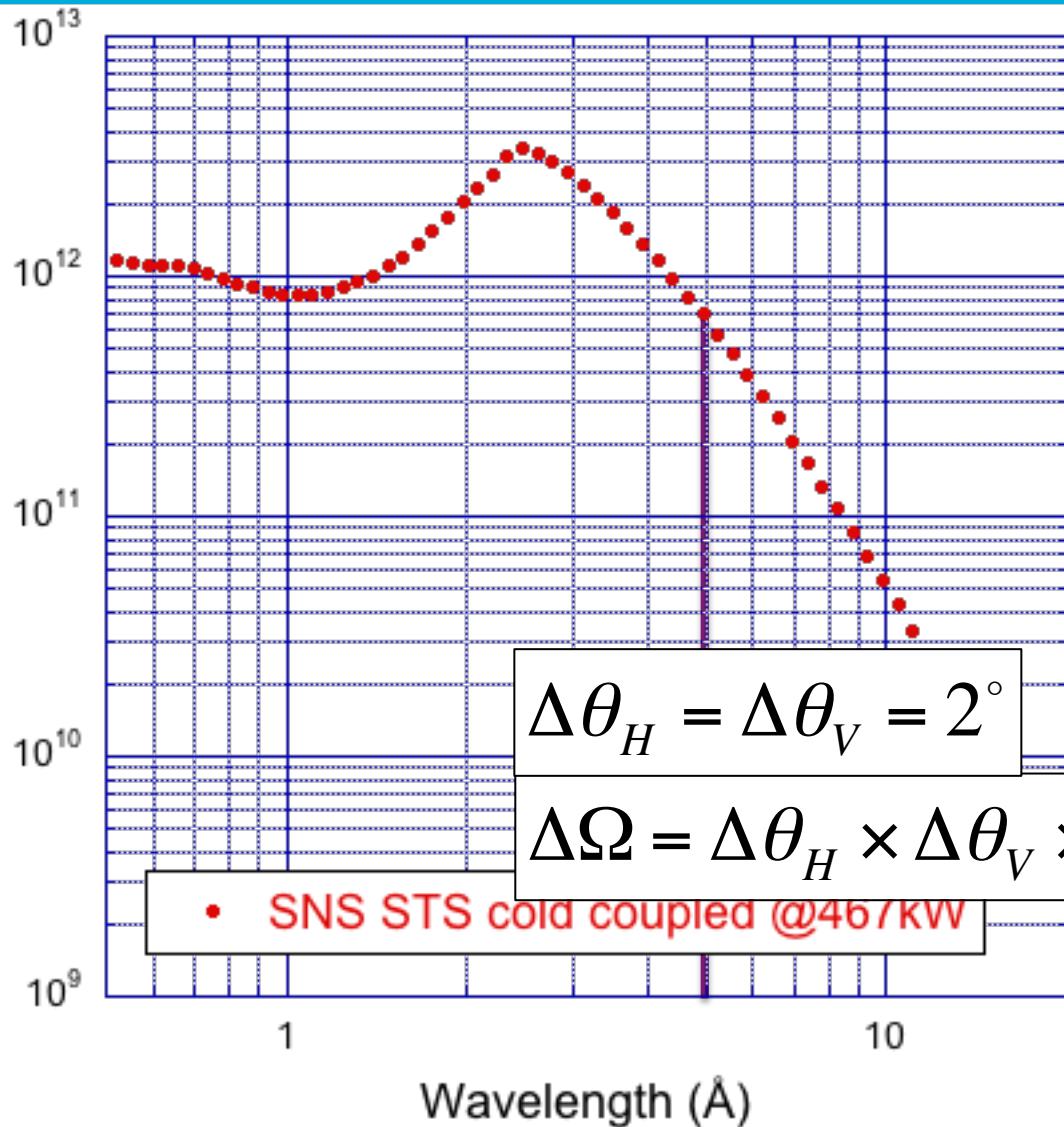
$$\Delta\lambda = 1.6 \times 10^{-3} \text{\AA}$$

$$\int_{\lambda_{\min}}^{\lambda_{\max}} B_{av} d\lambda \text{ at } \lambda = 5 \text{\AA}$$

$$\approx 1.1 \times 10^9 \text{ n/cm}^2/\text{s/sr}$$

# Example: SNS STS Cold Chopper Spectrometer

Time-Average Brightness ( $\text{n/cm}^2/\text{s/sr}/\text{\AA}$ )



$$\Phi = \int_{\lambda_{\min}}^{\lambda_{\max}} B_{av} d\lambda \times \Delta\Omega$$

$$\Delta\lambda = 1.6 \times 10^{-3} \text{ \AA}$$

$$\int_{\lambda_{\min}}^{\lambda_{\max}} B_{av} d\lambda \text{ at } \lambda = 5 \text{ \AA}$$

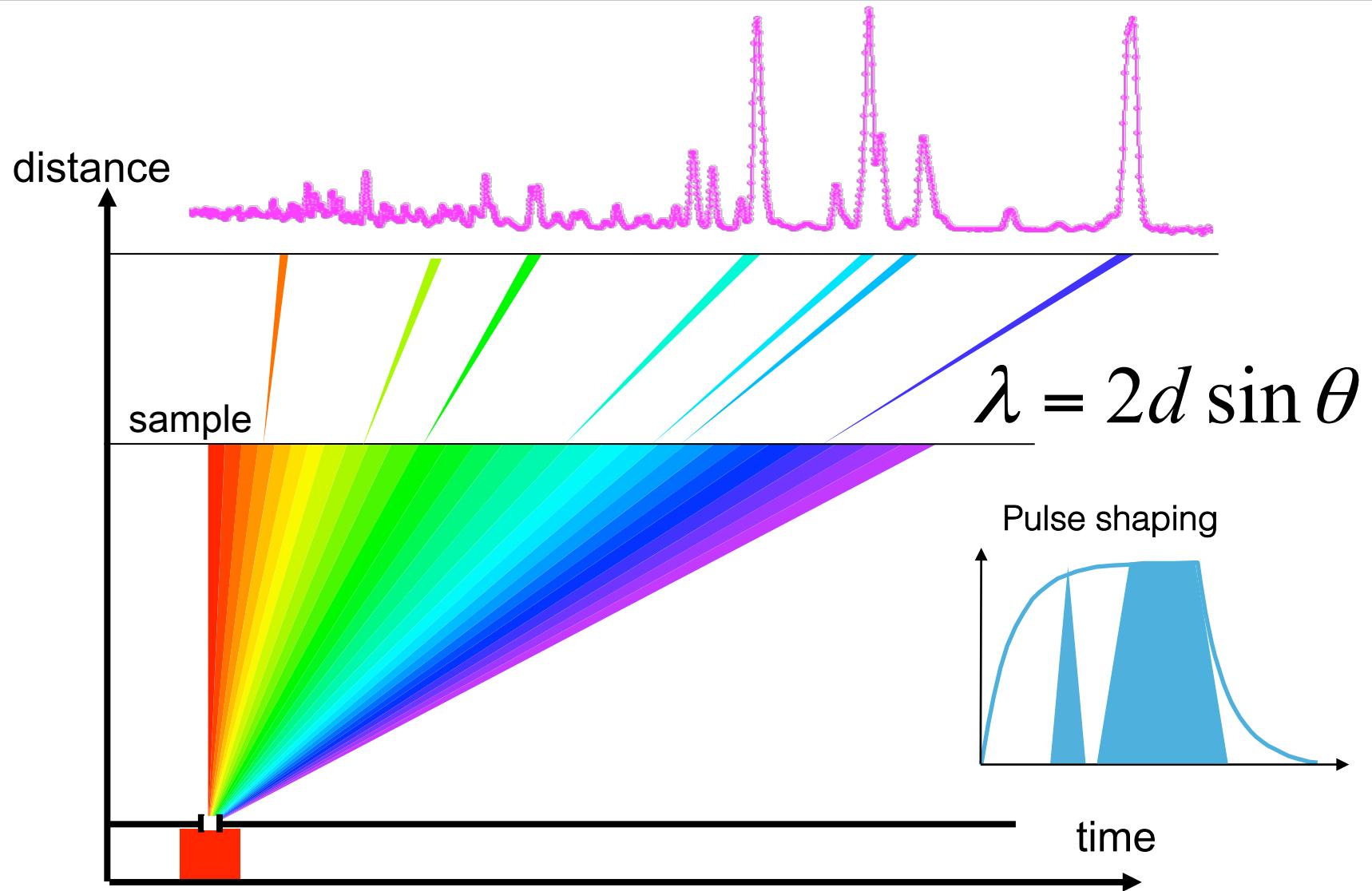
$$\approx 1.1 \times 10^9 \text{ n/cm}^2/\text{s/sr}$$

$$\Phi \approx 7 \times 10^5 \text{ n/cm}^2/\text{s}$$

# Resolution Calculations

- Calculation of instrument parameters given resolution requirement
- Decide on the relevant quantity
- Express how it is calculated
- Calculate its uncertainty
  - requires the partial derivatives
- Match the various contributions
  - maximises flux for a given resolution

# Example: ESS Powder Diffractometer



# Example: ESS Powder Diffractometer

$$d = \frac{\lambda}{2 \sin \theta}$$

# Example: ESS Powder Diffractometer

$$d = \frac{\lambda}{2 \sin \theta}$$

$$\Delta d^2 = \left( \frac{\partial d}{\partial \lambda} \Delta \lambda \right)^2 + \left( \frac{\partial d}{\partial \theta} \Delta \theta \right)^2$$

# Example: ESS Powder Diffractometer

$$d = \frac{\lambda}{2 \sin \theta}$$

$$\Delta d^2 = \left( \frac{\partial d}{\partial \lambda} \Delta \lambda \right)^2 + \left( \frac{\partial d}{\partial \theta} \Delta \theta \right)^2$$

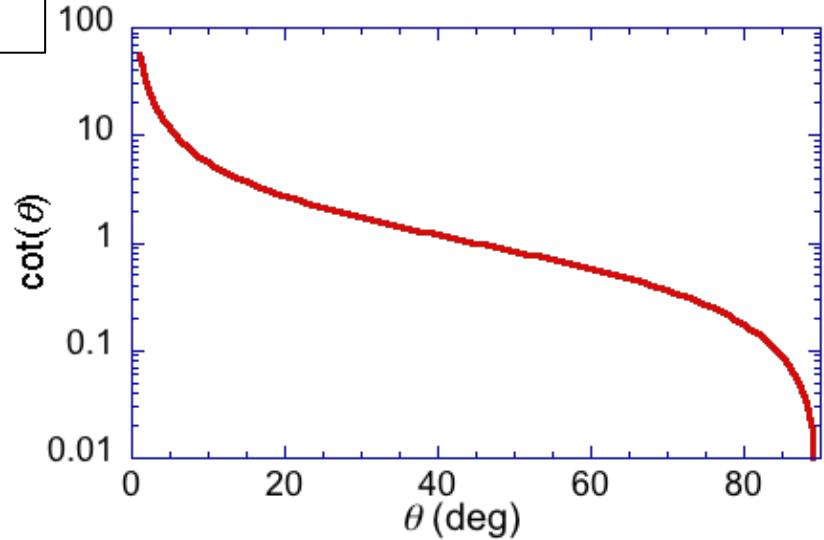
$$\begin{aligned}
 (\Delta d/d)^2 &= (\Delta \lambda / \lambda)^2 + (\cot \theta \Delta \theta)^2 \\
 &= (\Delta t/t)^2 + (\cot \theta \Delta \theta)^2
 \end{aligned}$$

# Example: ESS Powder Diffractometer

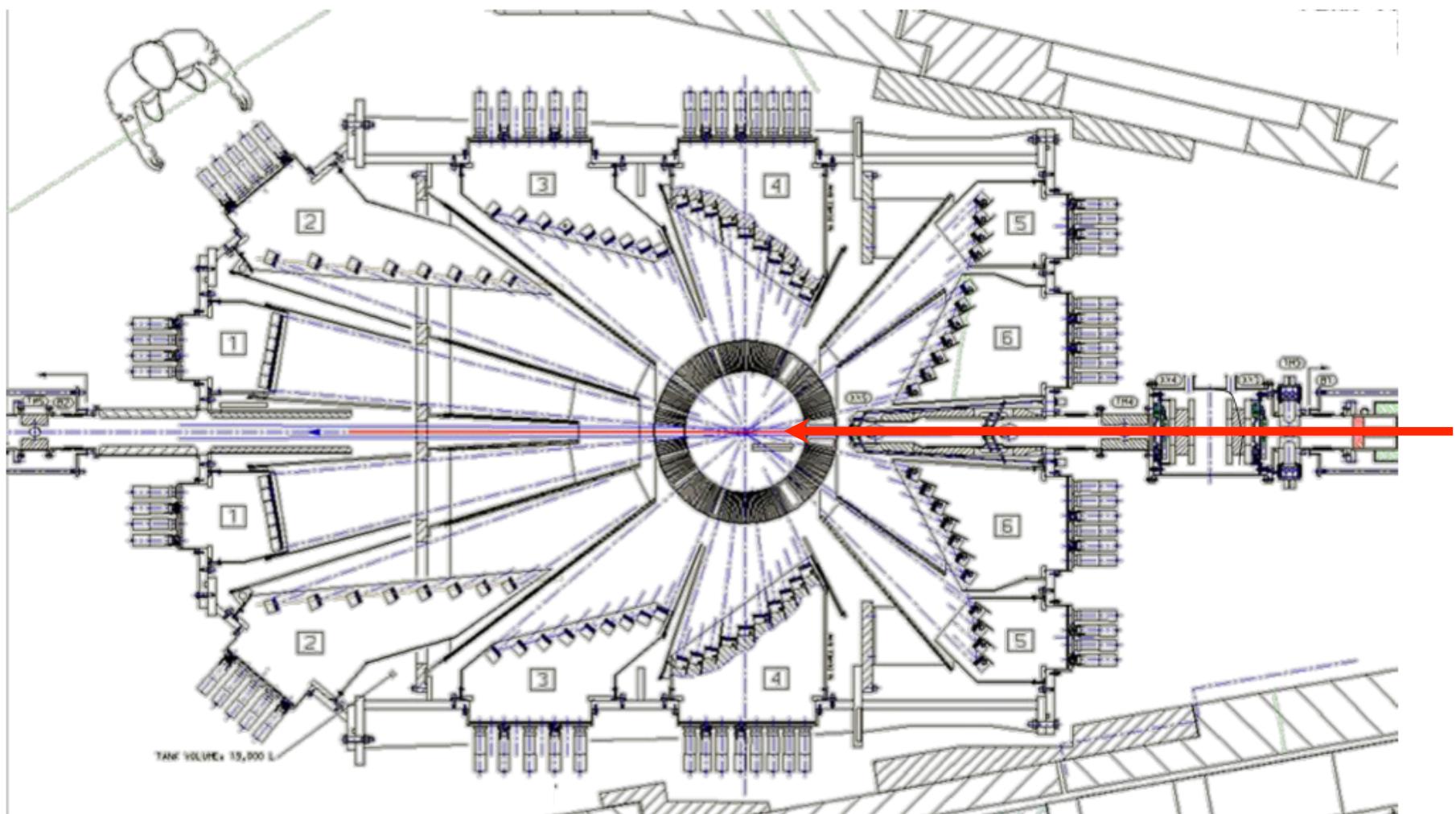
$$d = \frac{\lambda}{2 \sin \theta}$$

$$\Delta d^2 = \left( \frac{\partial d}{\partial \lambda} \Delta \lambda \right)^2 + \left( \frac{\partial d}{\partial \theta} \Delta \theta \right)^2$$

$$\begin{aligned} (\Delta d/d)^2 &= (\Delta \lambda / \lambda)^2 + (\cot \theta \Delta \theta)^2 \\ &= (\Delta t / t)^2 + (\cot \theta \Delta \theta)^2 \end{aligned}$$



# Example: ESS Powder Diffractometer



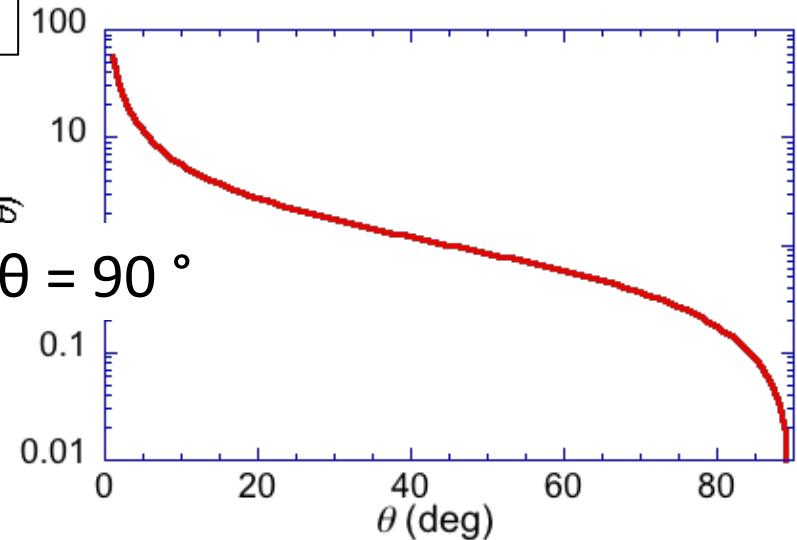
# Example: ESS Powder Diffractometer

$$d = \frac{\lambda}{2 \sin \theta}$$

$$\Delta d^2 = \left( \frac{\partial d}{\partial \lambda} \Delta \lambda \right)^2 + \left( \frac{\partial d}{\partial \theta} \Delta \theta \right)^2$$

$$\begin{aligned} (\Delta d/d)^2 &= (\Delta \lambda / \lambda)^2 + (\cot \theta \Delta \theta)^2 \\ &= (\Delta t / t)^2 + (\cot \theta \Delta \theta)^2 \end{aligned}$$

choose to match resolution terms for  $2\theta = 90^\circ$



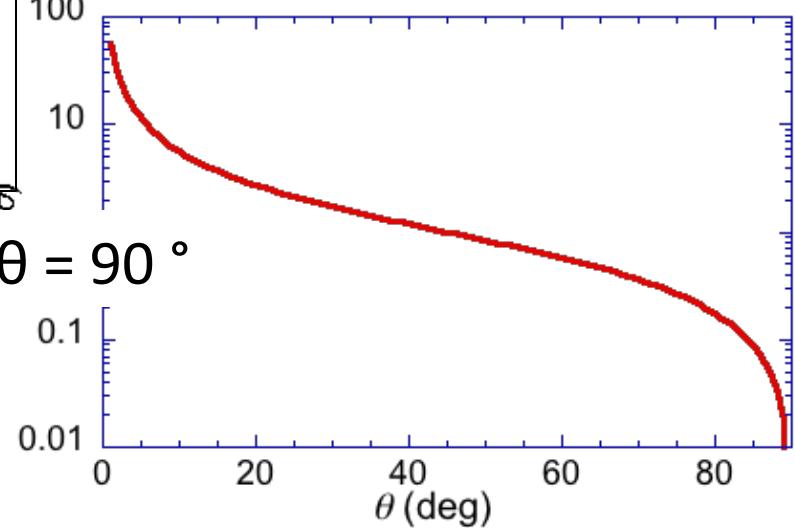
# Example: ESS Powder Diffractometer

$$d = \frac{\lambda}{2 \sin \theta}$$

$$\Delta d^2 = \left( \frac{\partial d}{\partial \lambda} \Delta \lambda \right)^2 + \left( \frac{\partial d}{\partial \theta} \Delta \theta \right)^2$$

$$\begin{aligned} (\Delta d/d)^2 &= (\Delta \lambda / \lambda)^2 + (\cot \theta \Delta \theta)^2 \\ &= (\Delta t / t)^2 + (\cot \theta \Delta \theta)^2 \\ &= (\Delta t / t)^2 + \Delta \theta^2 \end{aligned}$$

choose to match resolution terms for  $2\theta = 90^\circ$



# Example: ESS Powder Diffractometer

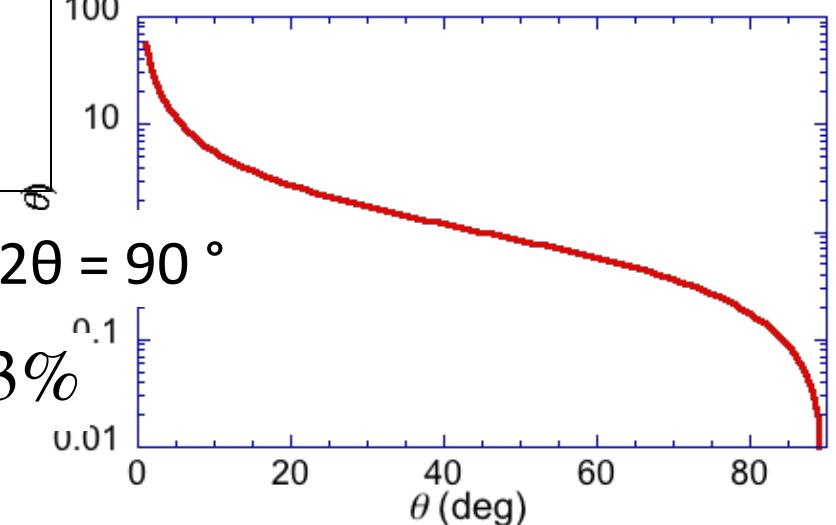
$$d = \frac{\lambda}{2 \sin \theta}$$

$$\Delta d^2 = \left( \frac{\partial d}{\partial \lambda} \Delta \lambda \right)^2 + \left( \frac{\partial d}{\partial \theta} \Delta \theta \right)^2$$

$$\begin{aligned} (\Delta d/d)^2 &= (\Delta \lambda / \lambda)^2 + (\cot \theta \Delta \theta)^2 \\ &= (\Delta t / t)^2 + (\cot \theta \Delta \theta)^2 \\ &= (\Delta t / t)^2 + \Delta \theta^2 \end{aligned}$$

choose to match resolution terms for  $2\theta = 90^\circ$

science case: optimise for  $\Delta d/d = 0.3\%$



# Example: ESS Powder Diffractometer

$$d = \frac{\lambda}{2 \sin \theta}$$

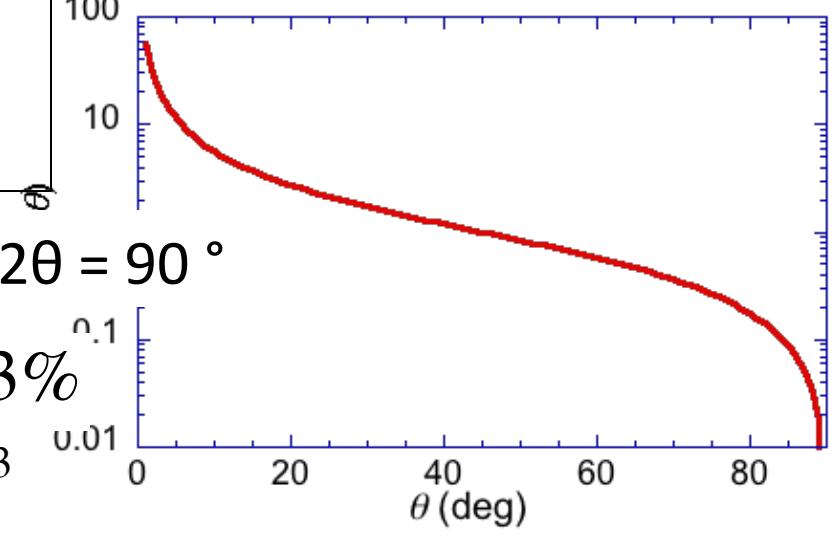
$$\Delta d^2 = \left( \frac{\partial d}{\partial \lambda} \Delta \lambda \right)^2 + \left( \frac{\partial d}{\partial \theta} \Delta \theta \right)^2$$

$$\begin{aligned} (\Delta d/d)^2 &= (\Delta \lambda / \lambda)^2 + (\cot \theta \Delta \theta)^2 \\ &= (\Delta t / t)^2 + (\cot \theta \Delta \theta)^2 \\ &= (\Delta t / t)^2 + \Delta \theta^2 \end{aligned}$$

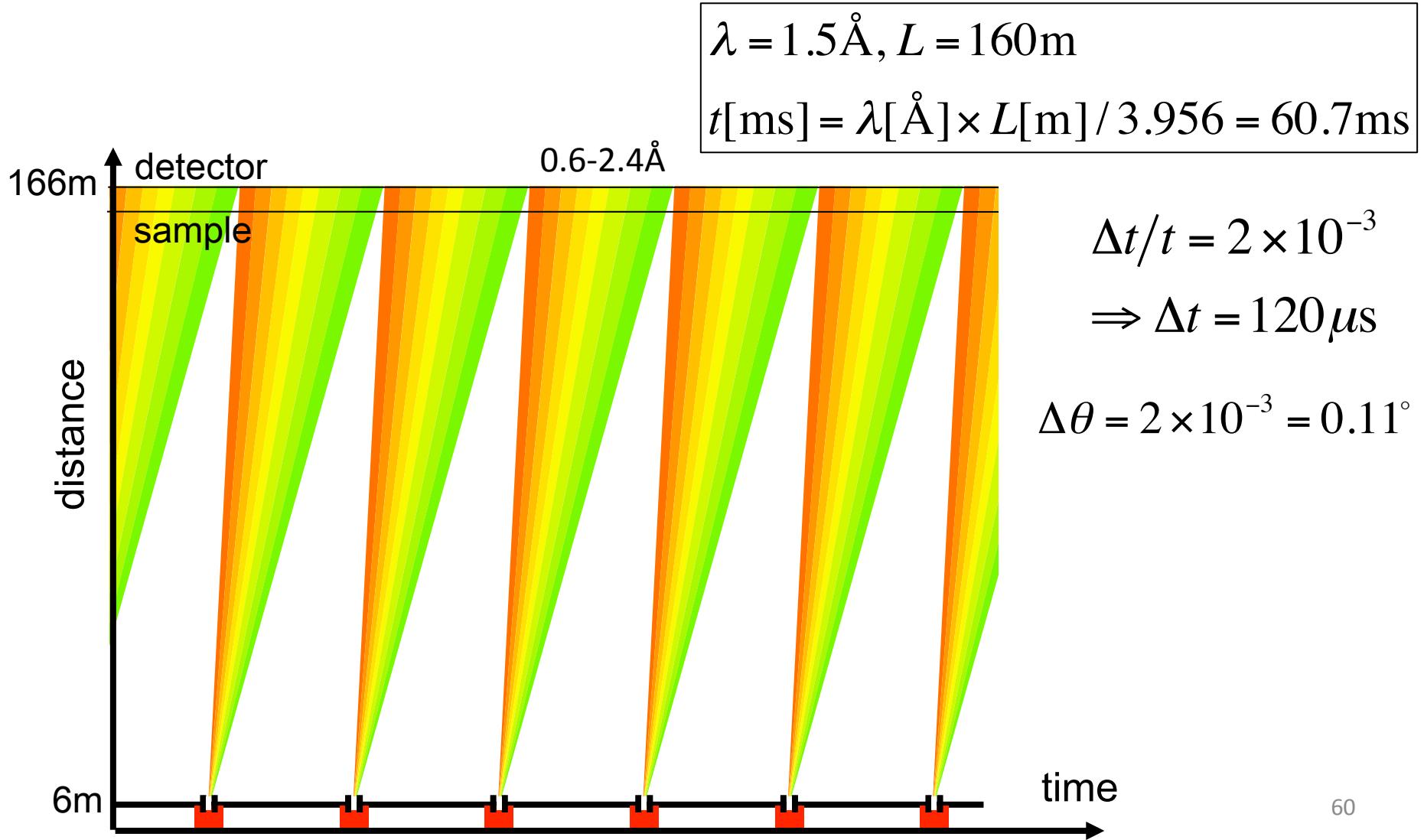
choose to match resolution terms for  $2\theta = 90^\circ$

science case: optimise for  $\Delta d/d = 0.3\%$

$\Rightarrow \Delta t/t = 2 \times 10^{-3}, \Delta \theta = 2 \times 10^{-3}$

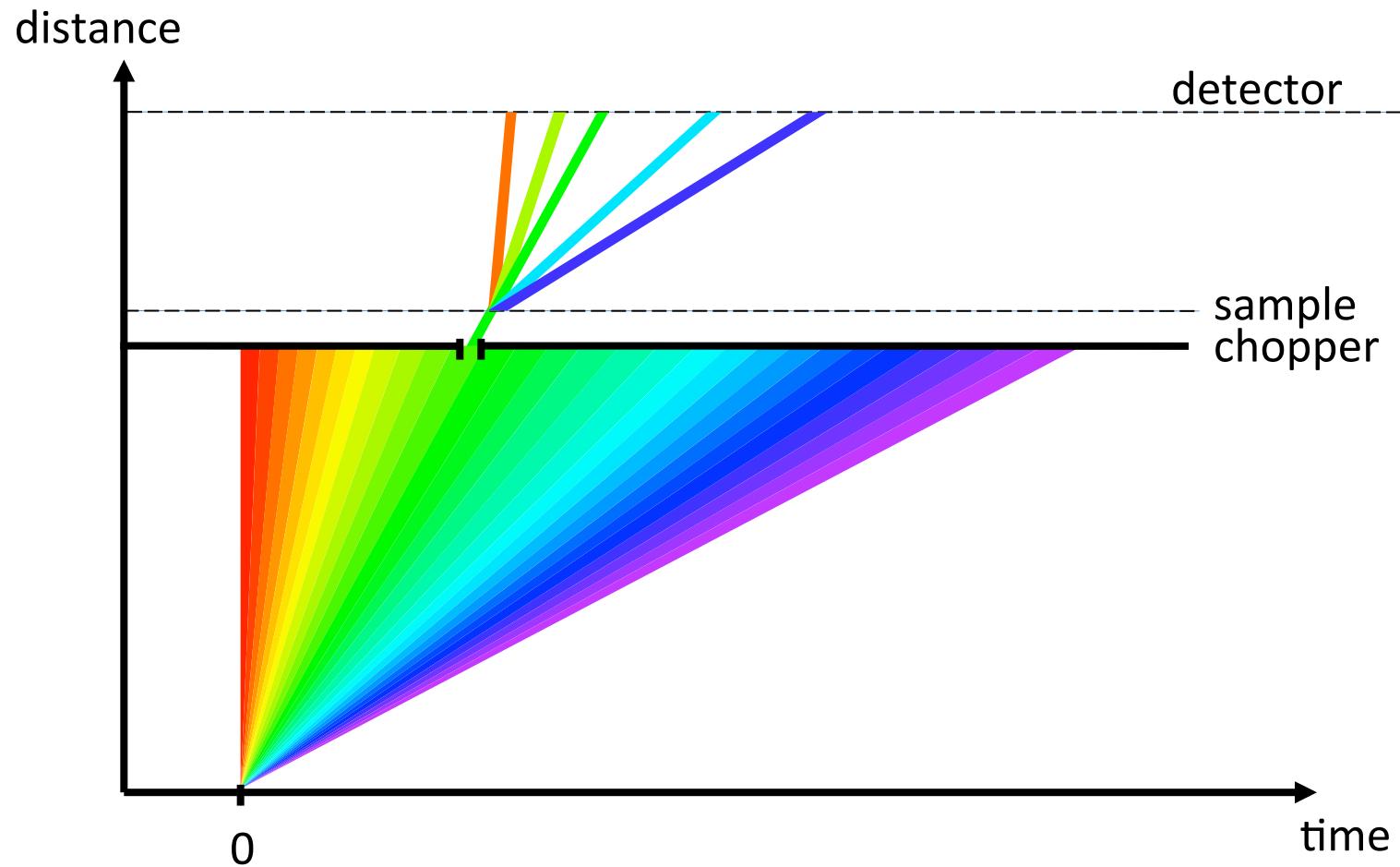


# Example: ESS Powder Diffractometer



# Example: SNS STS Cold Chopper Spectrometer

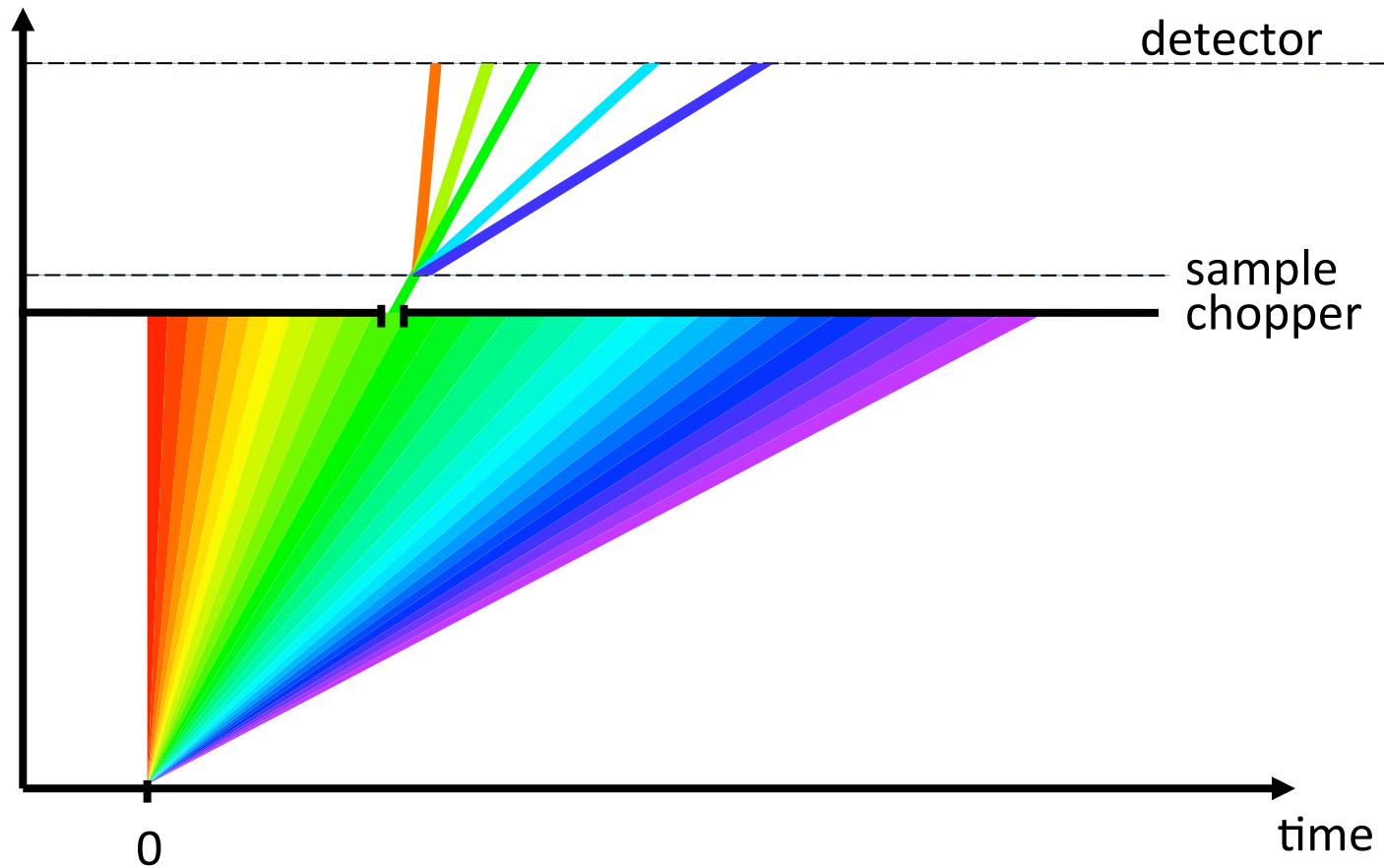
$$\hbar\omega = E_i - E_f$$



# Example: SNS STS Cold Chopper Spectrometer

$$\hbar\omega = E_i - E_f$$

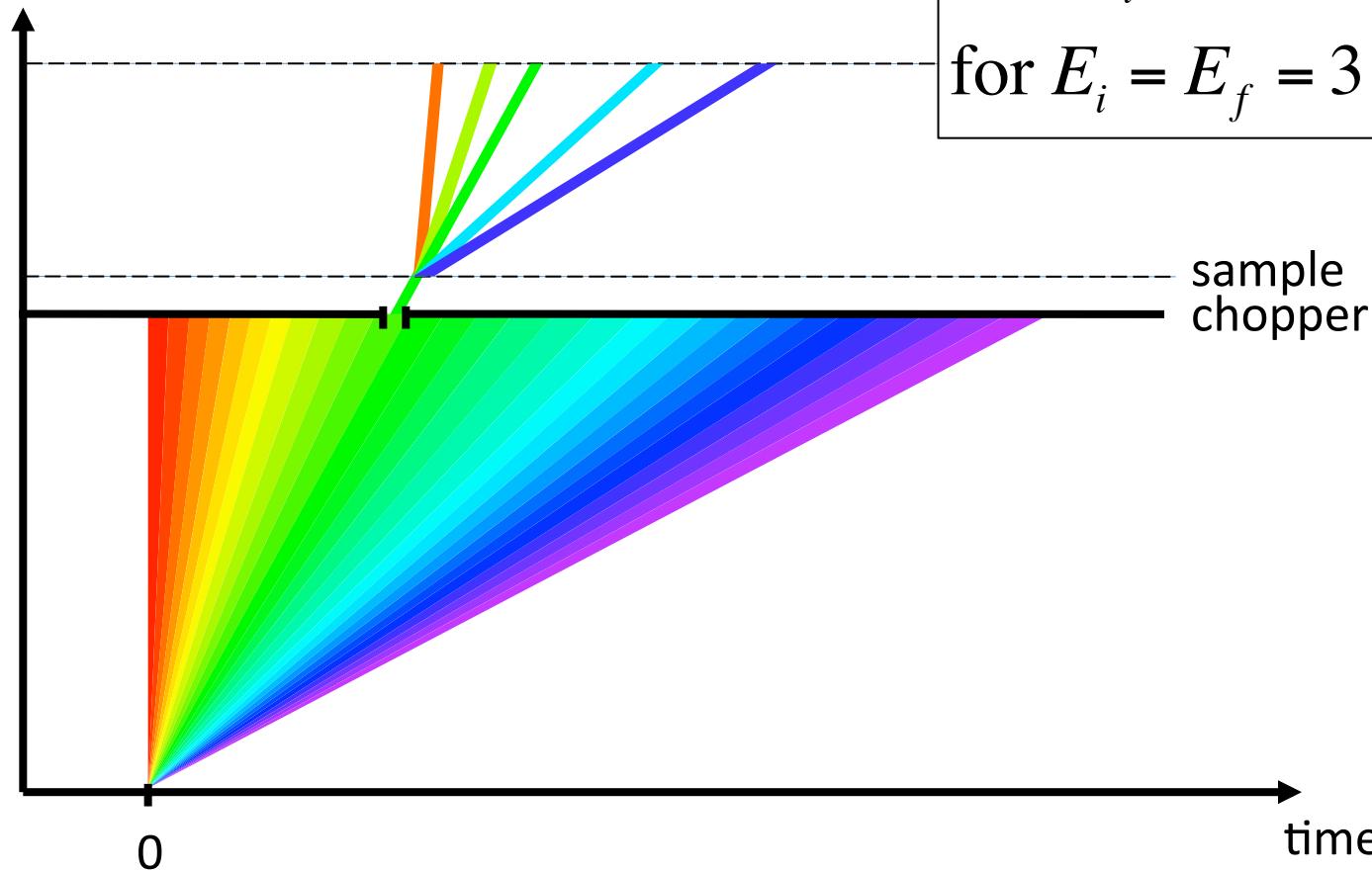
science case: “1% resolution at  $\lambda=5\text{\AA}$ ”



# Example: SNS STS Cold Chopper Spectrometer

$$\hbar\omega = E_i - E_f$$

science case: “1% resolution at  $\lambda=5\text{\AA}$ ”



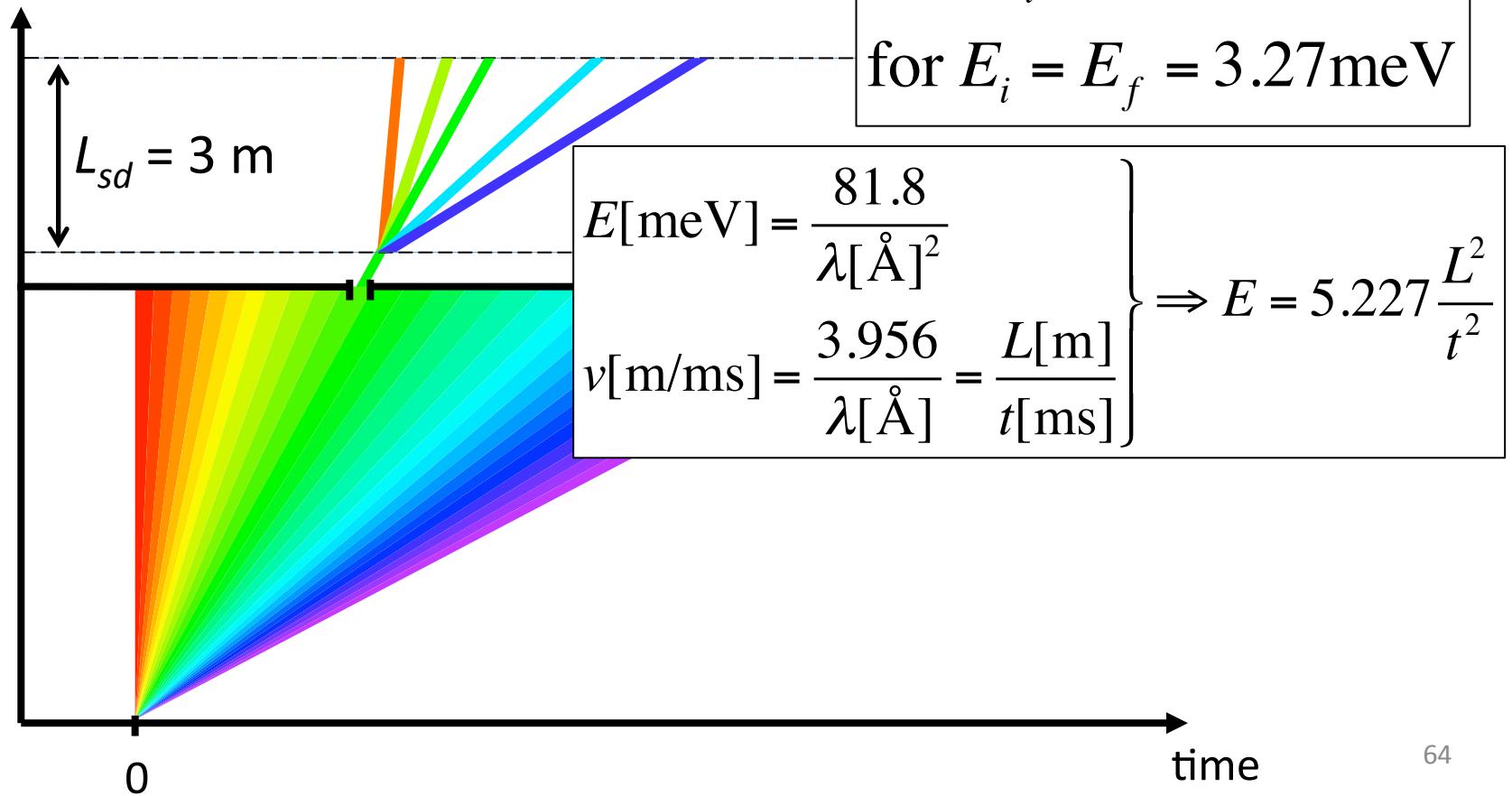
$$\Rightarrow \frac{\Delta\hbar\omega}{E_i} = 0.01$$

$$\text{for } E_i = E_f = 3.27\text{meV}$$

# Example: SNS STS Cold Chopper Spectrometer

$$\hbar\omega = E_i - E_f$$

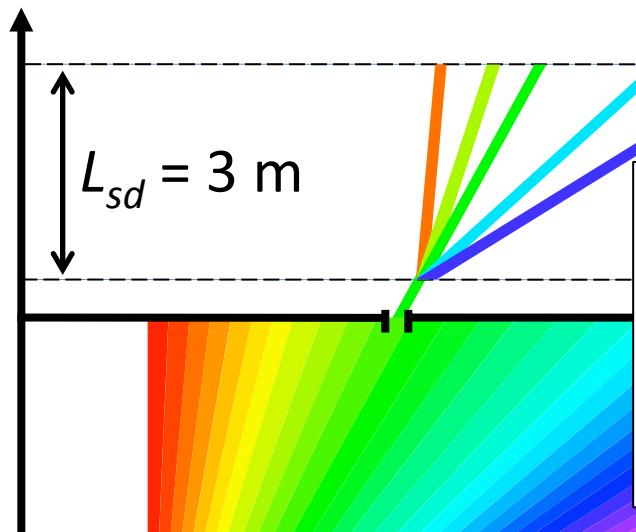
science case: “1% resolution at  $\lambda=5\text{\AA}$ ”



# Example: SNS STS Cold Chopper Spectrometer

$$\hbar\omega = E_i - E_f$$

science case: “1% resolution at  $\lambda=5\text{\AA}$ ”



$$\Rightarrow \frac{\Delta\hbar\omega}{E_i} = 0.01$$

$$\text{for } E_i = E_f = 3.27\text{ meV}$$

$$E[\text{meV}] = \frac{81.8}{\lambda[\text{\AA}]^2}$$

$$v[\text{m/ms}] = \frac{3.956}{\lambda[\text{\AA}]} = \frac{L[\text{m}]}{t[\text{ms}]}$$

$$\Rightarrow E = 5.227 \frac{L^2}{t^2}$$

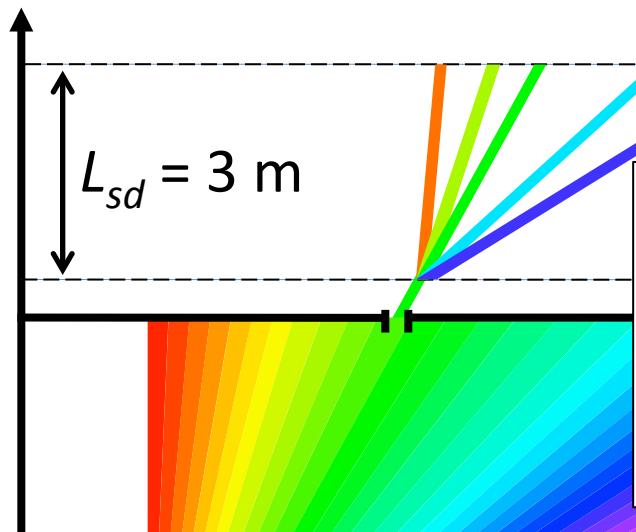
$$\frac{\Delta E}{E_i} = -2 \frac{\Delta t}{t} = \frac{\sqrt{E_i[\text{meV}]}}{1142 L_{sd}[\text{m}]} \Delta t_{\text{det}}[\mu\text{s}]$$

M. Arai et al., ICANS-X Proc. 297 (1988)

# Example: SNS STS Cold Chopper Spectrometer

$$\hbar\omega = E_i - E_f$$

science case: “1% resolution at  $\lambda=5\text{\AA}$ ”



$$\Rightarrow \frac{\Delta\hbar\omega}{E_i} = 0.01$$

$$\text{for } E_i = E_f = 3.27\text{ meV}$$

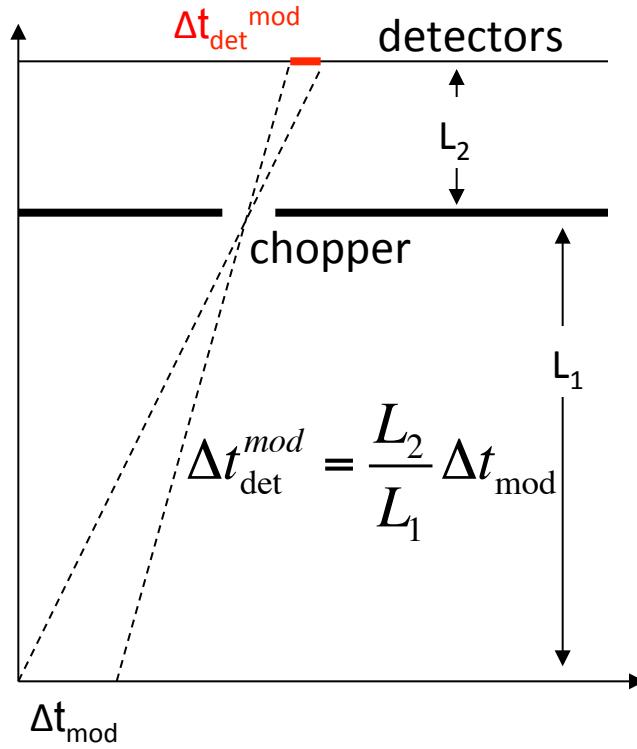
$$\left. \begin{aligned} E[\text{meV}] &= \frac{81.8}{\lambda[\text{\AA}]^2} \\ v[\text{m/ms}] &= \frac{3.956}{\lambda[\text{\AA}]} = \frac{L[\text{m}]}{t[\text{ms}]} \end{aligned} \right\} \Rightarrow E = 5.227 \frac{L^2}{t^2}$$

$$\frac{\Delta E}{E_i} = -2 \frac{\Delta t}{t} = \frac{\sqrt{E_i[\text{meV}]}}{1142 L_{sd}[\text{m}]} \Delta t_{\text{det}}[\mu\text{s}] \Rightarrow \Delta t_{\text{det}} = 19\mu\text{s}$$

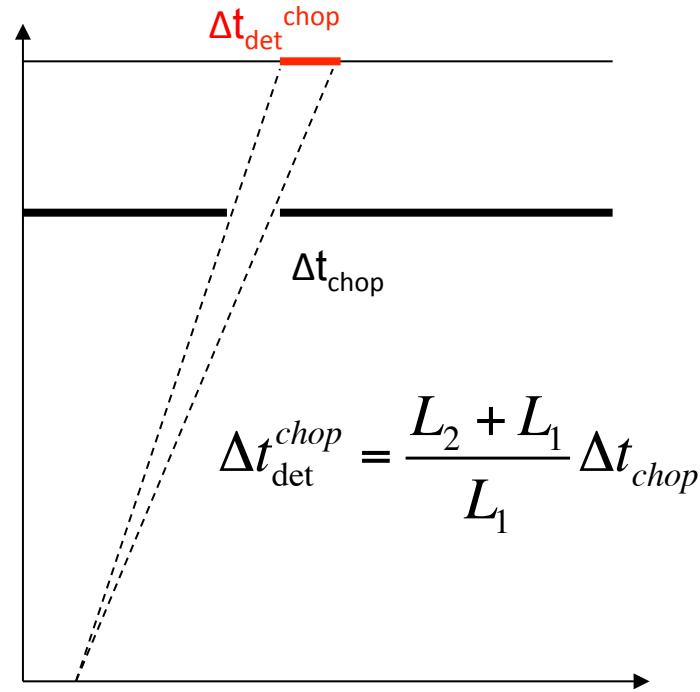
M. Arai et al., ICANS-X Proc. 297 (1988)

# Example: SNS STS Cold Chopper Spectrometer

$$\Delta t_{\text{det}} = \sqrt{(\Delta t_{\text{det}}^{\text{mod}})^2 + (\Delta t_{\text{det}}^{\text{chop}})^2}$$



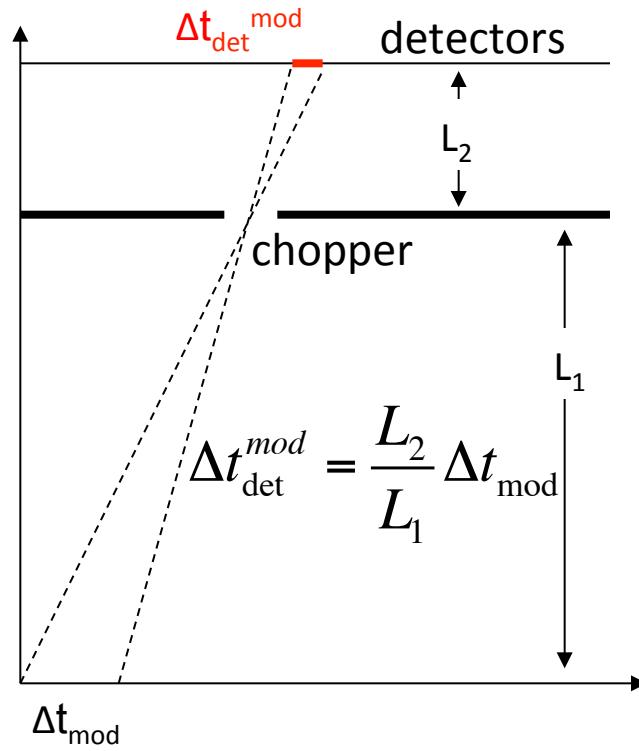
$$\Delta t_{\text{det}}^{\text{mod}} = \frac{L_2}{L_1} \Delta t_{\text{mod}}$$



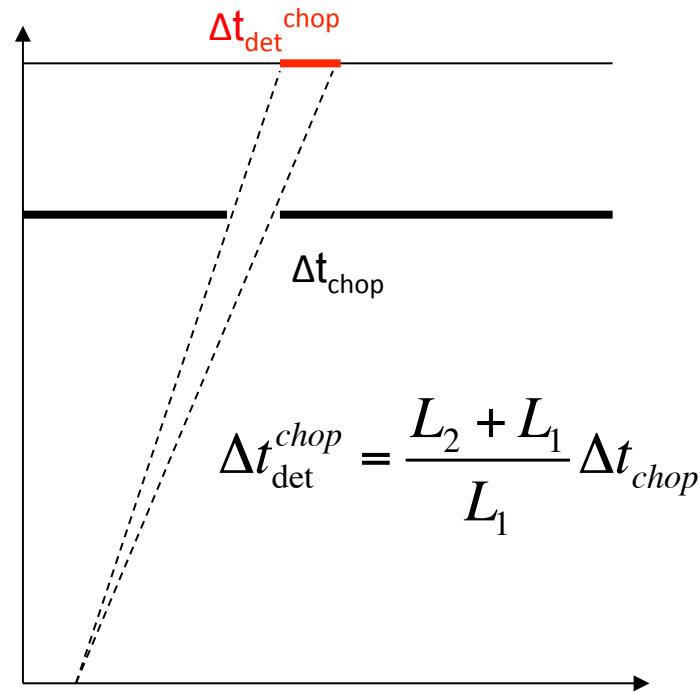
$$\Delta t_{\text{det}}^{\text{chop}} = \frac{L_2 + L_1}{L_1} \Delta t_{\text{chop}}$$

# Example: SNS STS Cold Chopper Spectrometer

$$\Delta t_{\text{det}} = \sqrt{(\Delta t_{\text{det}}^{\text{mod}})^2 + (\Delta t_{\text{det}}^{\text{chop}})^2}$$



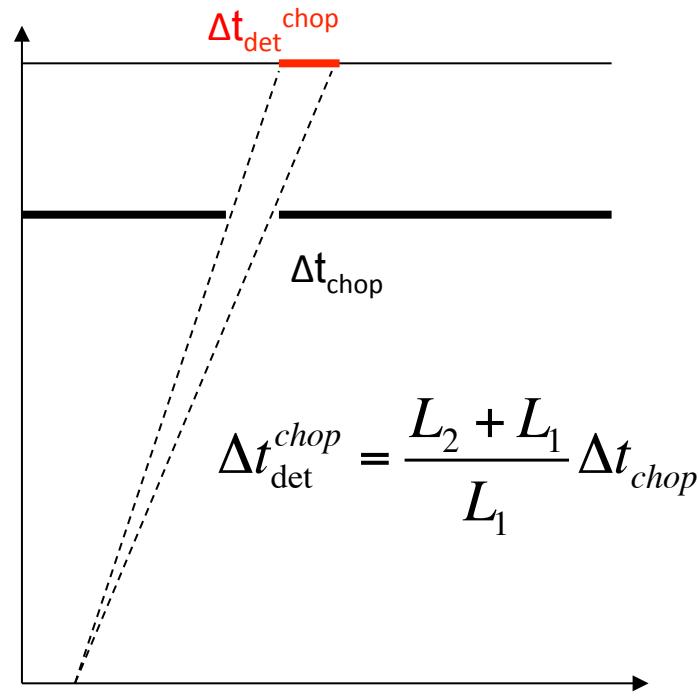
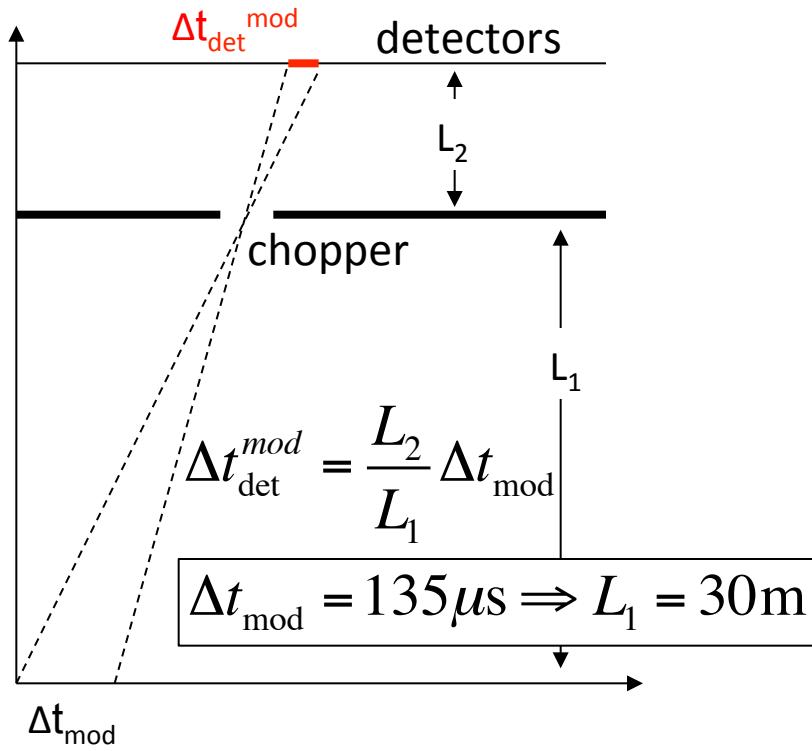
$$\Delta t_{\text{det}}^{\text{mod}} = \Delta t_{\text{det}}^{\text{chop}} = \frac{1}{\sqrt{2}} 19 \mu\text{s} = 13.4 \mu\text{s}$$



# Example: SNS STS Cold Chopper Spectrometer

$$\Delta t_{\text{det}} = \sqrt{(\Delta t_{\text{det}}^{\text{mod}})^2 + (\Delta t_{\text{det}}^{\text{chop}})^2}$$

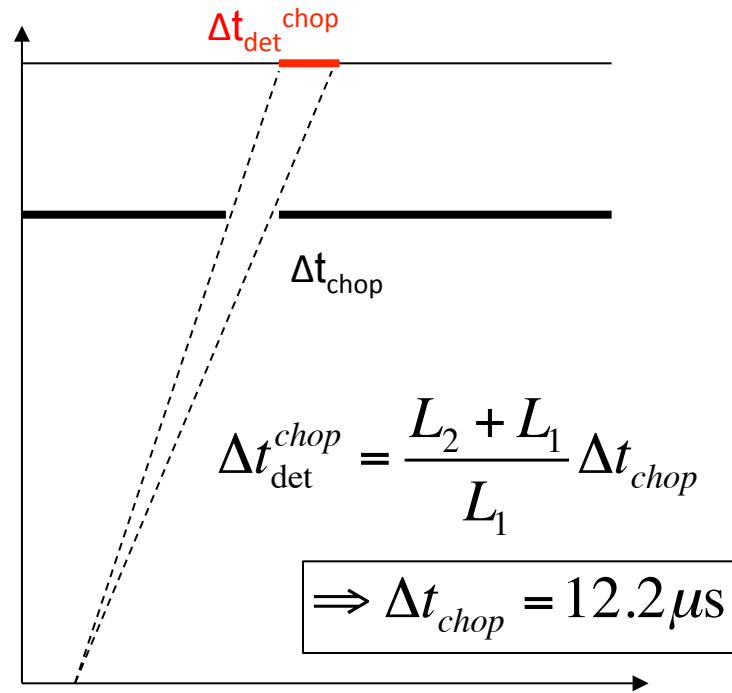
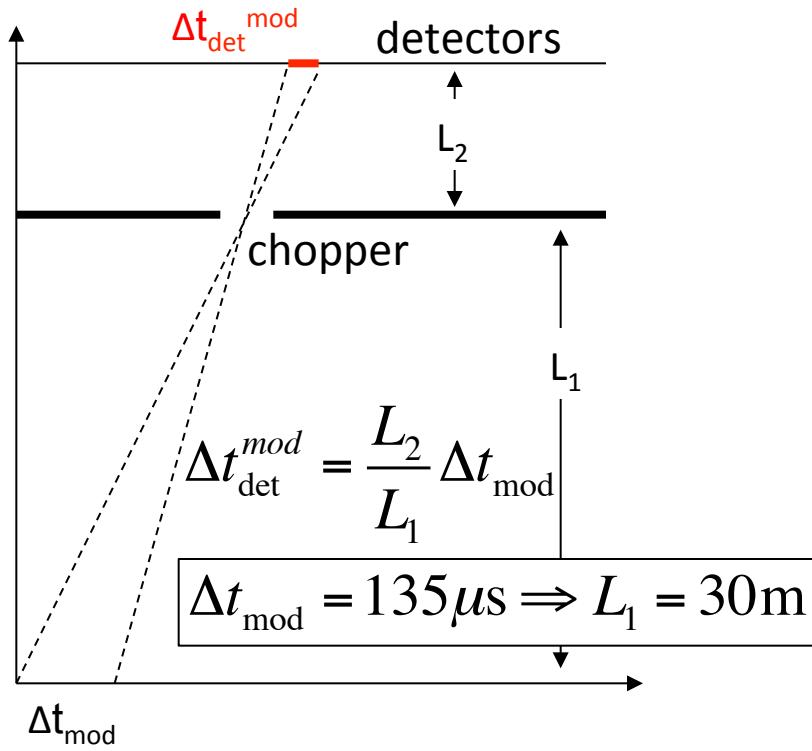
$$\Delta t_{\text{det}}^{\text{mod}} = \Delta t_{\text{det}}^{\text{chop}} = \frac{1}{\sqrt{2}} 19 \mu\text{s} = 13.4 \mu\text{s}$$



# Example: SNS STS Cold Chopper Spectrometer

$$\Delta t_{\text{det}} = \sqrt{(\Delta t_{\text{det}}^{\text{mod}})^2 + (\Delta t_{\text{det}}^{\text{chop}})^2}$$

$$\Delta t_{\text{det}}^{\text{mod}} = \Delta t_{\text{det}}^{\text{chop}} = \frac{1}{\sqrt{2}} 19 \mu\text{s} = 13.4 \mu\text{s}$$



# Summary

- Peak vs Time-Average Brightness
- Brilliance Transfer
  - if you don't know it, set it 50%
  - very useful metric when evaluating simulations
- Calculation of flux given instrument parameters
- Flux at sample: 
$$\Phi = \int_{\lambda_{\min}}^{\lambda_{\max}} B_{av} d\lambda \times \Delta\Omega$$
- Resolution calculations
  - calculation of instrument parameters given resolution requirement
  - formulate the measurement requirement
  - calculate partial differentials
  - match resolution contributions
  - example: ESS powder diffractometer
  - example: SNS STS chopper spectrometer

# Thank you!

