Neutron Optics Methods

"An Introduction to the Joys of Acceptance/Phase Space Diagrams"

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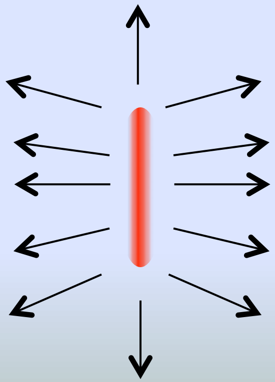
NIST

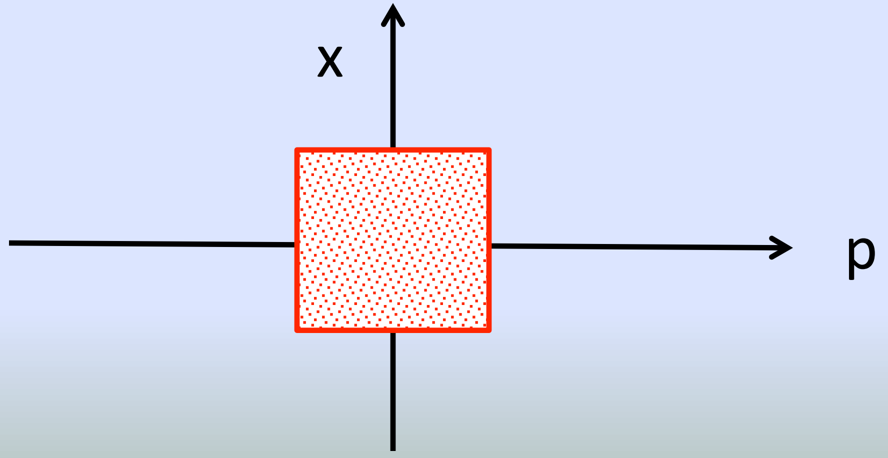
From John R. D. Copley "The Joy of Acceptance Diagrams", Journal of Neutron Research, Vol. 1, No. 2, pp. 21-36, 1993.

Phase Space

- Neutrons generated by the source can be parameterized by **p**, **x**, t, etc. This is a large dimensional space that is not possible to fully draw on paper, but consider the following as a useful representation for describing the acceptance of neutrons through slits, collimators, guides, etc.

Line source of neutrons



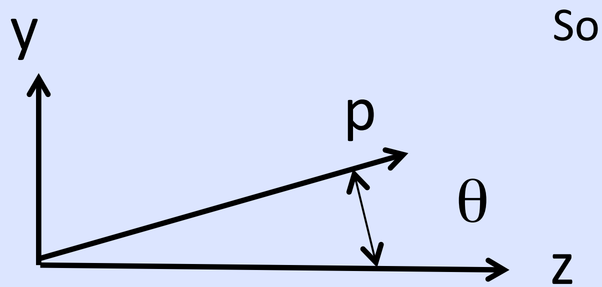


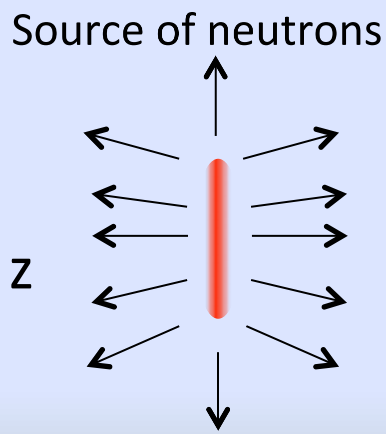
Liouville Theorem

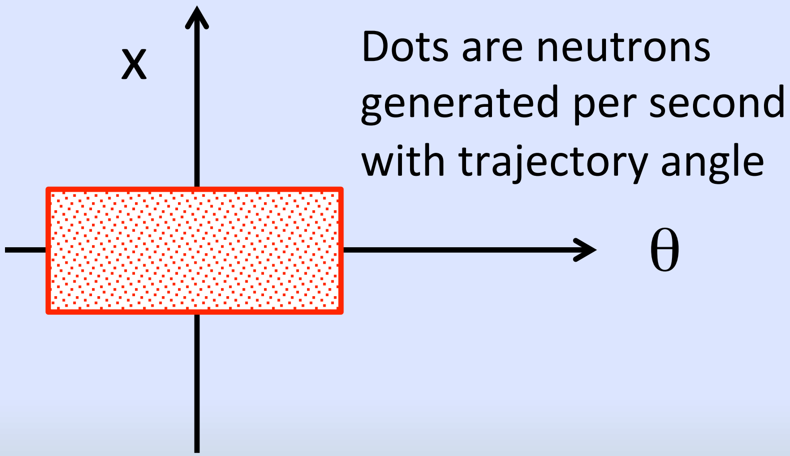
- Optical devices like mirrors and lenses are constrained by the Liouville theorem such that they can not increase the phase space density.
- Inefficiencies in the optical device can decrease the phase accepted phase space density.

Source

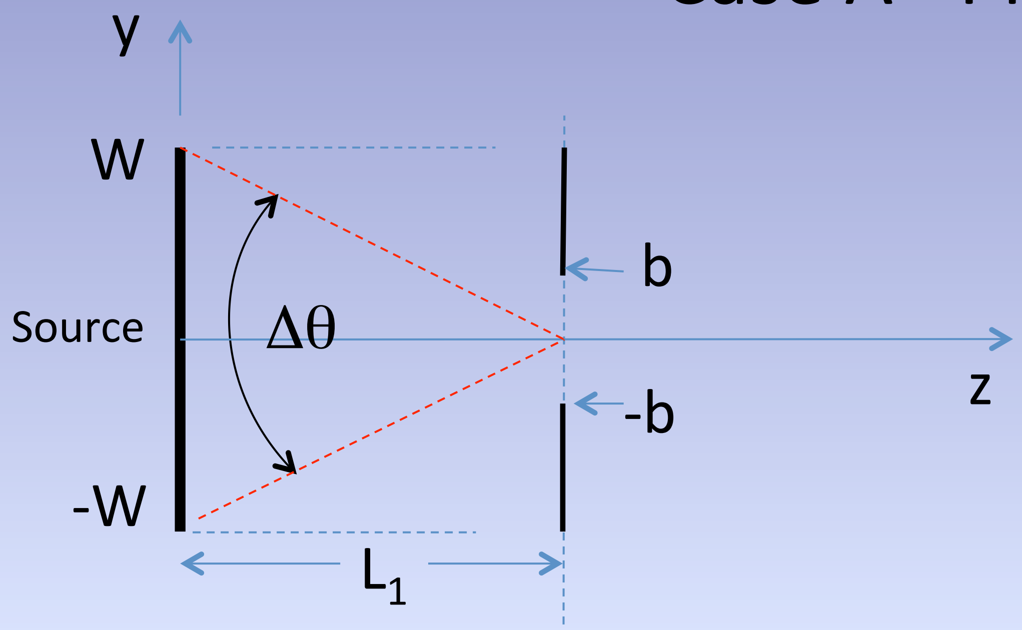
- Let's simplify things a bit more
  - Assume uniform emittance of source at all points in all directions
  - Use θ to describe the trajectory of the neutron
  - Trace rays through the system
  - The phase space is described by position coordinates and angle θ







Case A - Pinhole



Assume a square pinhole/slit and source.

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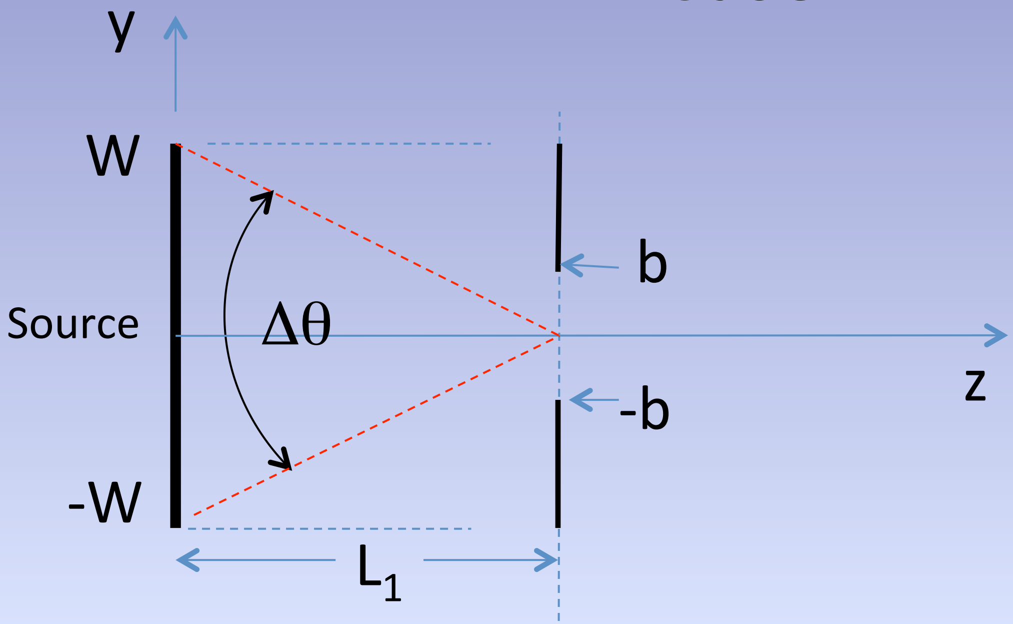

Assume a square pinhole/slit and source.
Assume small angle approximations are valid.

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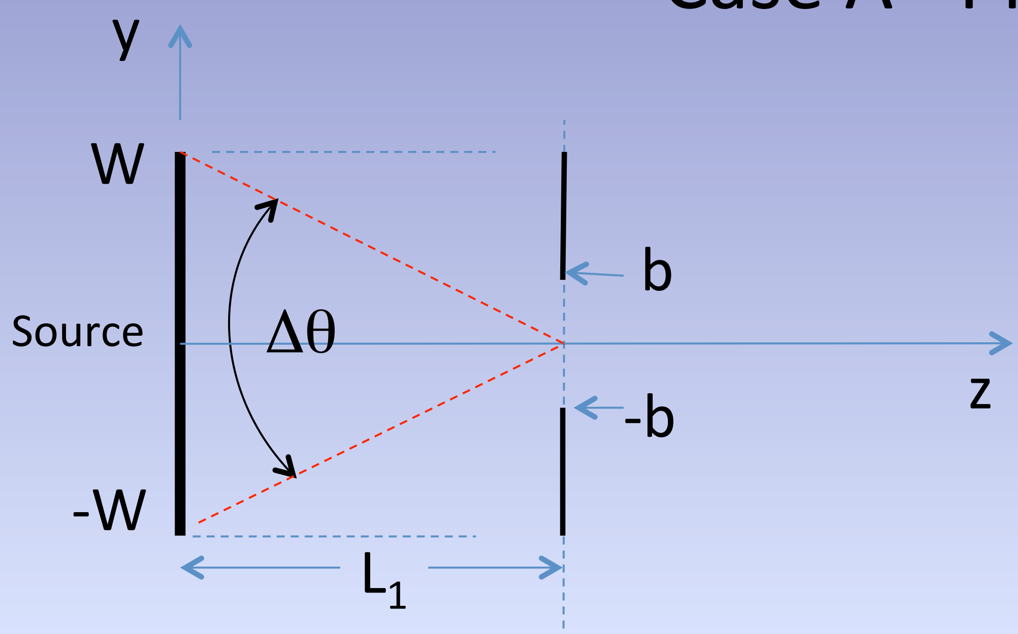

Assume a square pinhole/slit and source.
Assume small angle approximations are valid.
Intensity at a point in the plane of the slit is proportional to the solid angle subtended with the source.

$$\Delta\Omega = (2W/L_1)^2$$

$$B = d^4\phi/dA\,d\Omega\,dt\,d\lambda$$

$$\text{Fluence Rate} = B * \Delta\Omega$$

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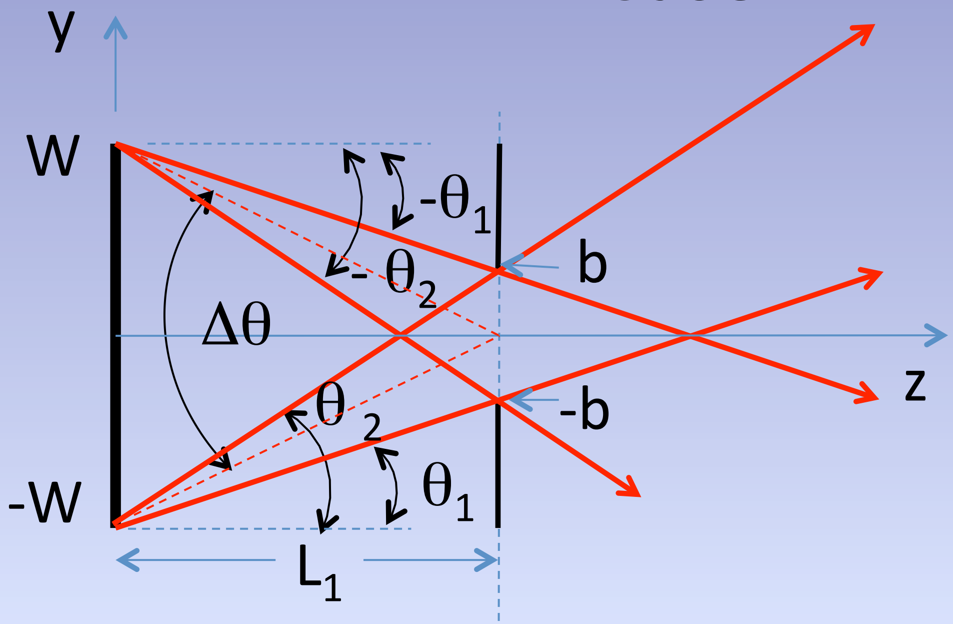

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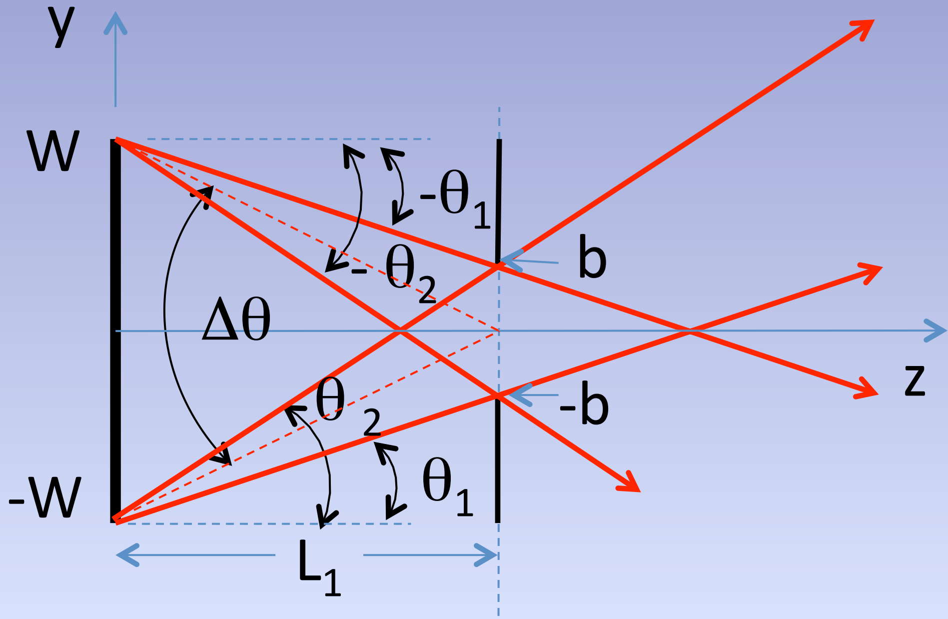

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$$B = d^4\phi/dA\,d\Omega\,dt\,d\lambda$$

$$\text{Fluence Rate} = B*\Delta\Omega$$

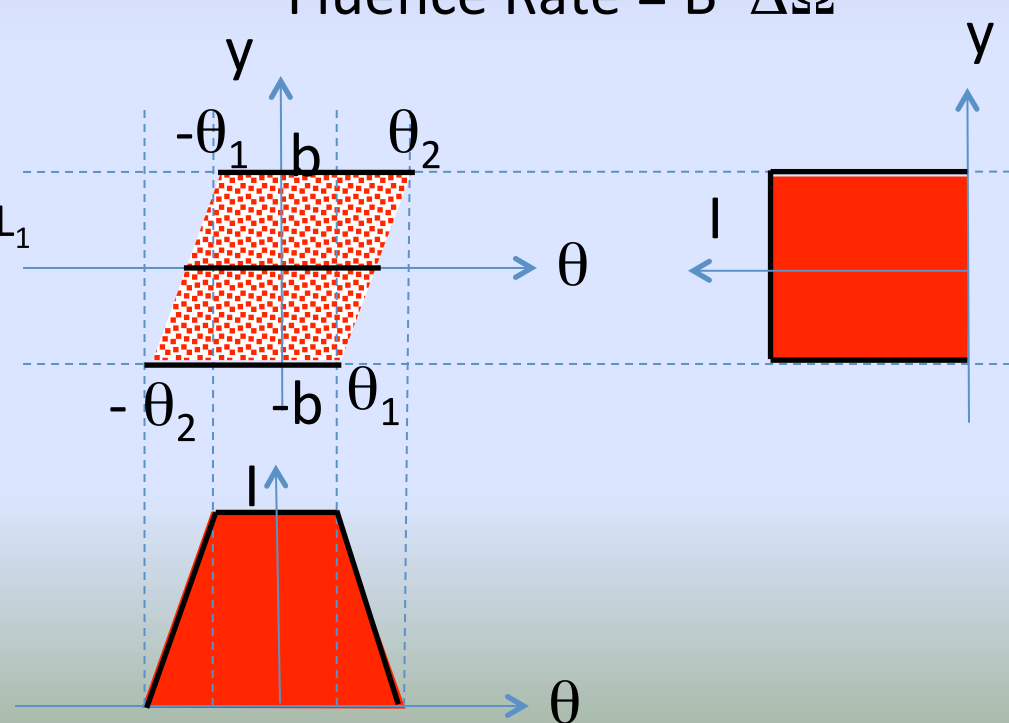
$$\theta_1 = \frac{W+b}{L_1}$$

$$\theta_2 = \frac{W+b}{L_1}$$

$$\Delta\theta = \frac{2W}{L_1}$$

$$\text{Area} = 4b*W/L_1$$

Phase space distribution accepted by pinhole at x = L_1



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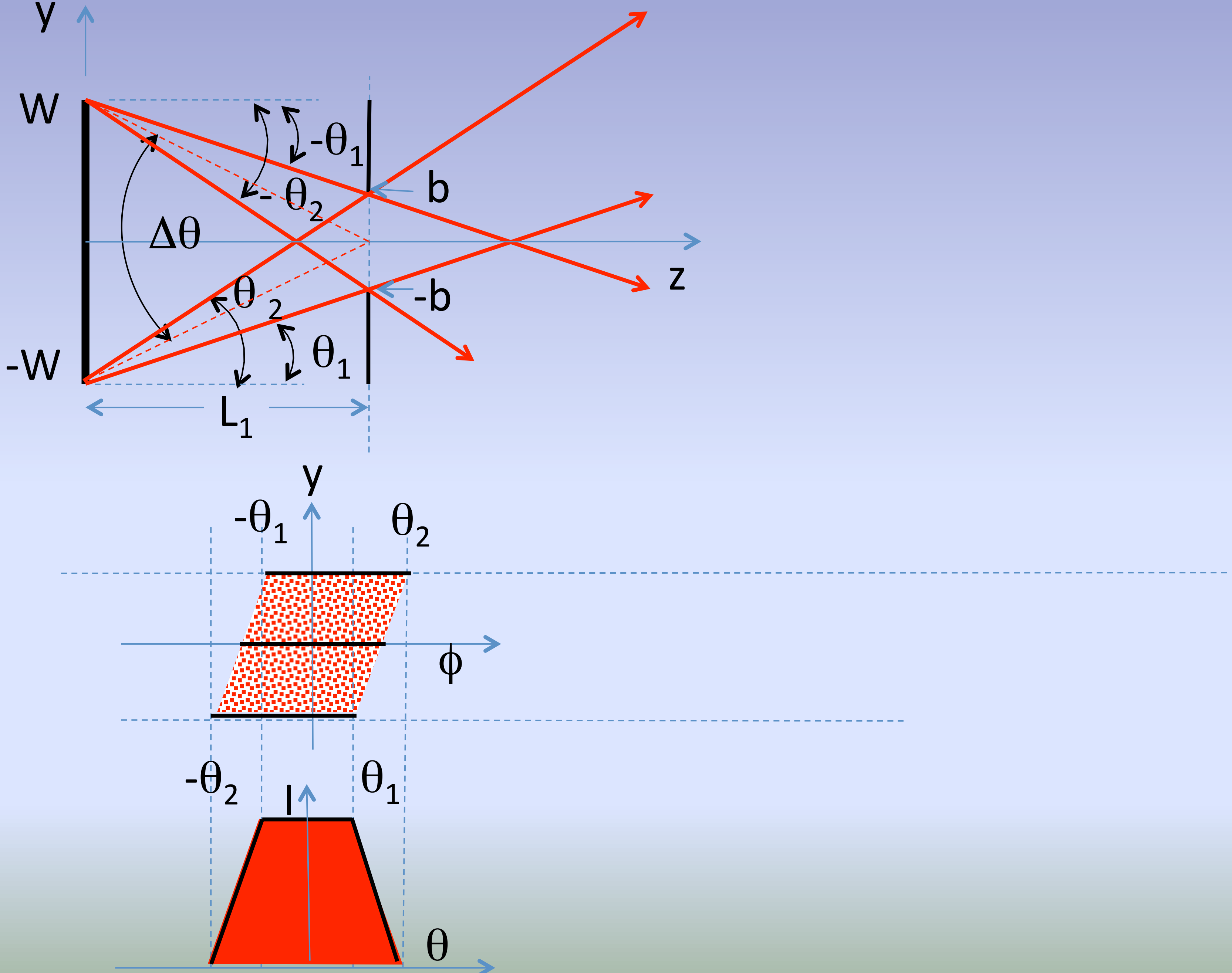

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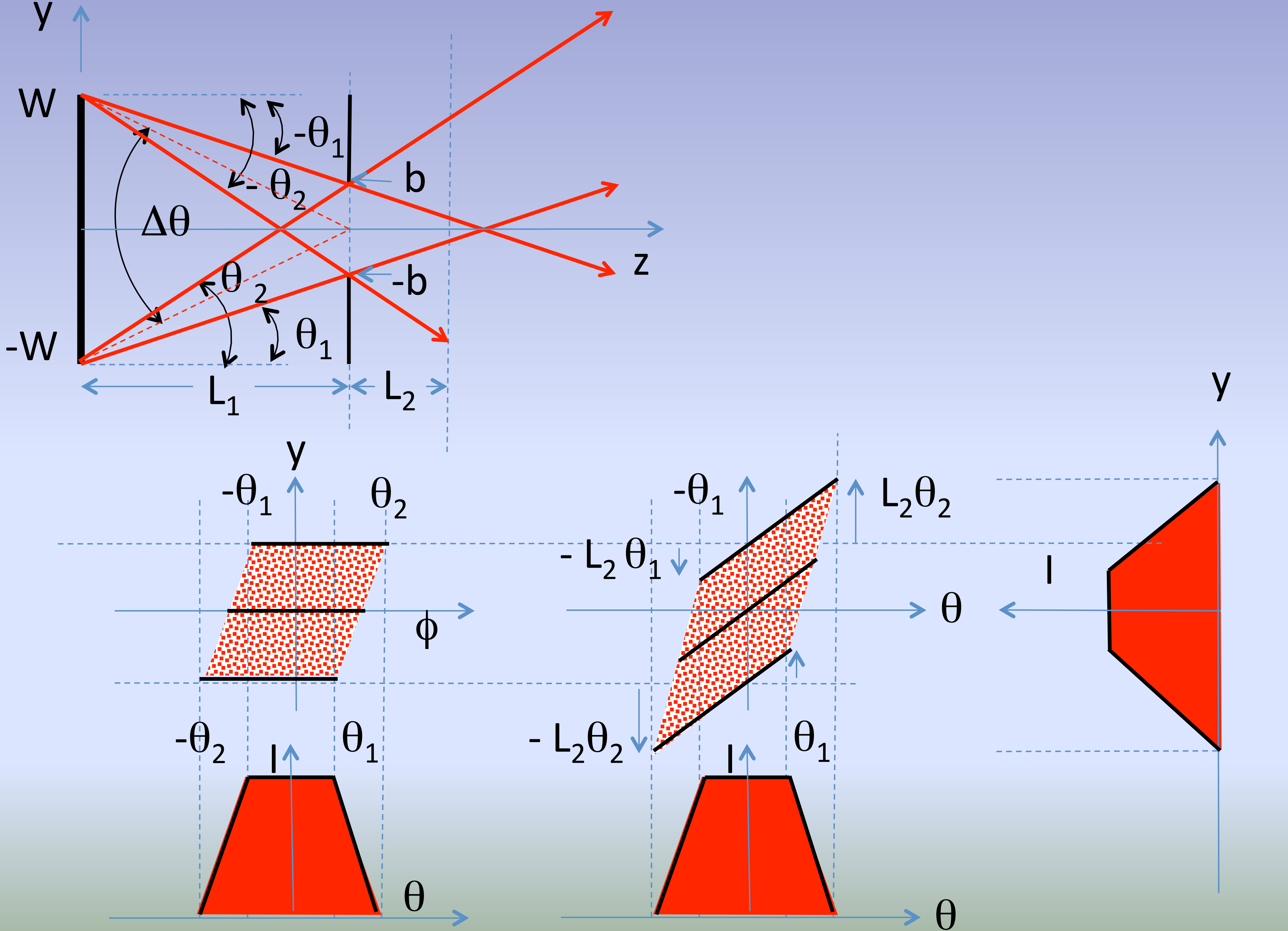

General rules

- Propagating in real space produces a shear of the phase space diagram but does not change the area of phase space diagram nor does it increase the phase space density.

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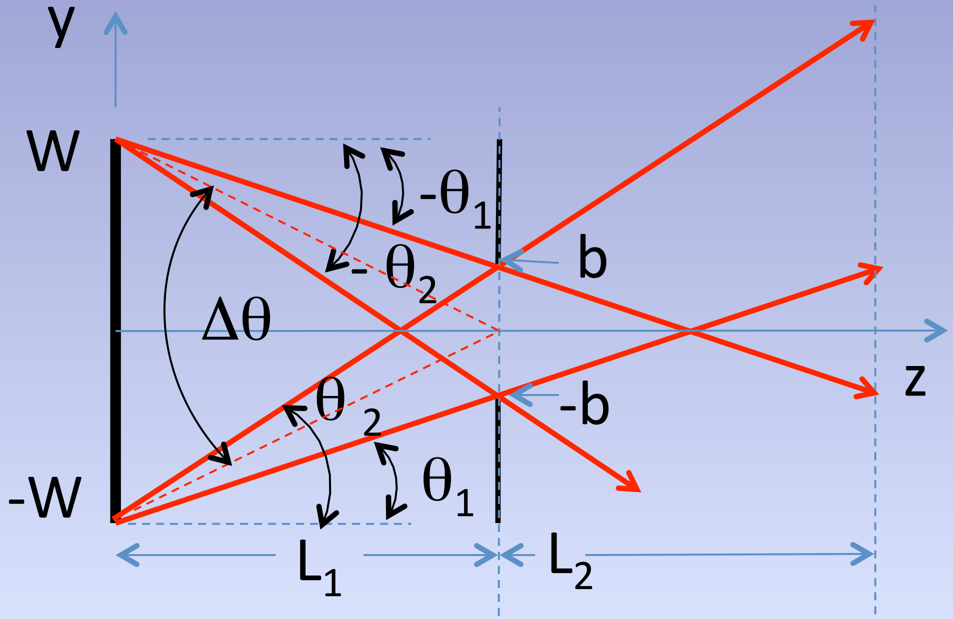
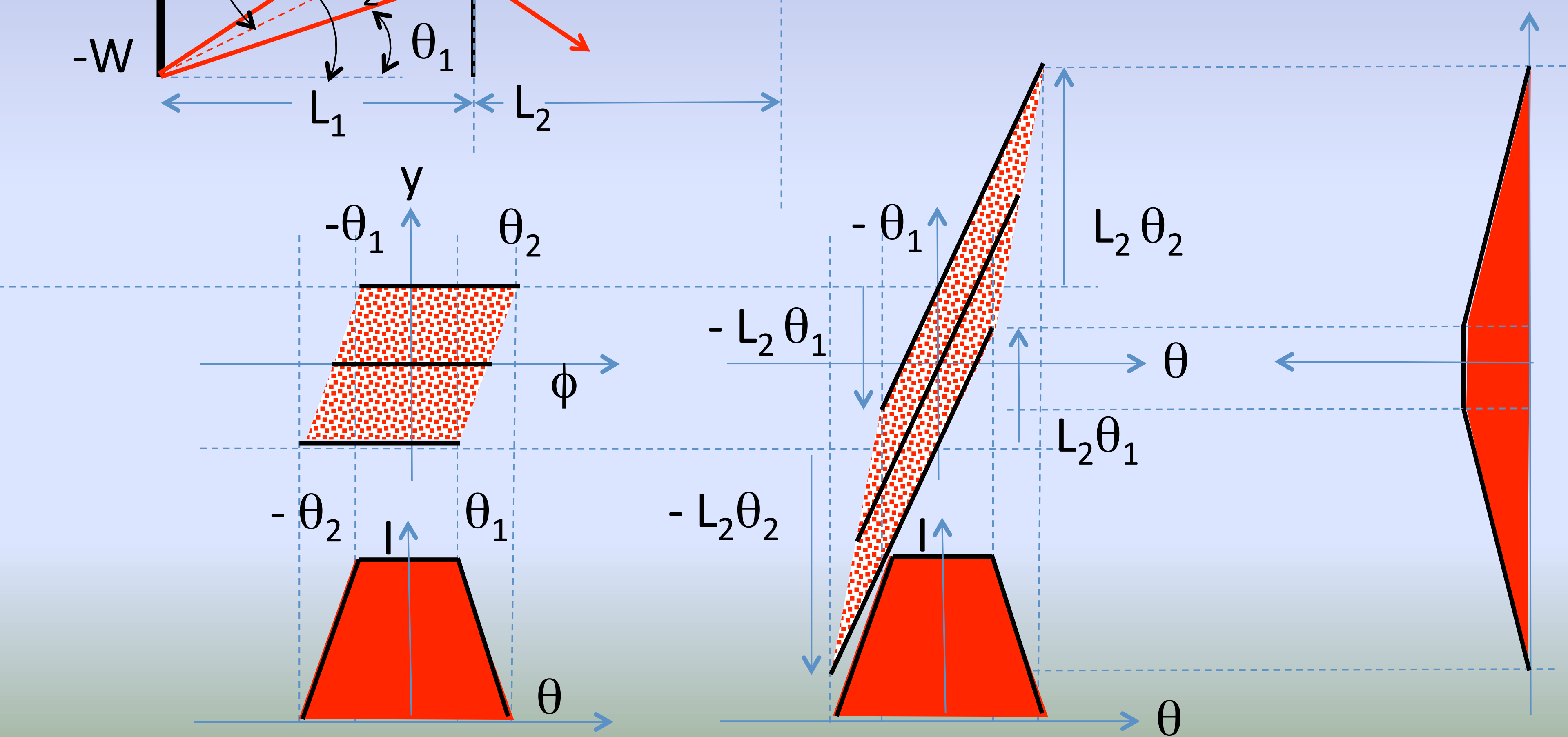
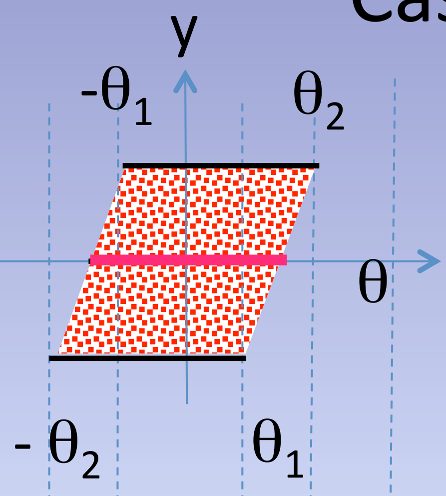


Propagating in real space produces a shear of the phase space diagram. Area of phase space is constant and the phase space density is constant.



Case A - Pinhole

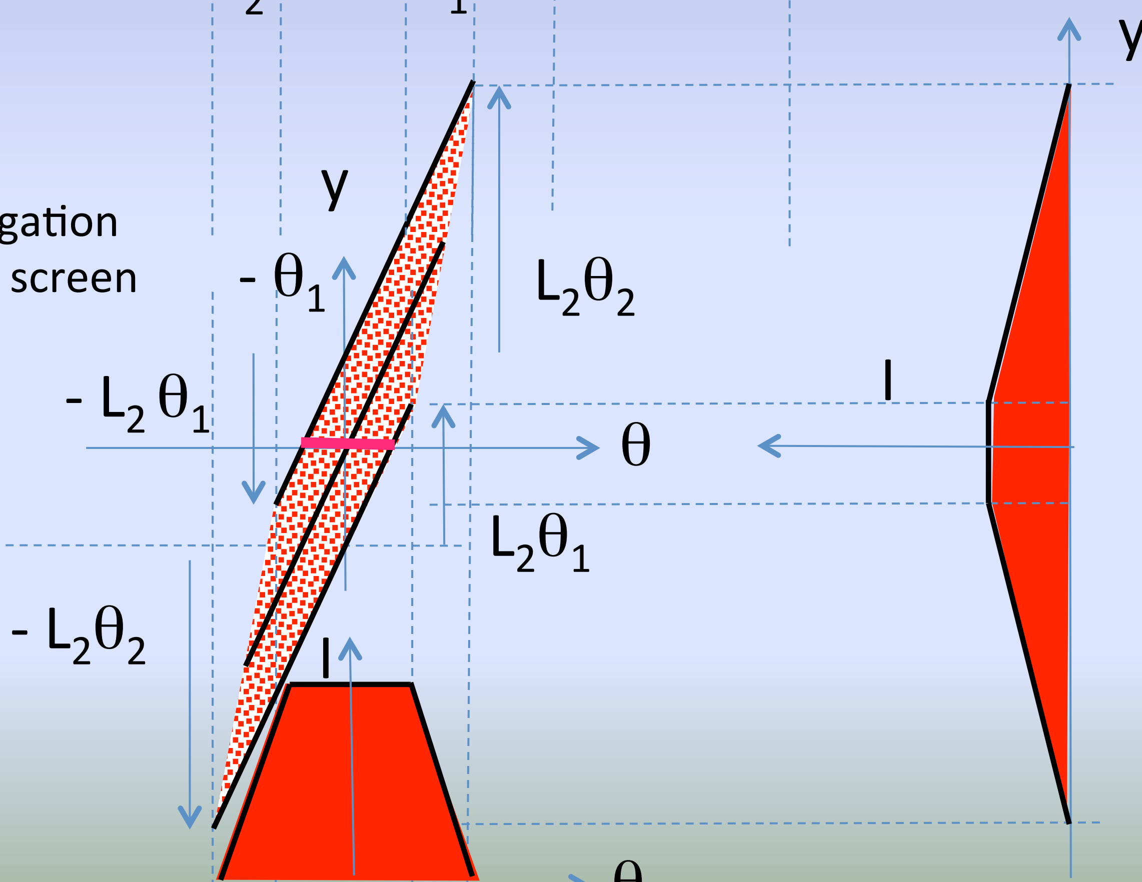
Entrance to the slit



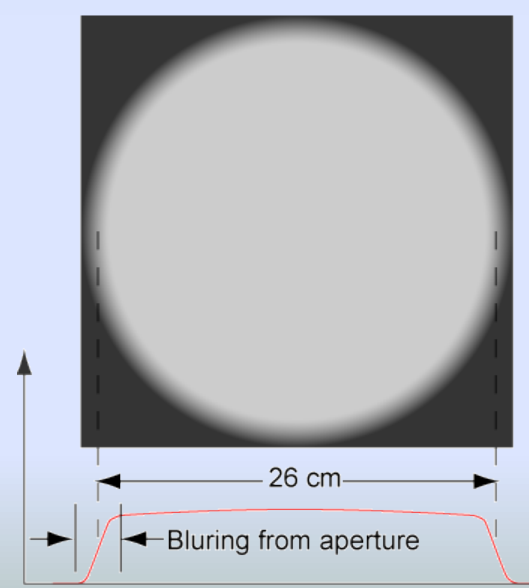
Density of phase space is the same however the integral over angle θ, which gives us the flux density, is down by the ratio of the length of the 2 magenta lines.

At L_2 the ratio = $(2b/L_2)^2$

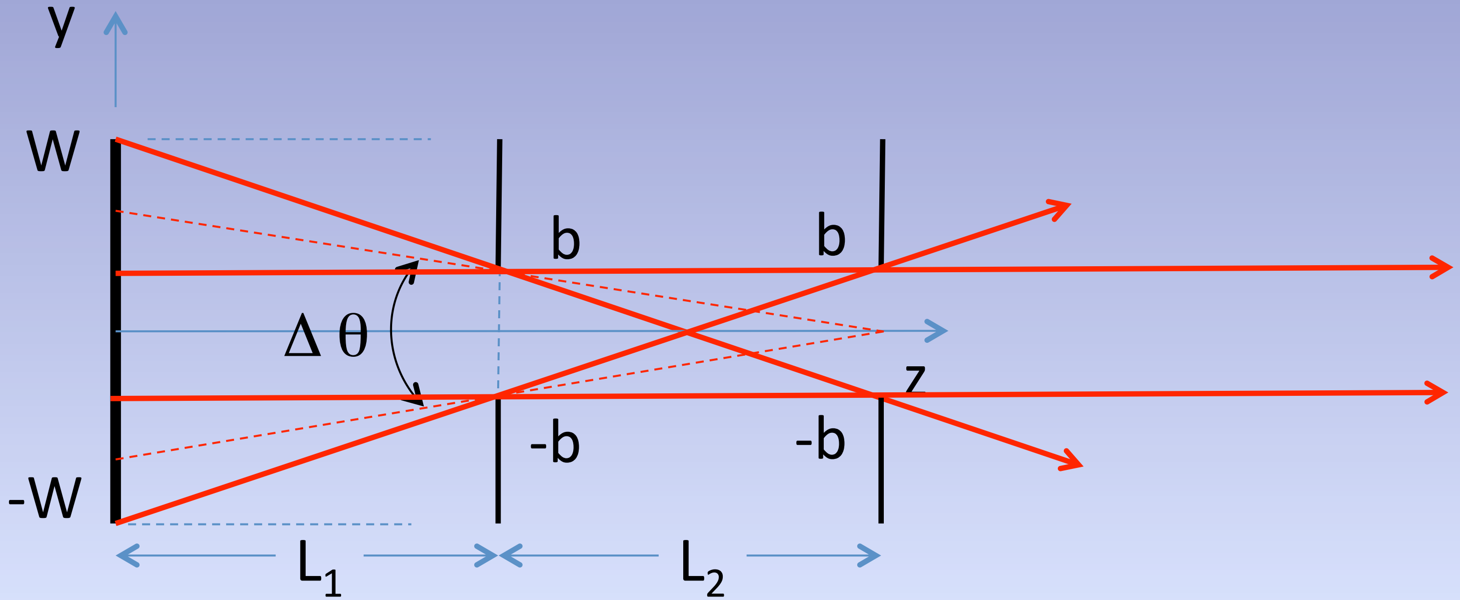
Propagation to the screen

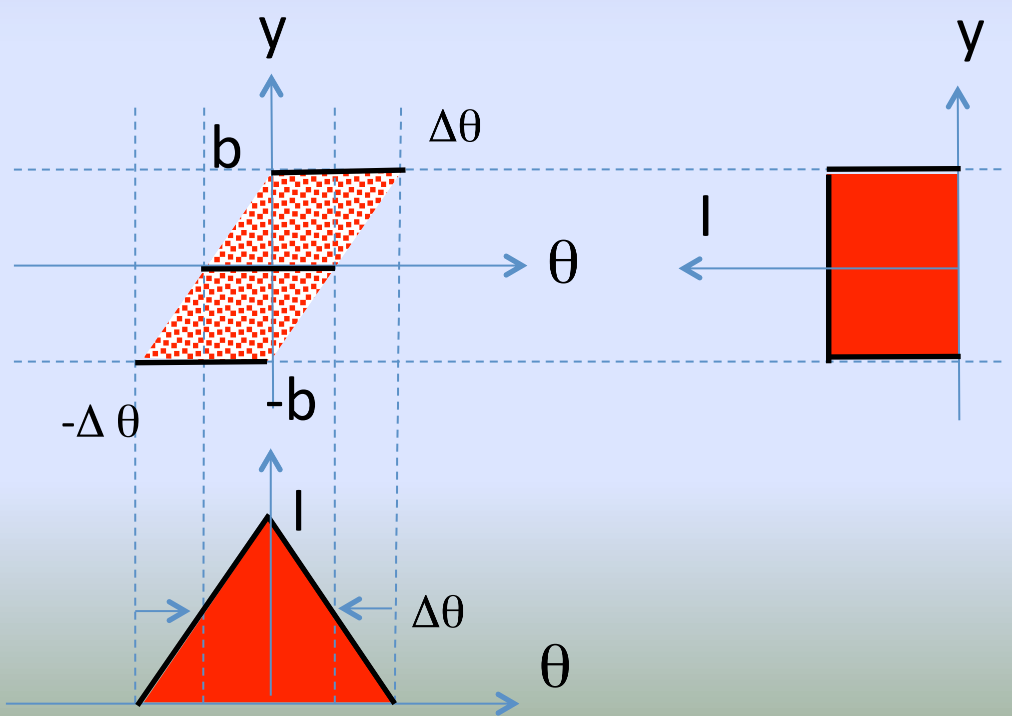


At 6 m



Case B – Double Slit





$$\Delta\theta = \frac{b}{L_1}$$

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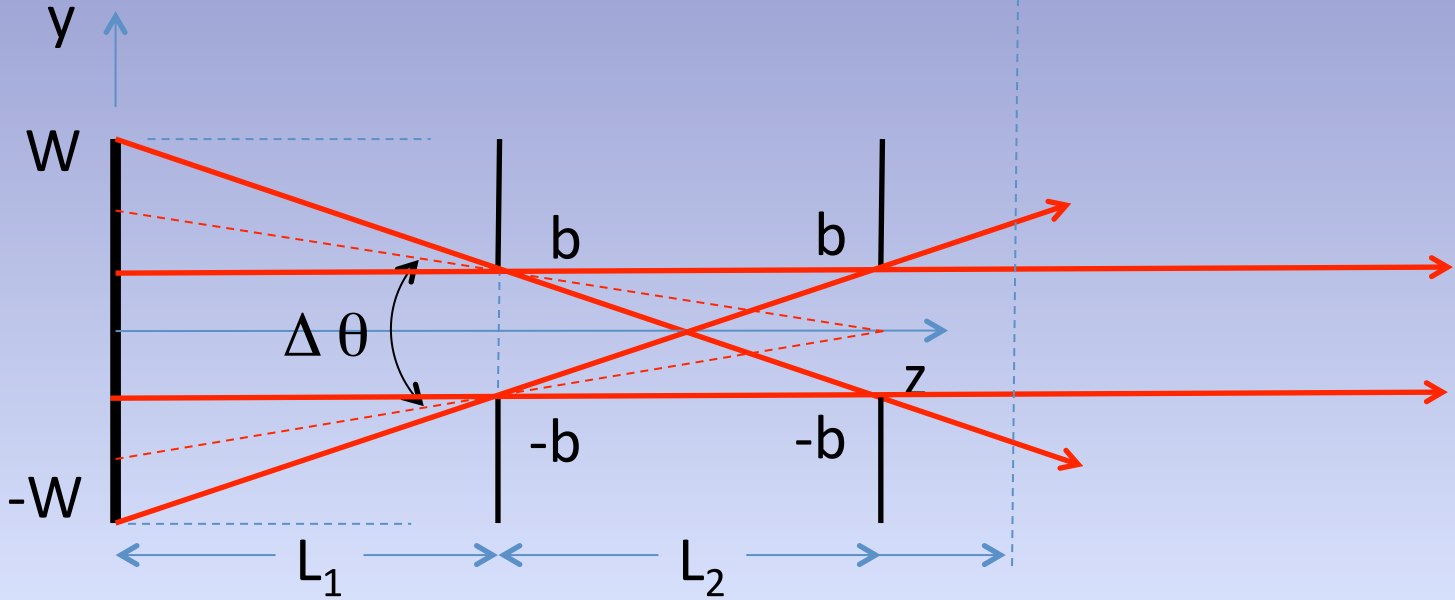

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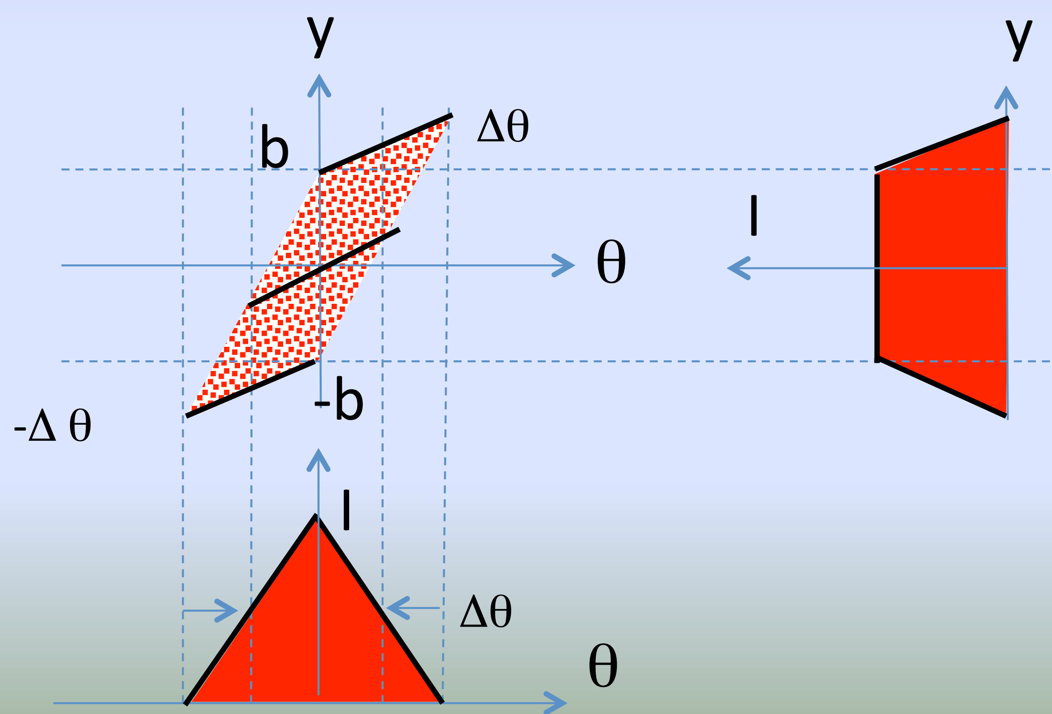
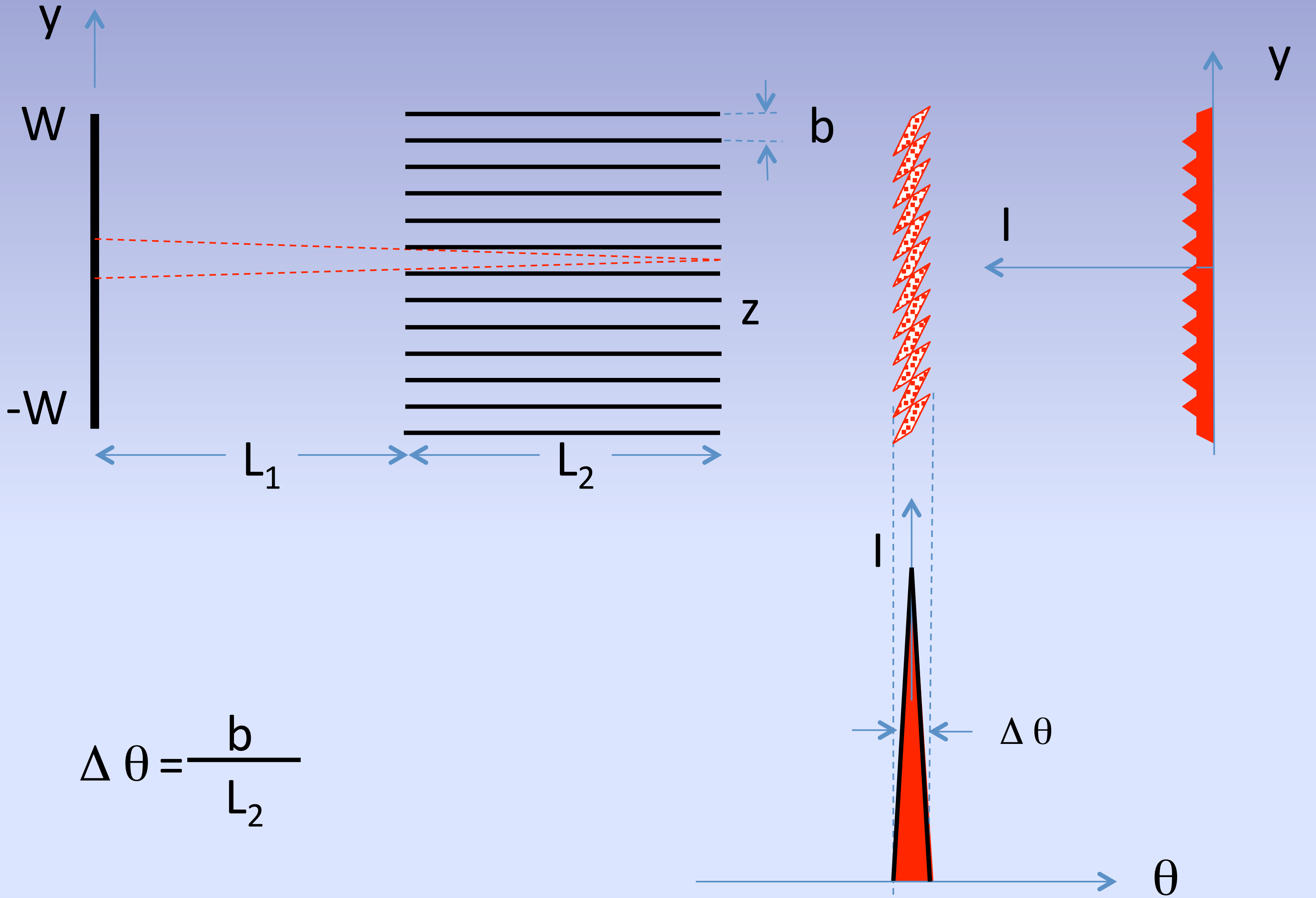
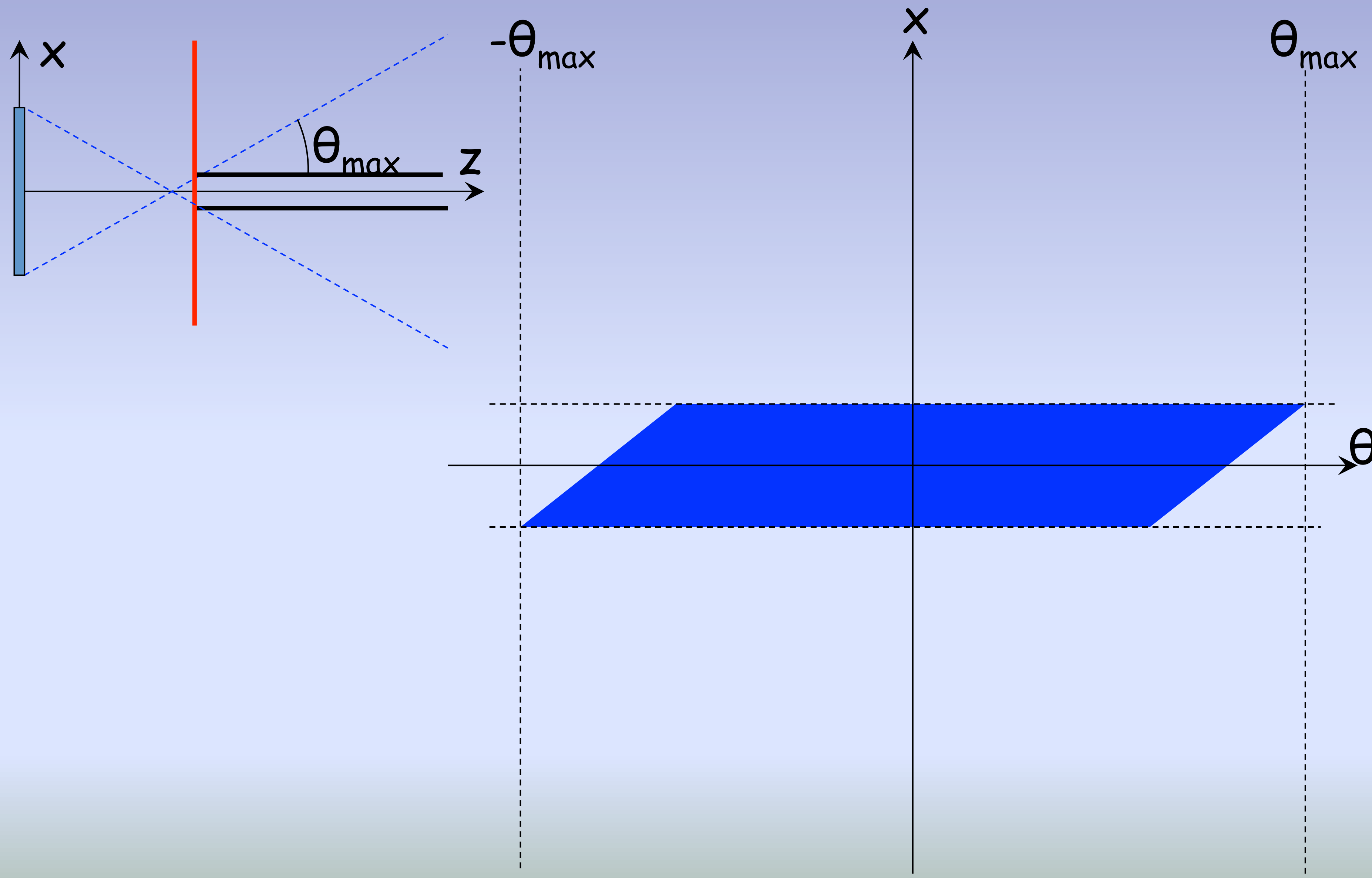


Case B – Soller collimator



$$\Delta\,\theta = \frac{b}{L_2}$$

Case C – Neutron Guide



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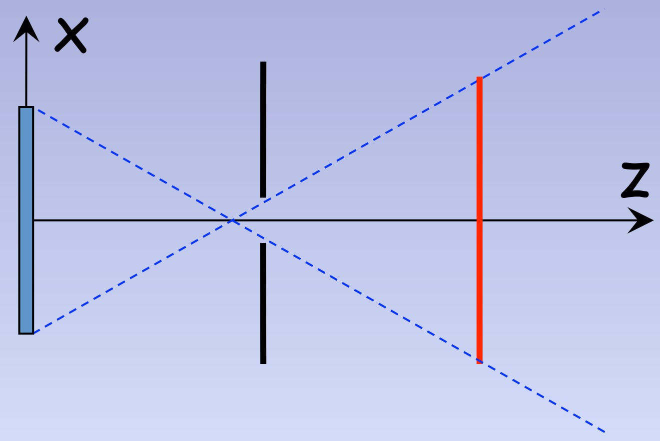

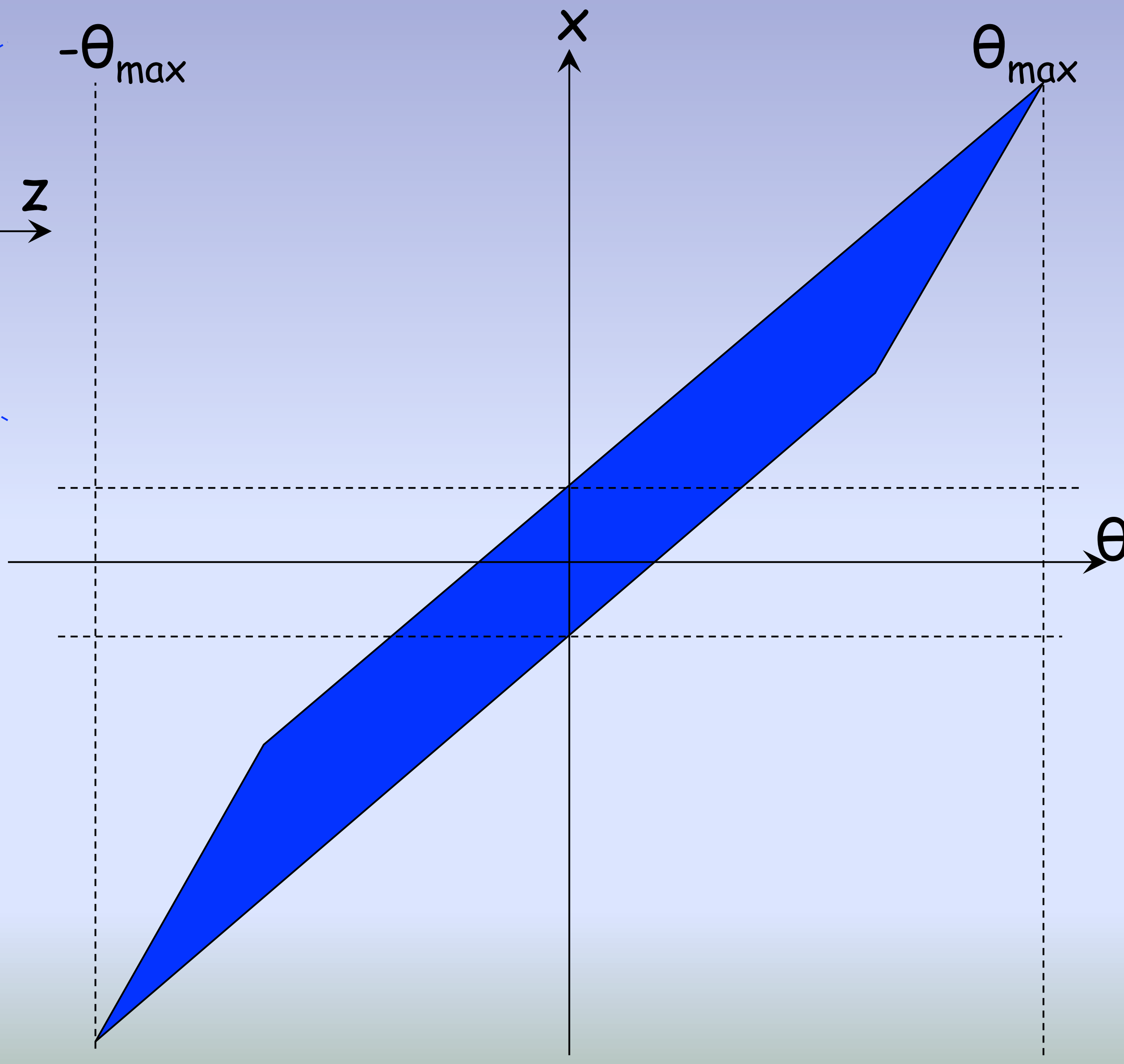

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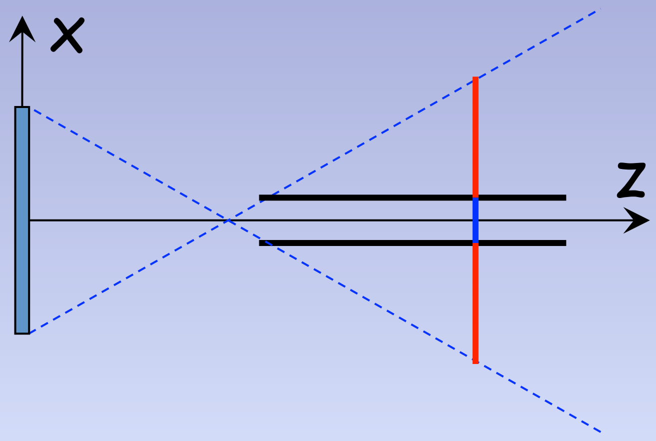

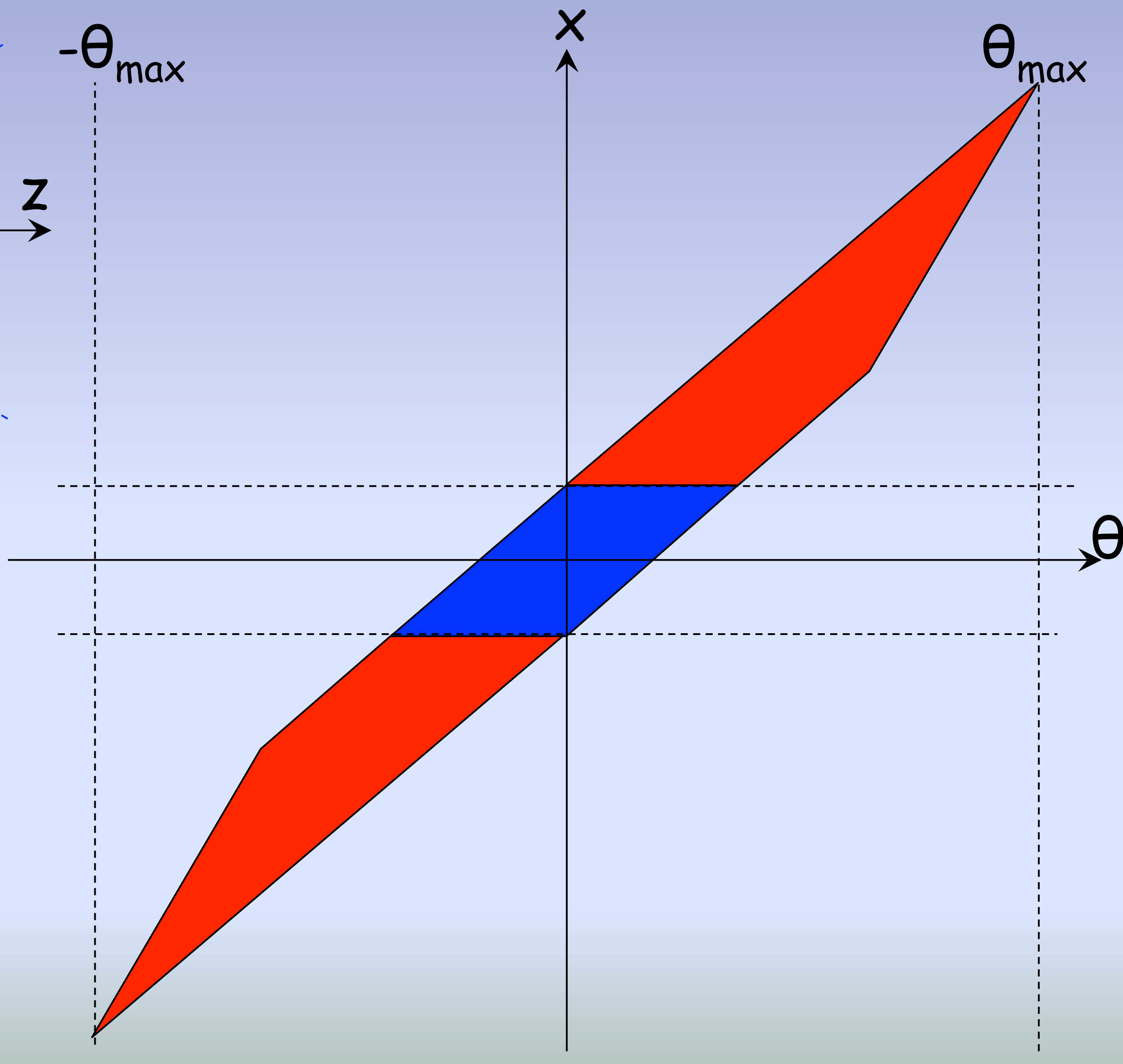

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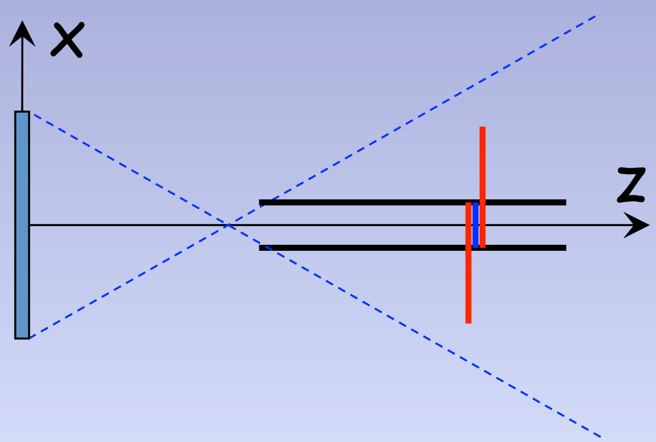

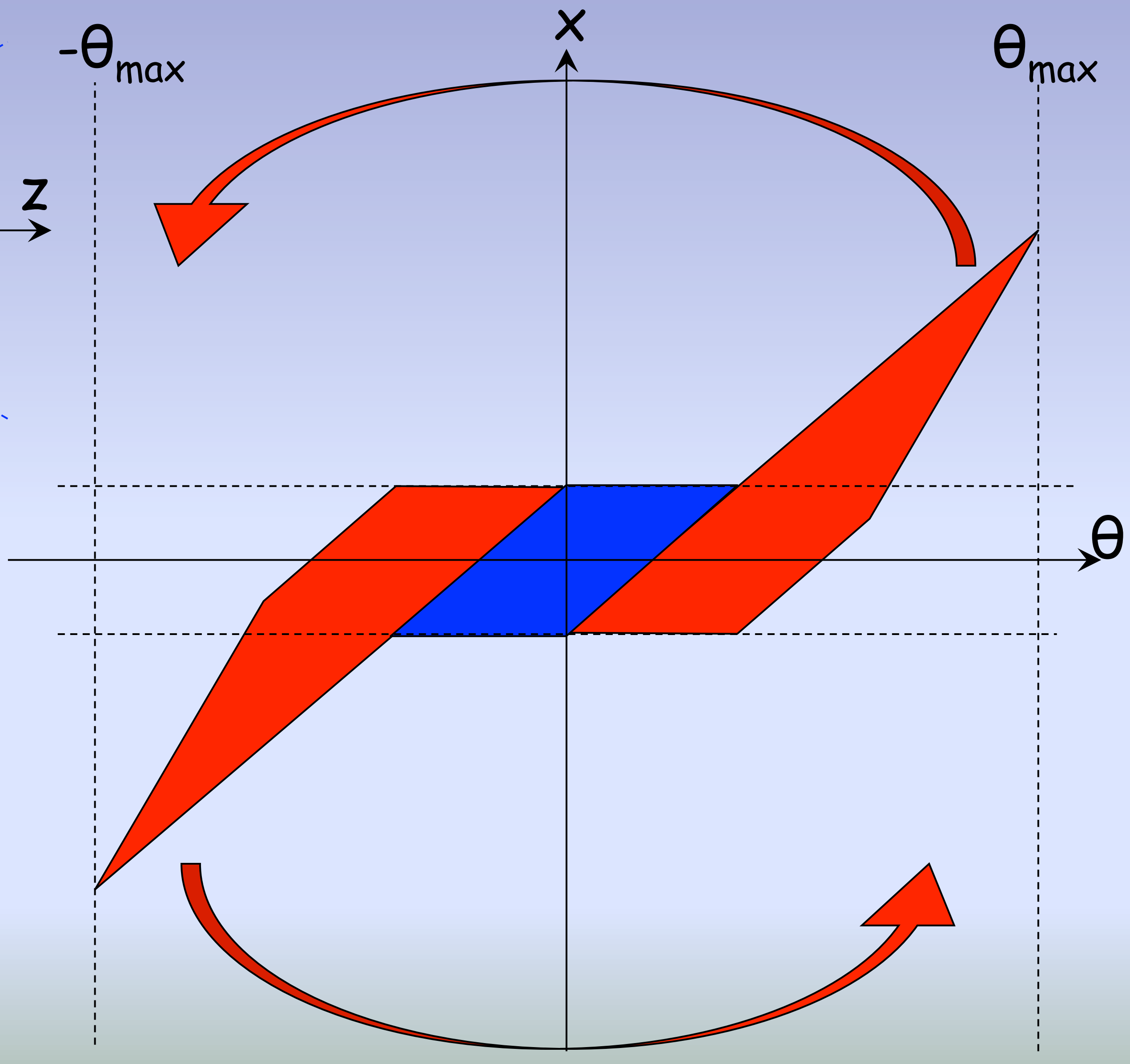

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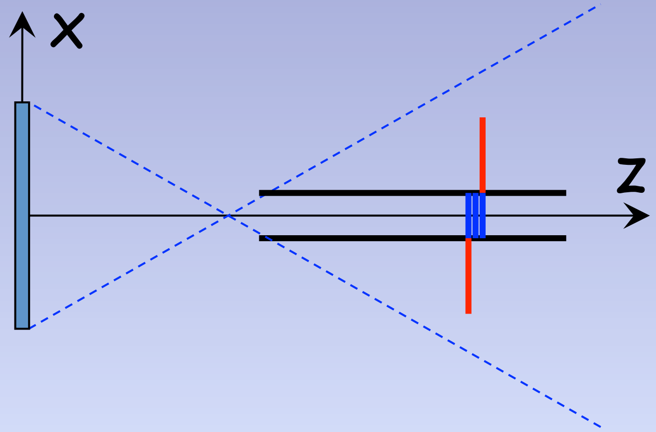

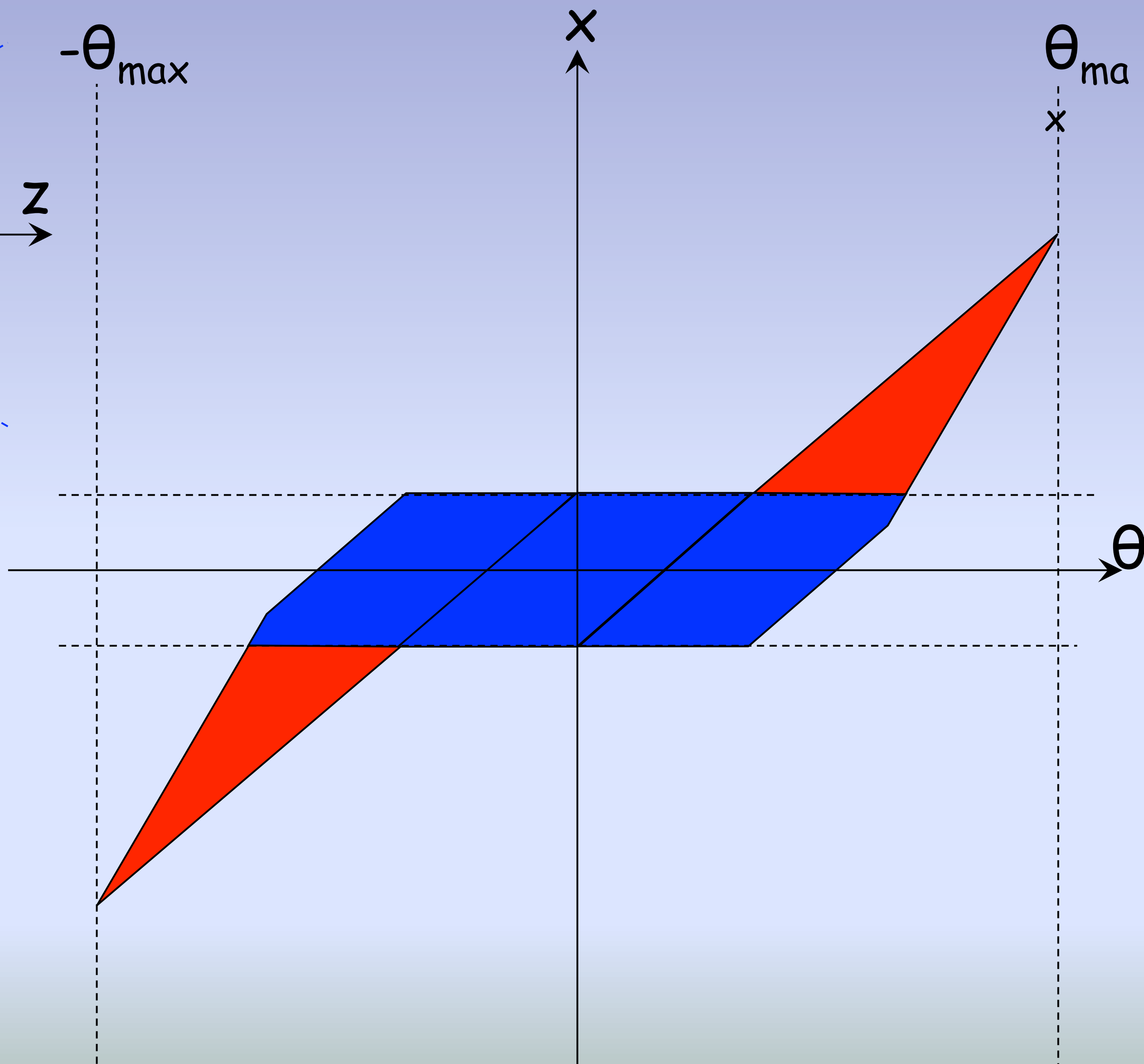

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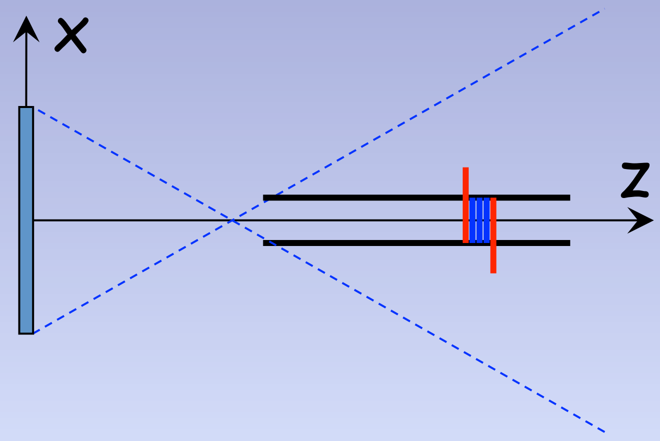

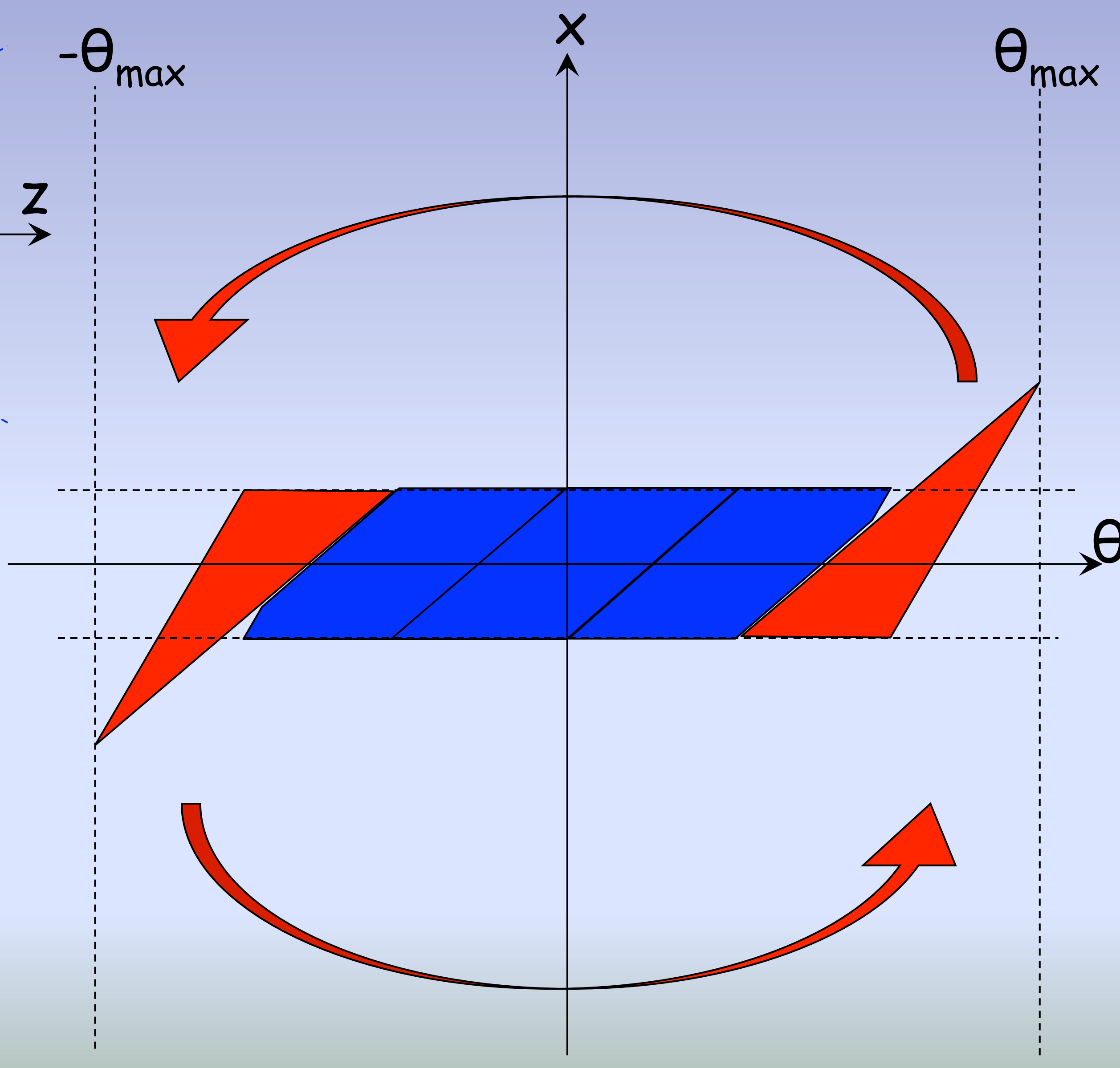

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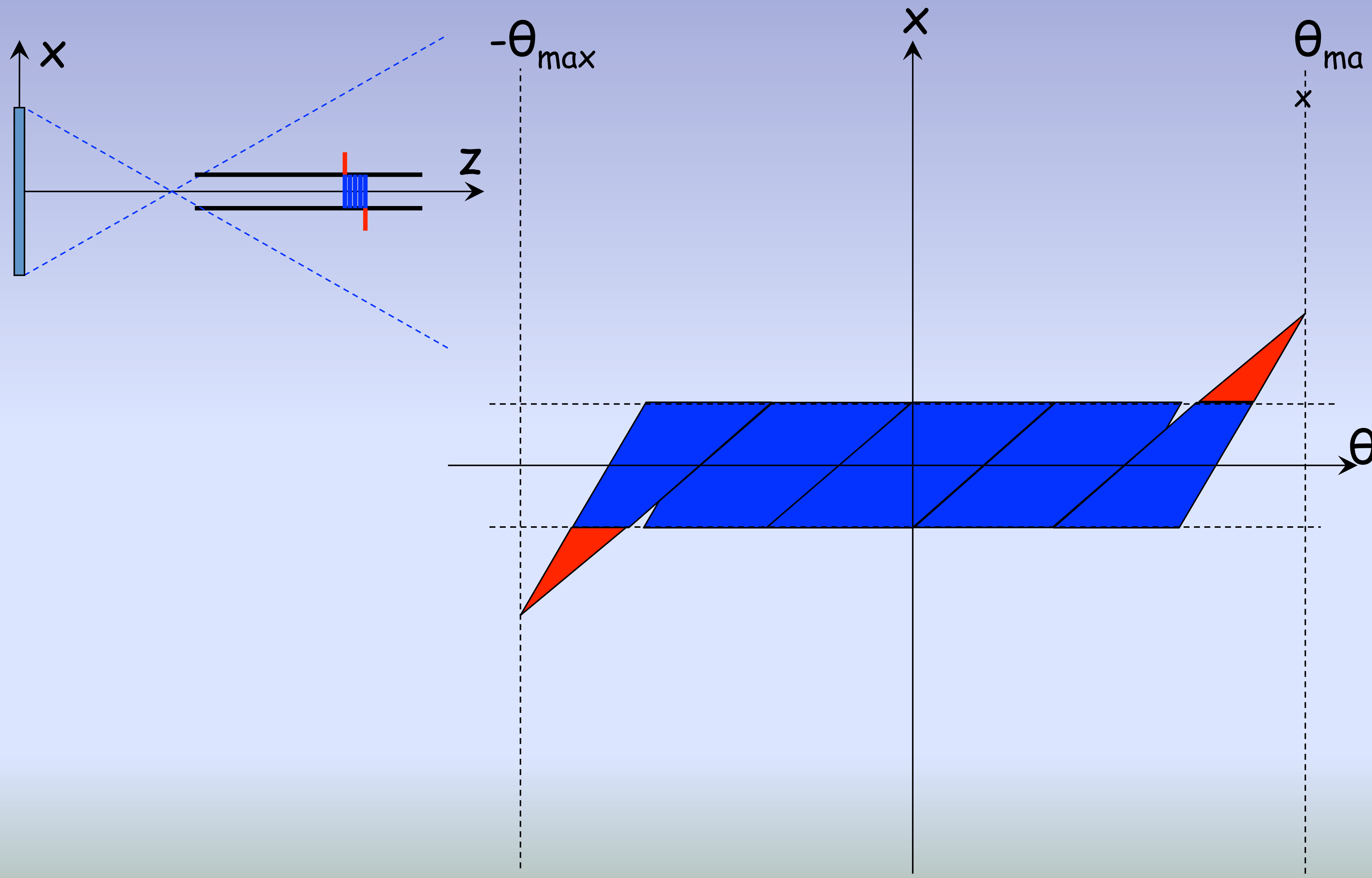

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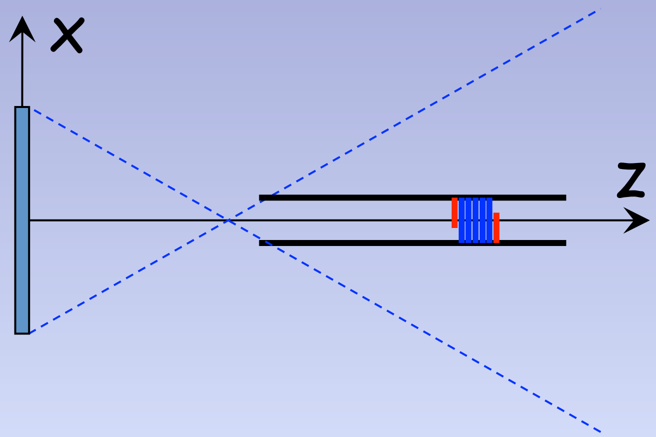

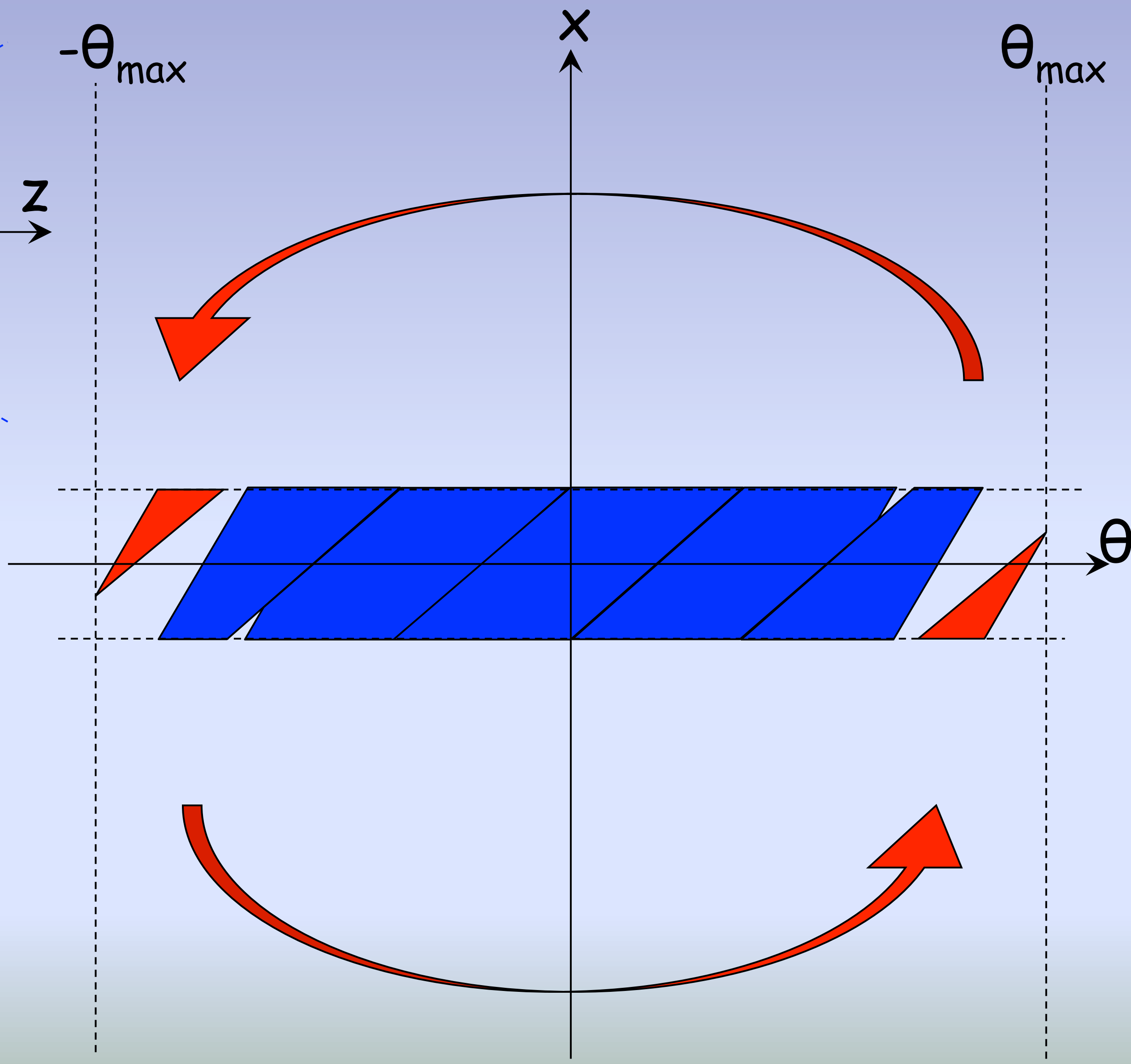

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- Guide dimension cuts the phase space distribution, and the mirror flips that section by 180°.

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- Guide dimension cuts the phase space distribution, and the mirror flips that section by 180°.
- Never flip a section of phase space onto another as this would violate the Liouville theorem.

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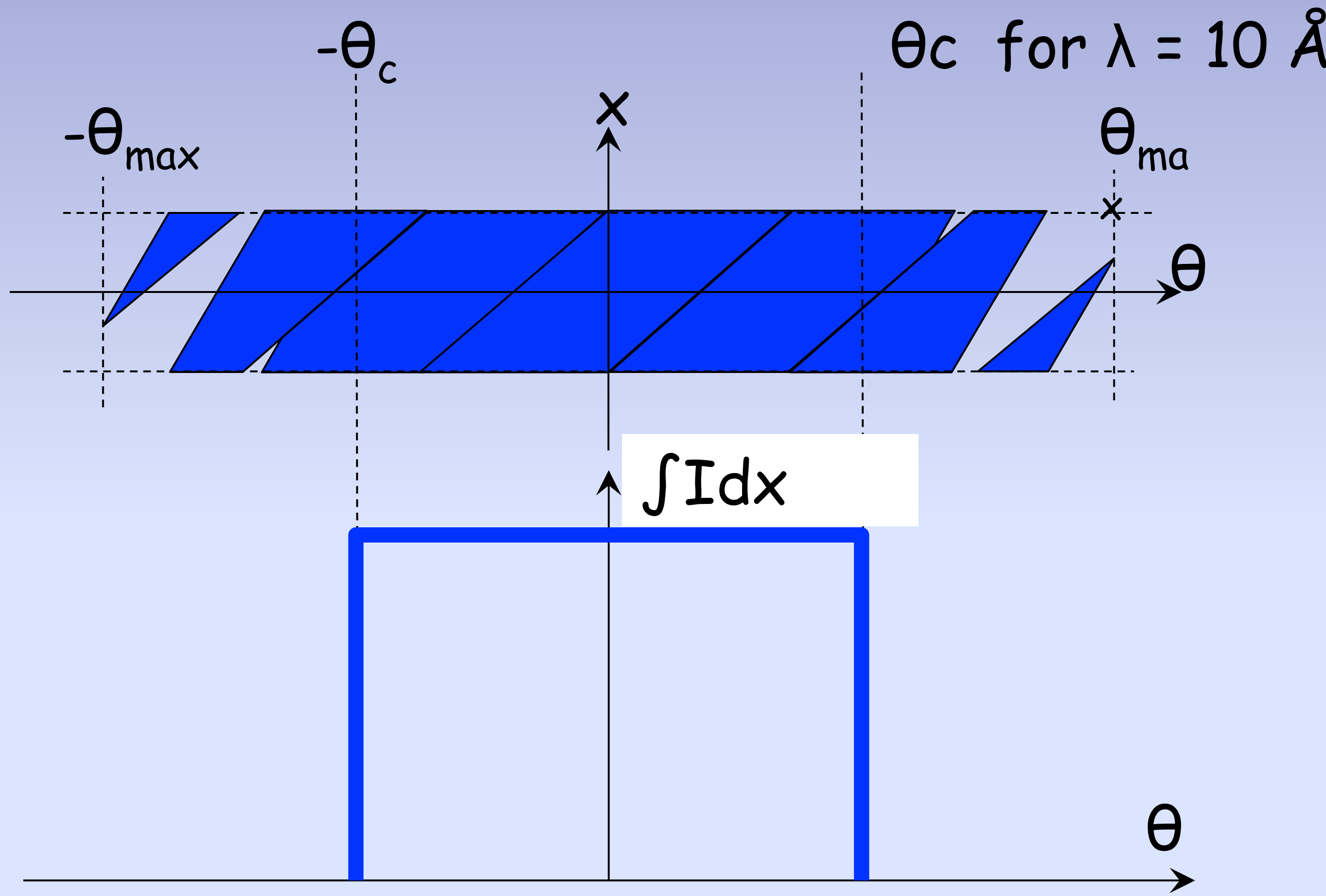
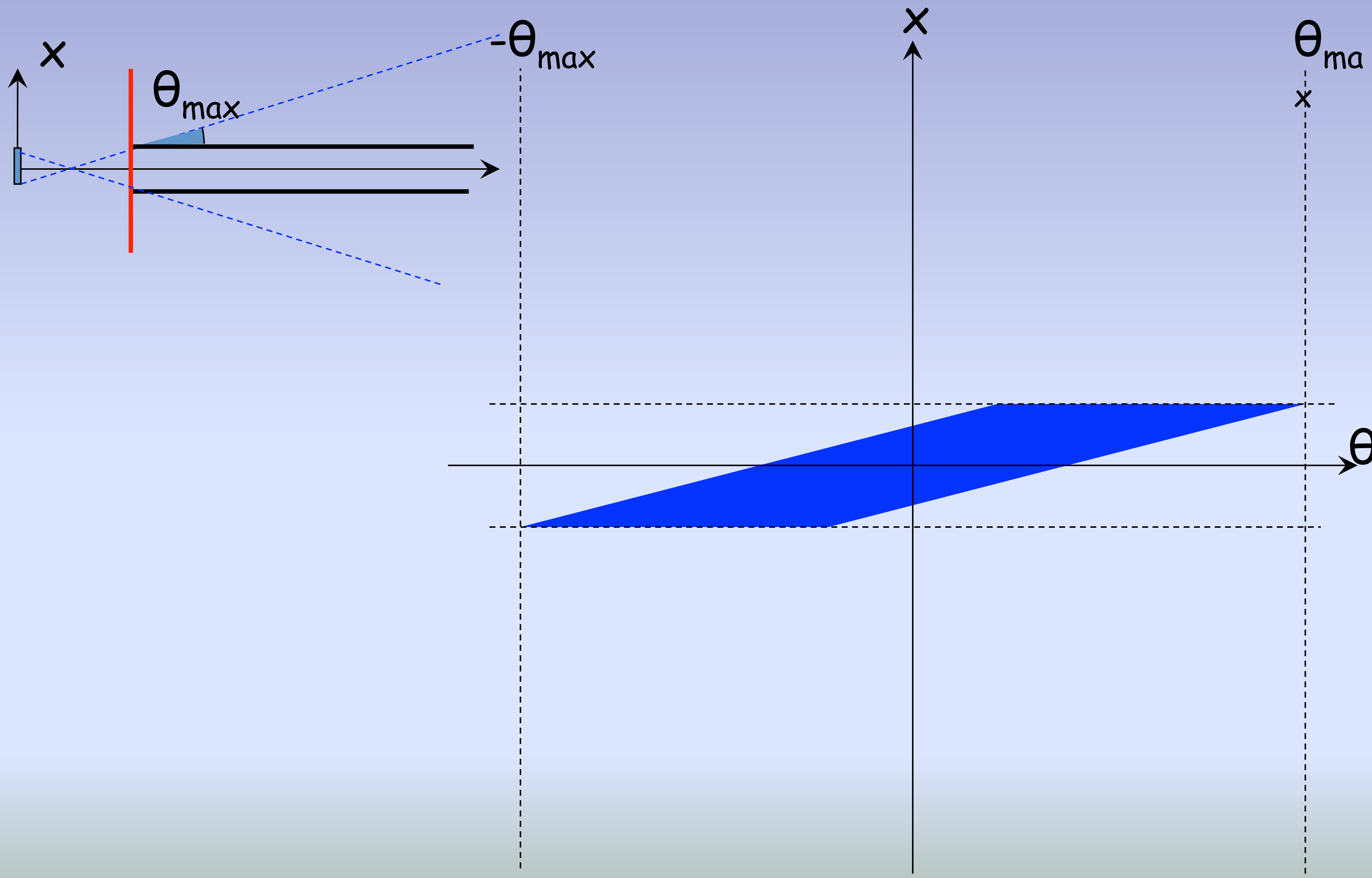


Case C - Neutron Guide Small Source



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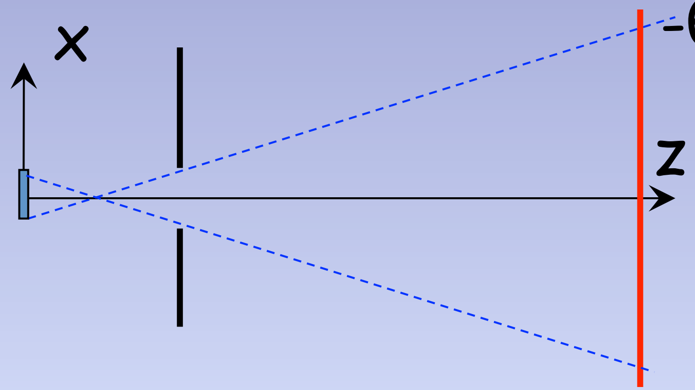

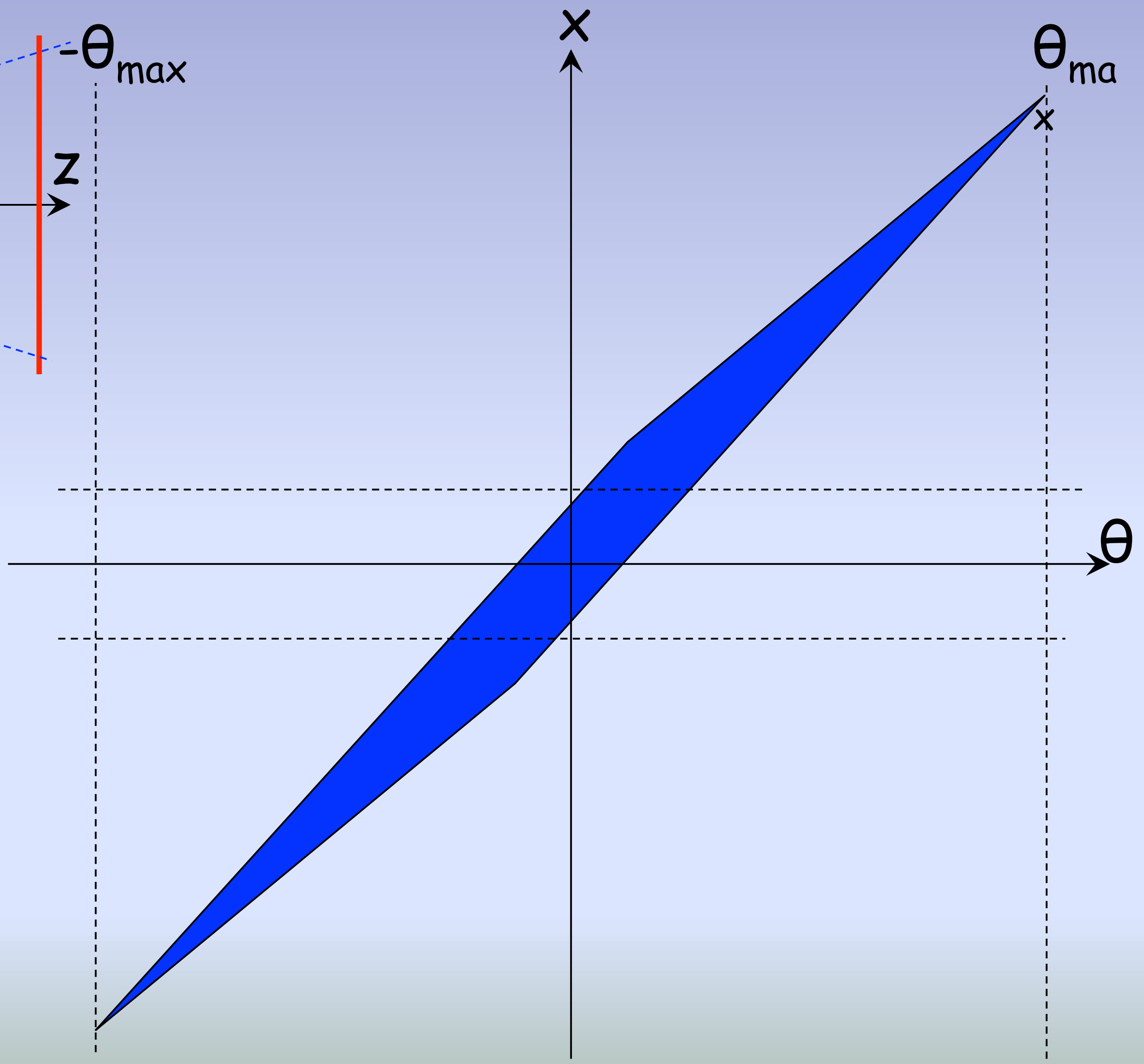

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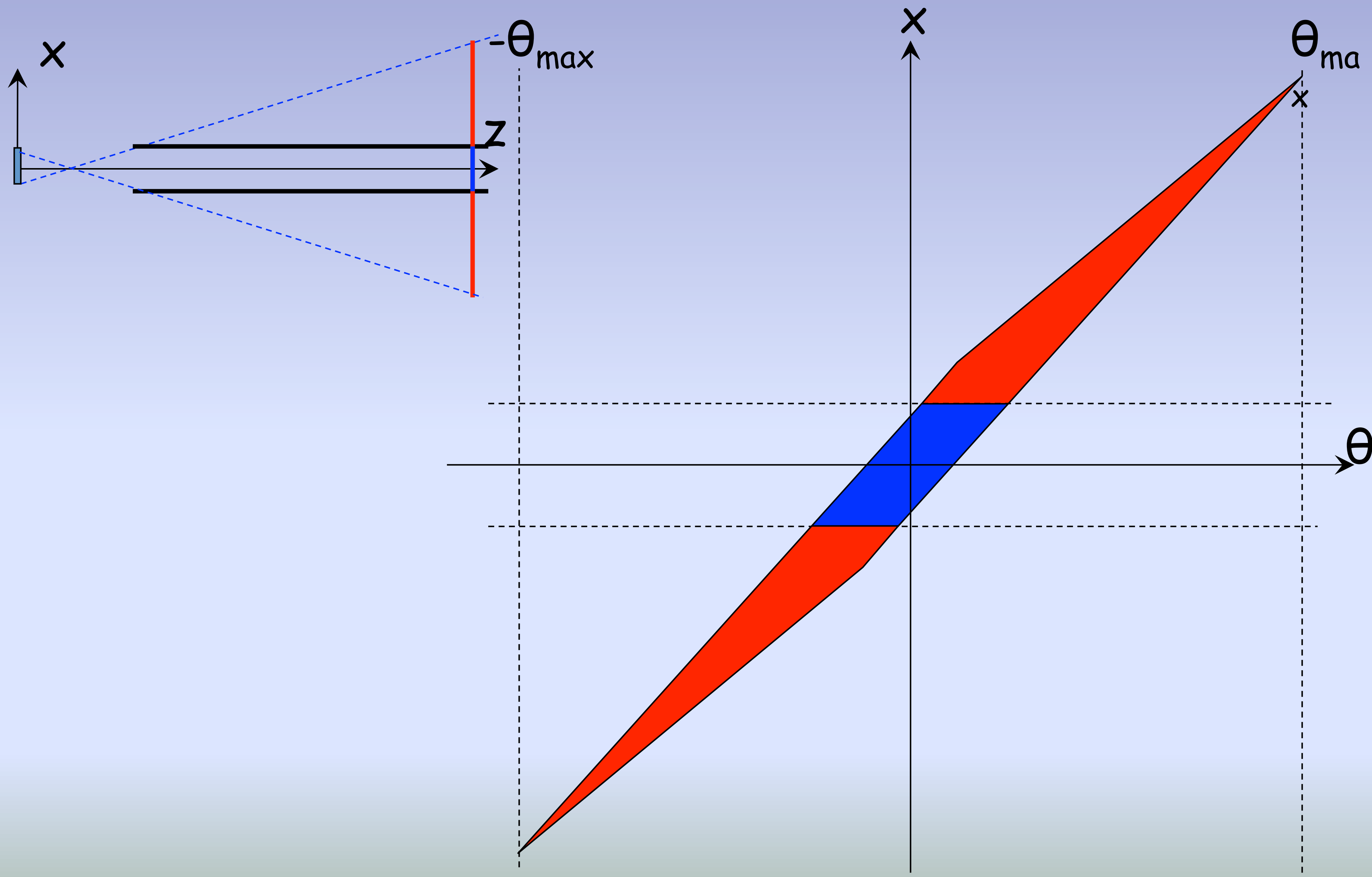

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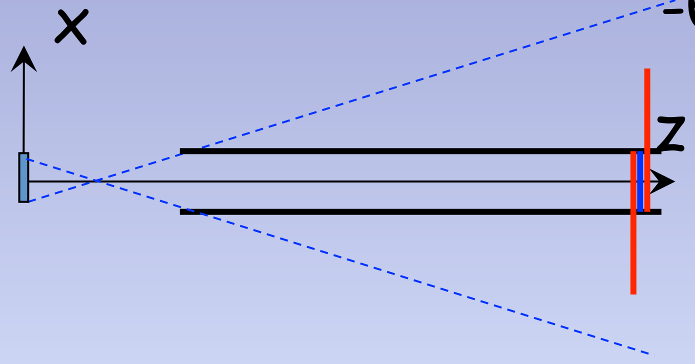

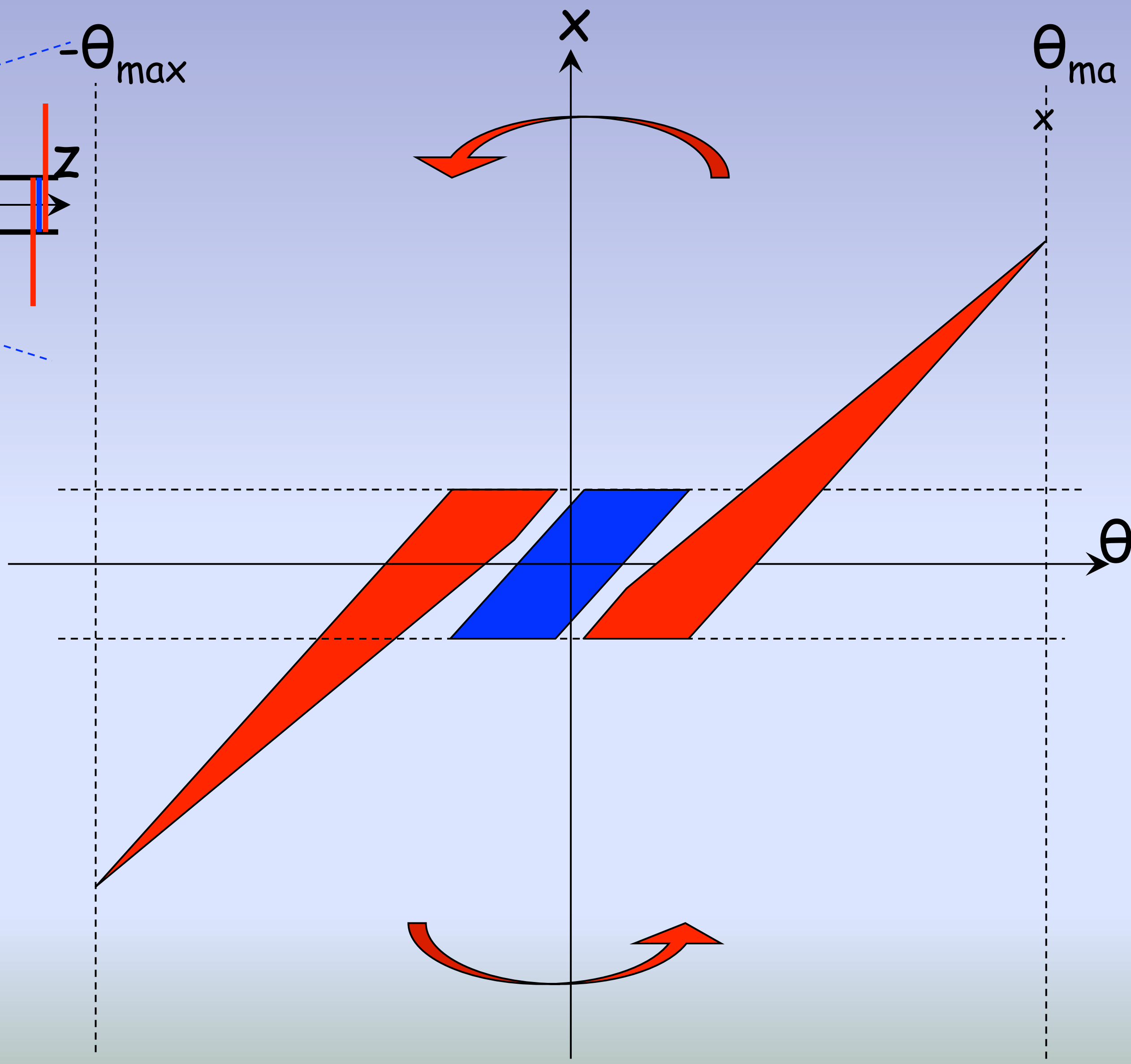

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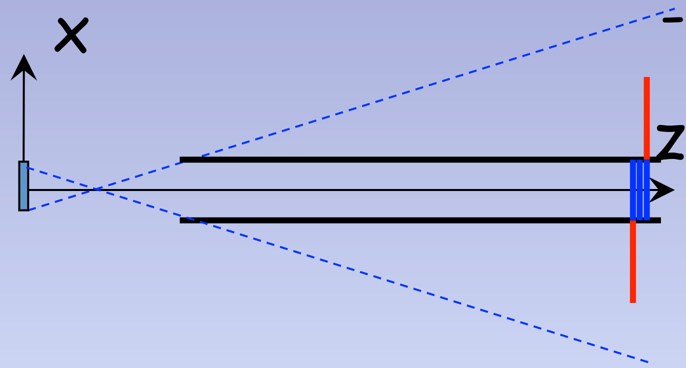

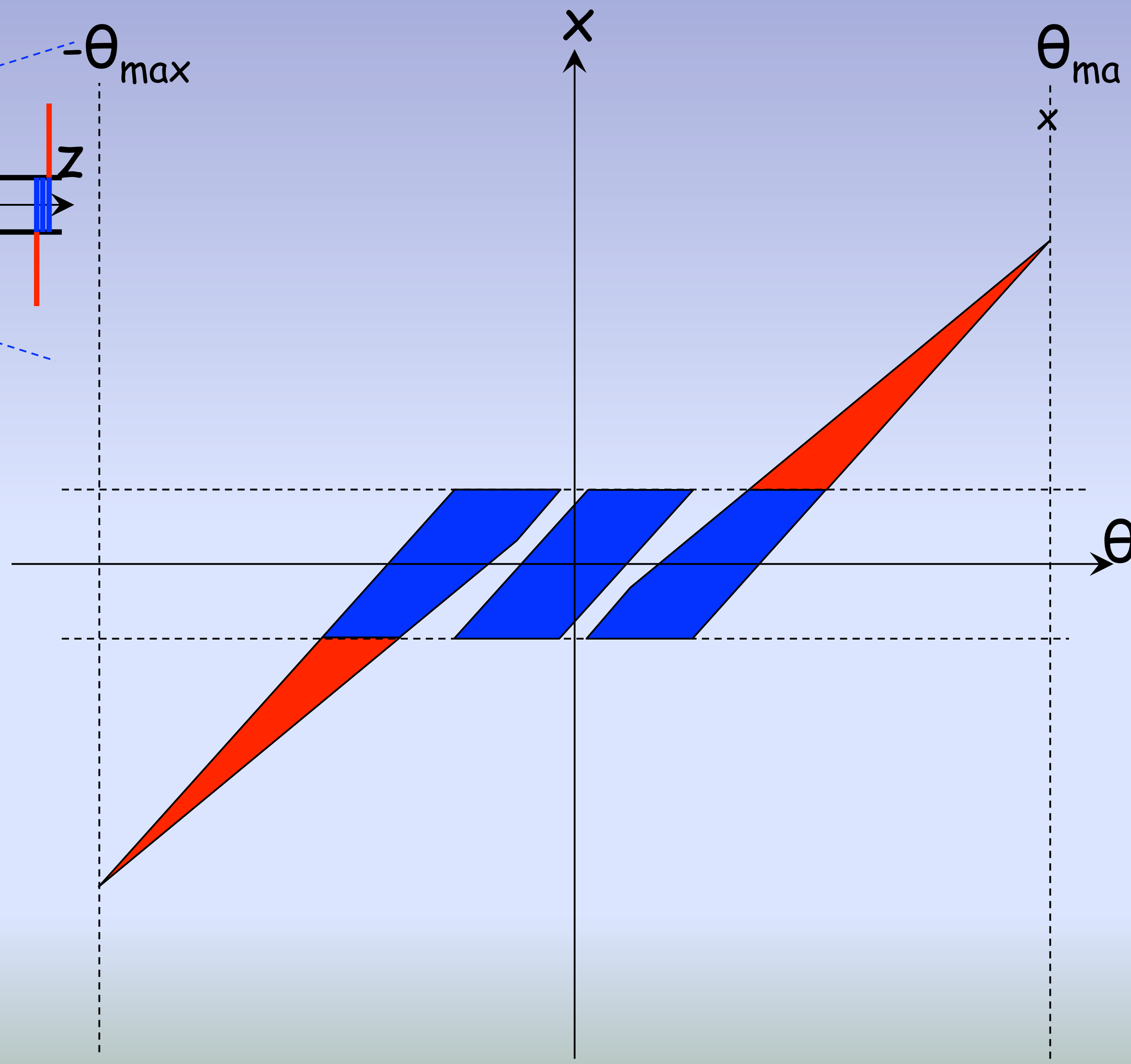

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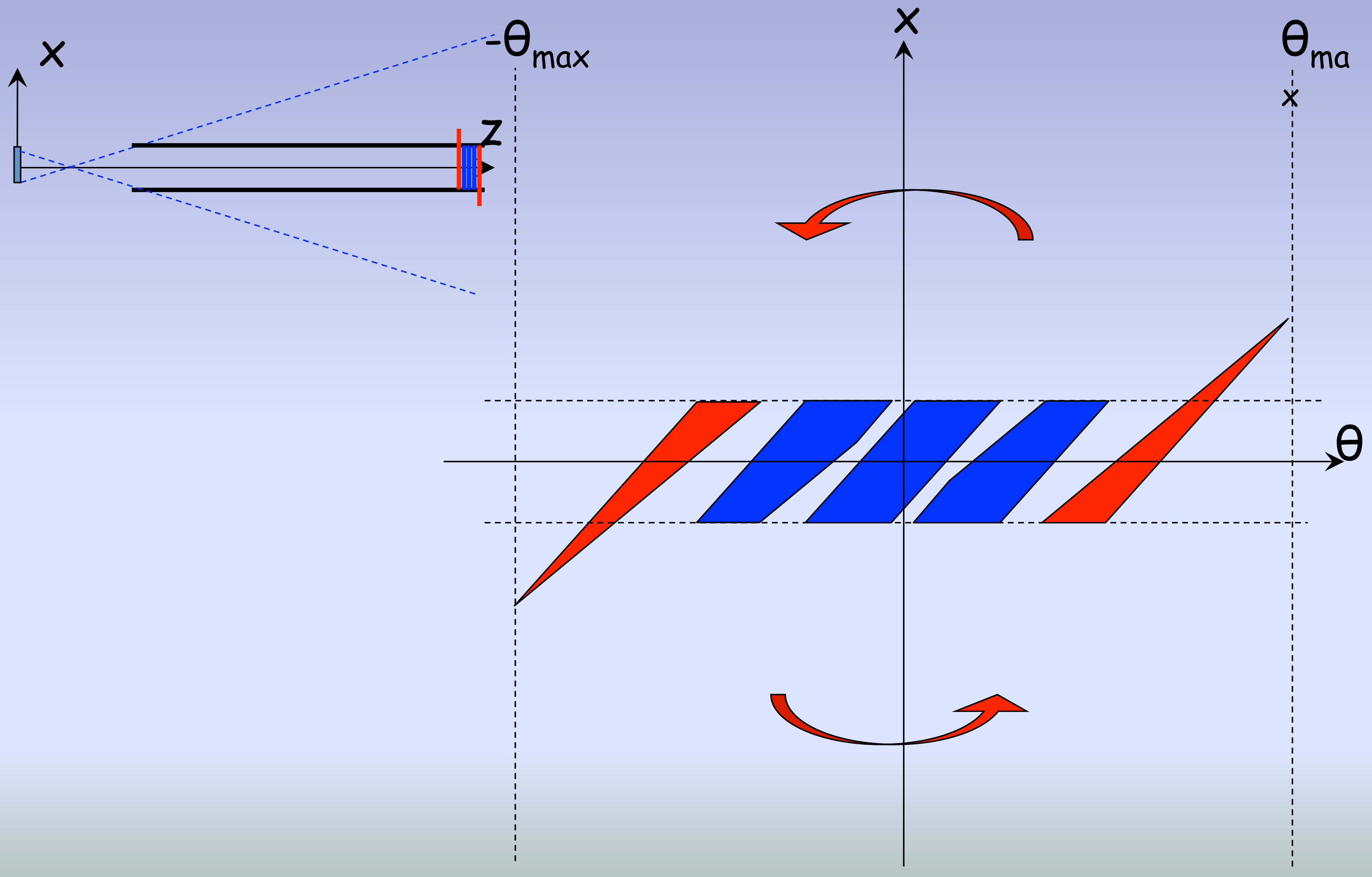

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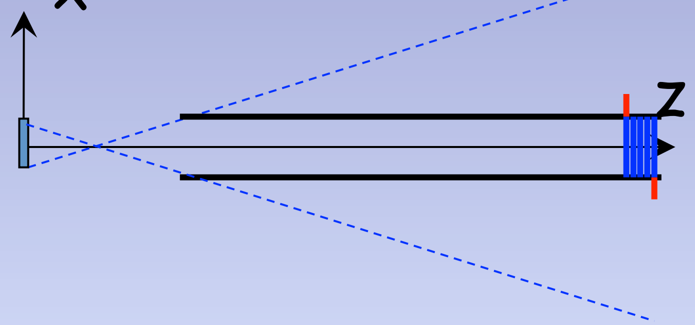

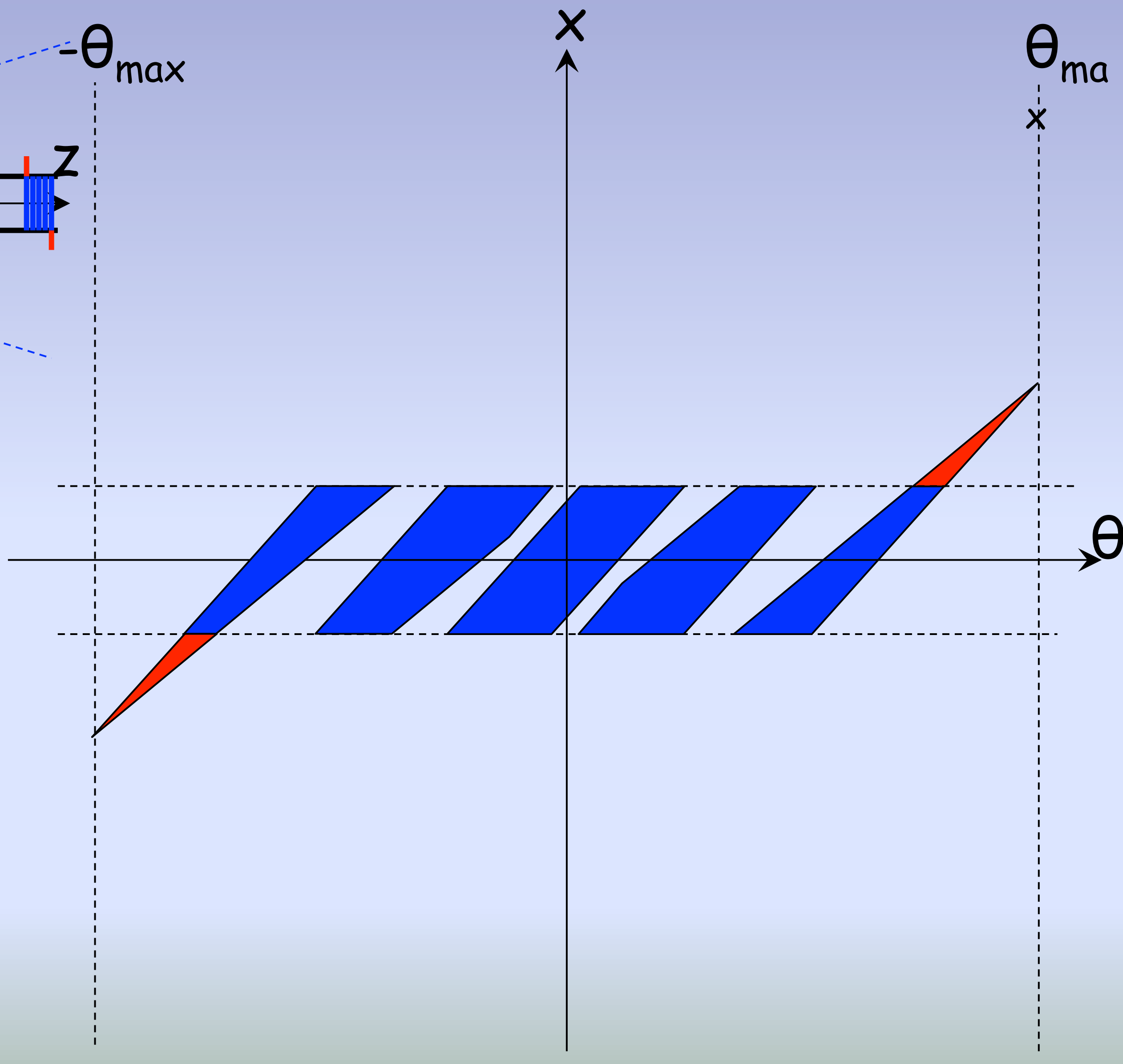

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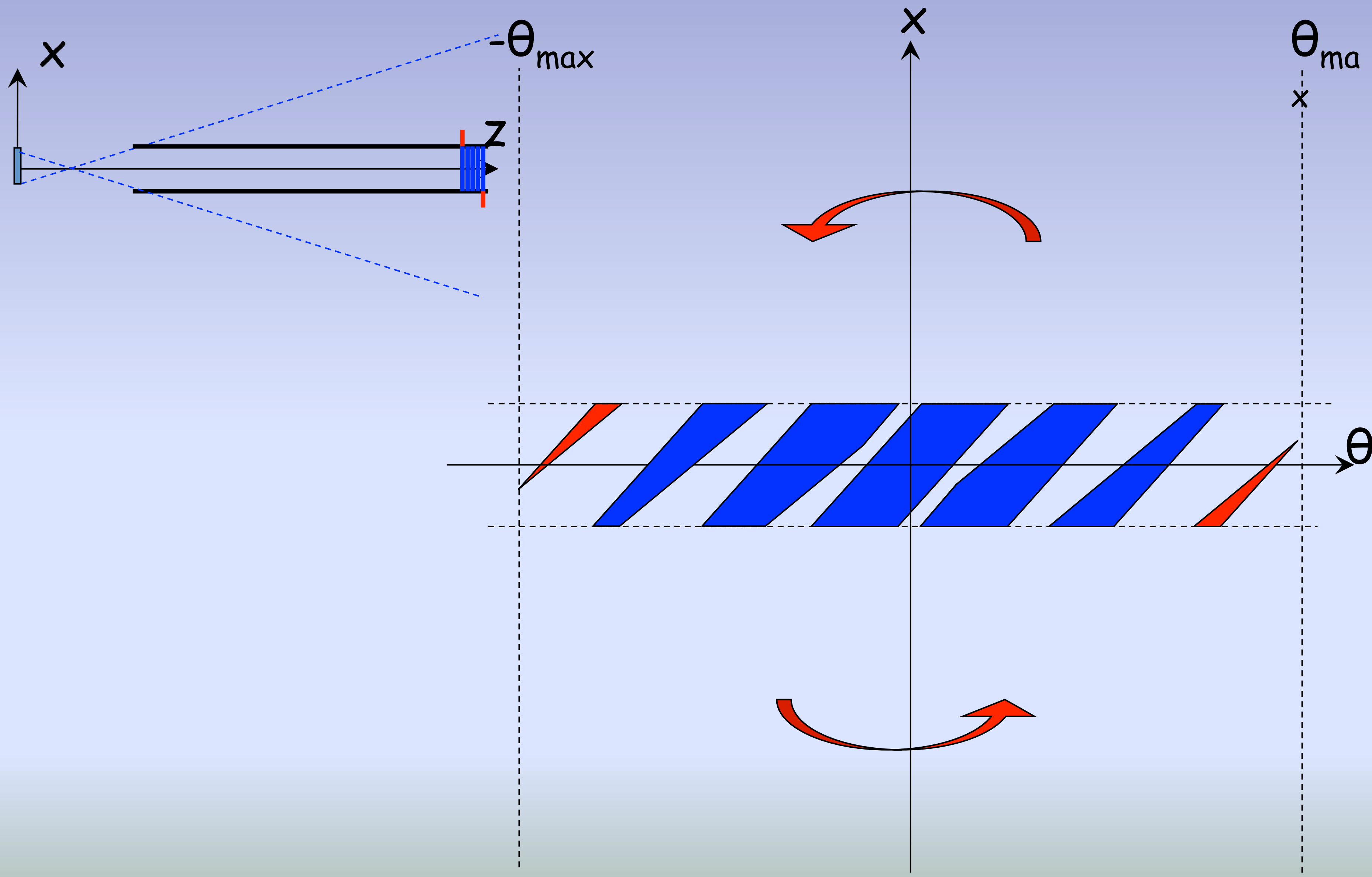
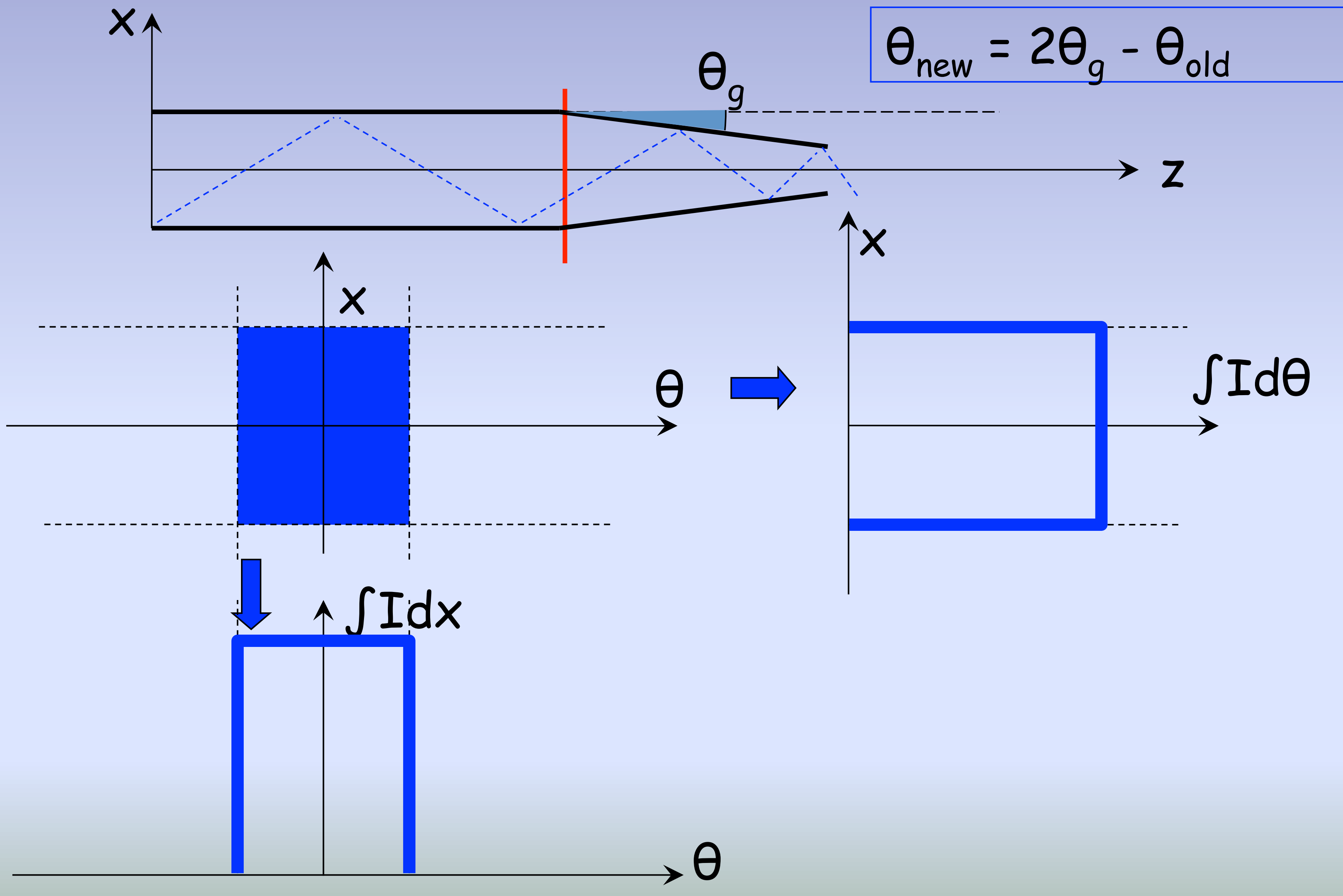


Case D - Focusing Guide



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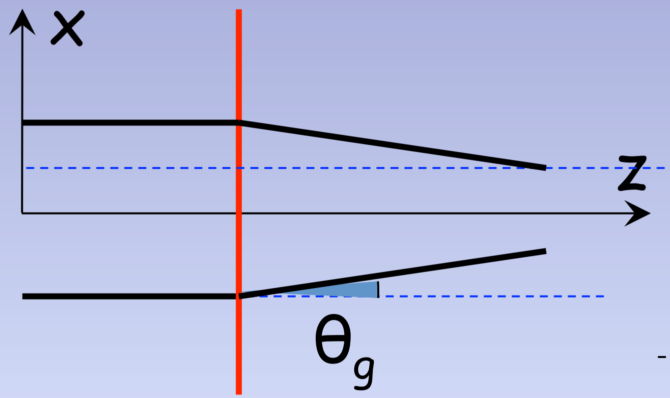
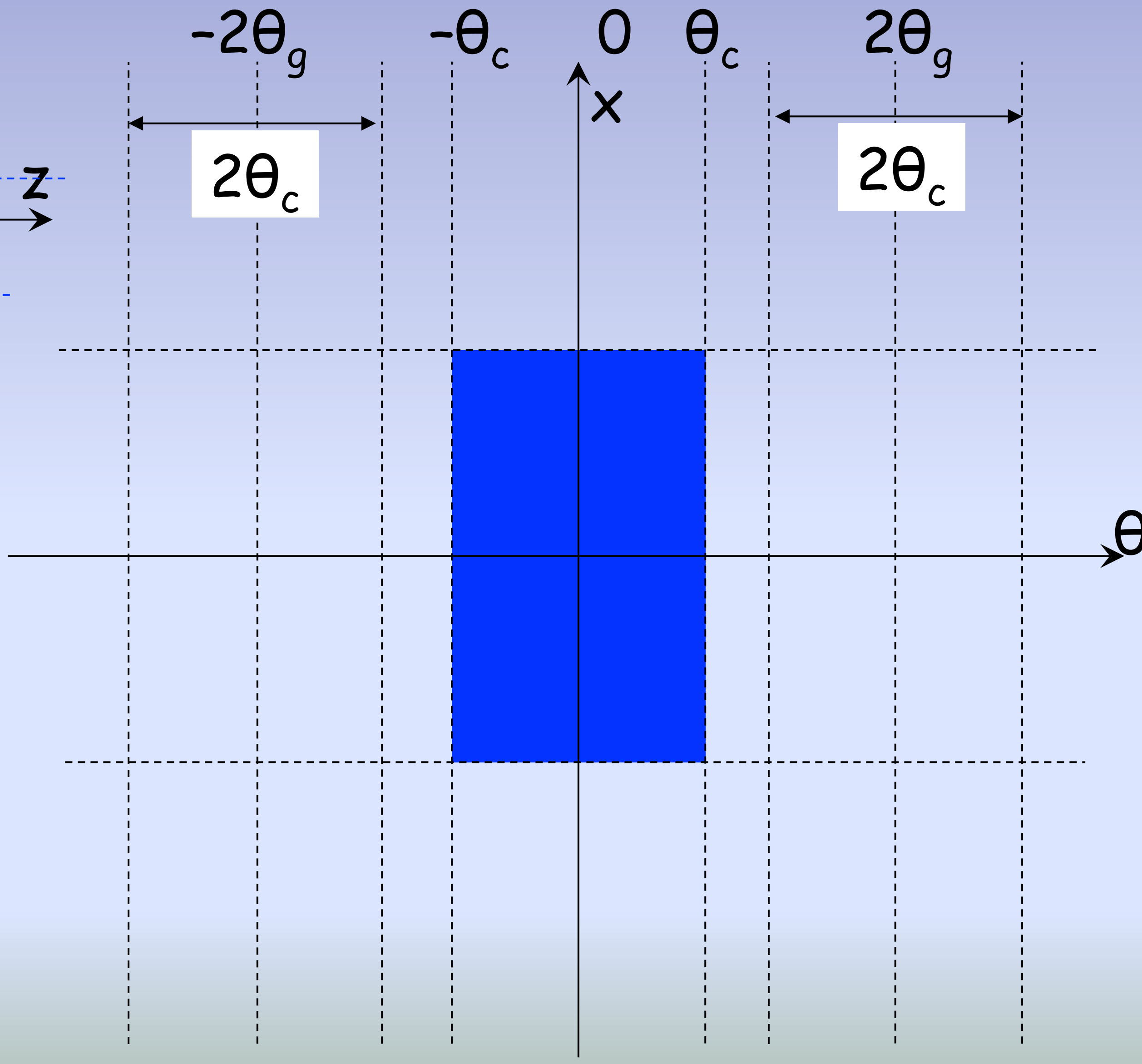


w_g = 10 - 5 cm

L_g = 2.5 m

θ_g = 0.6°

θ_c = 0.4°



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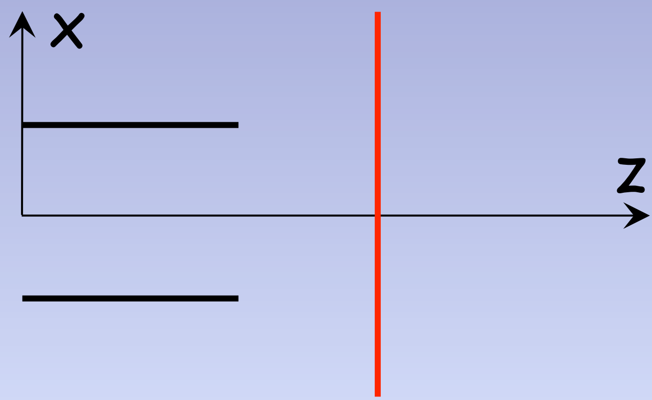

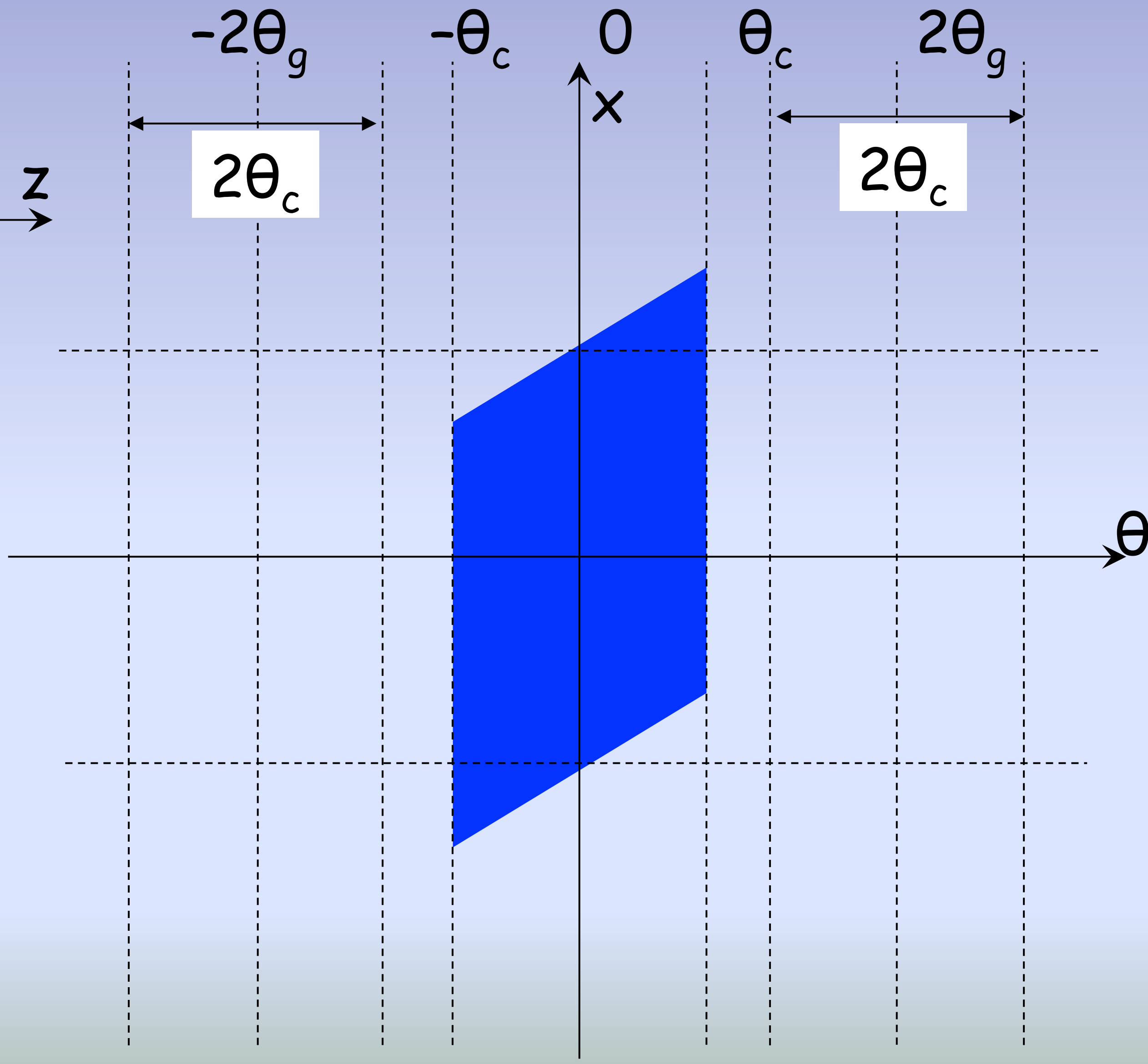

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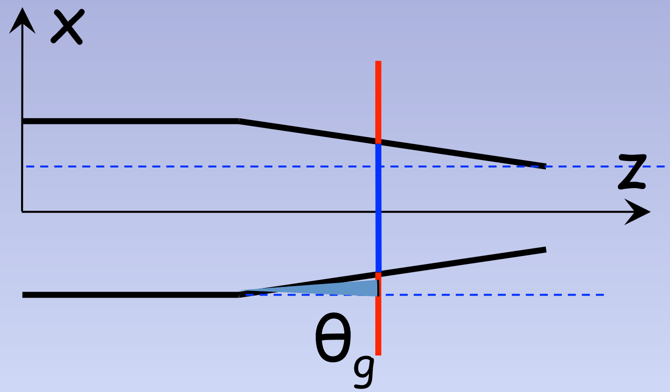

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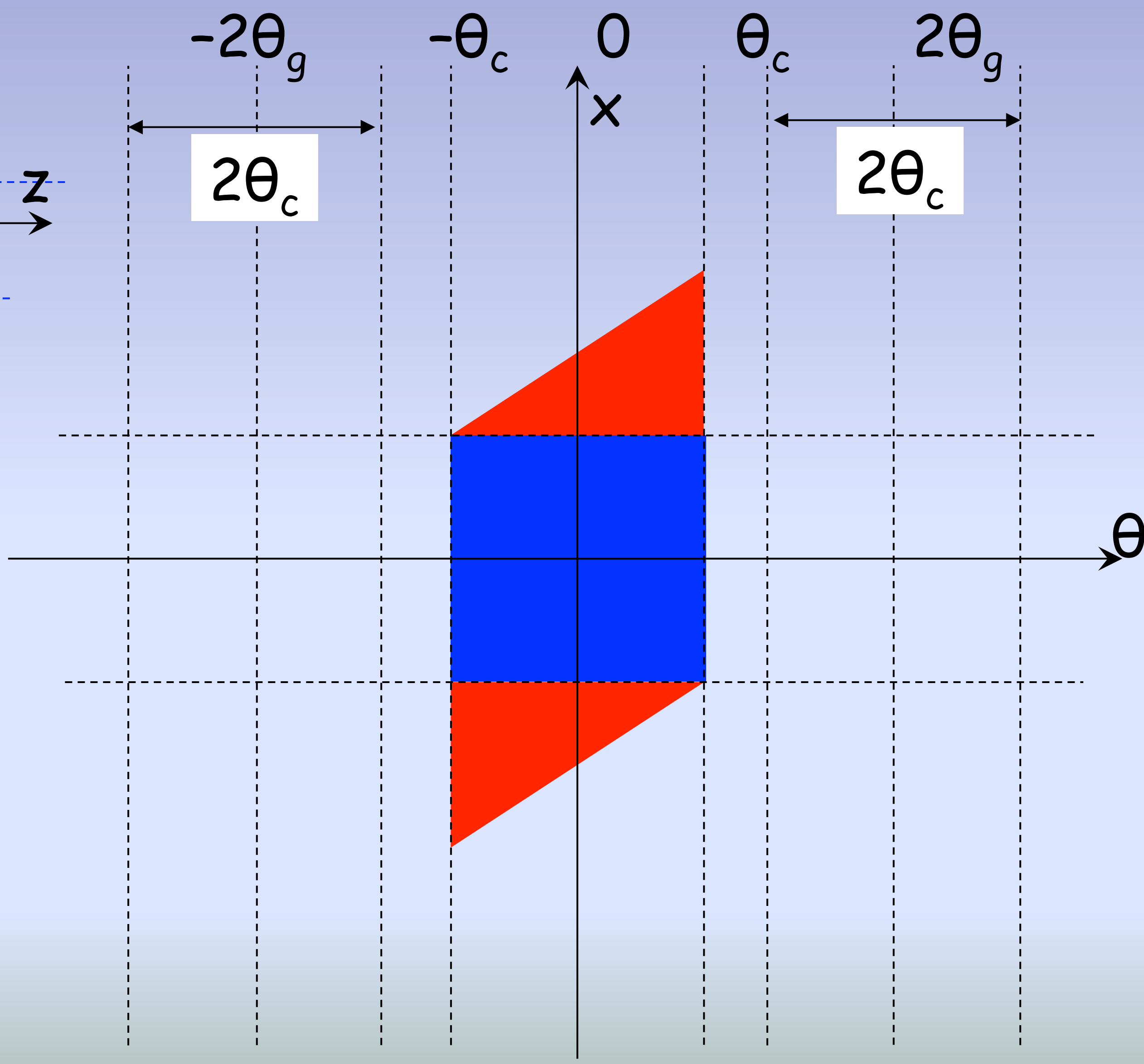

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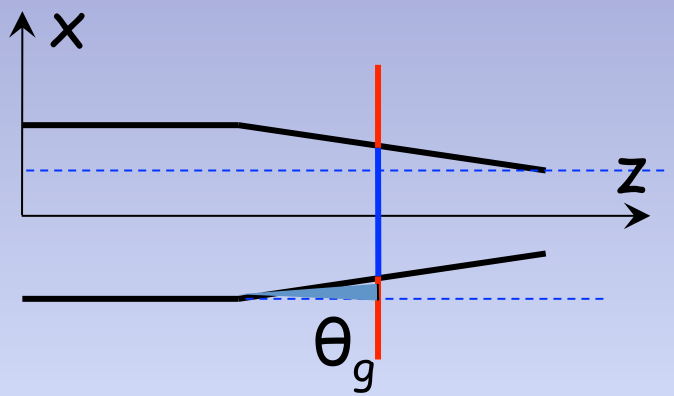

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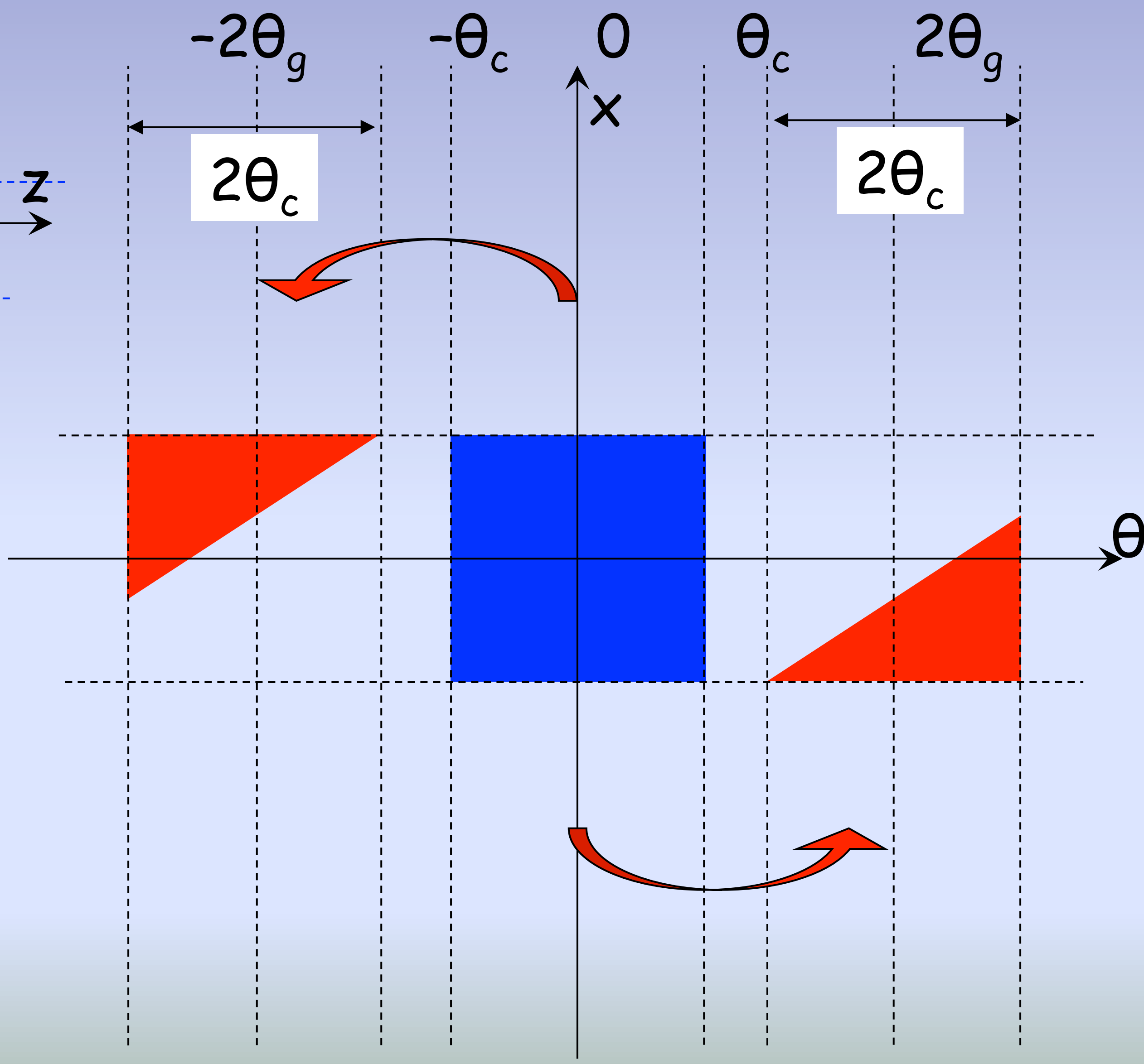

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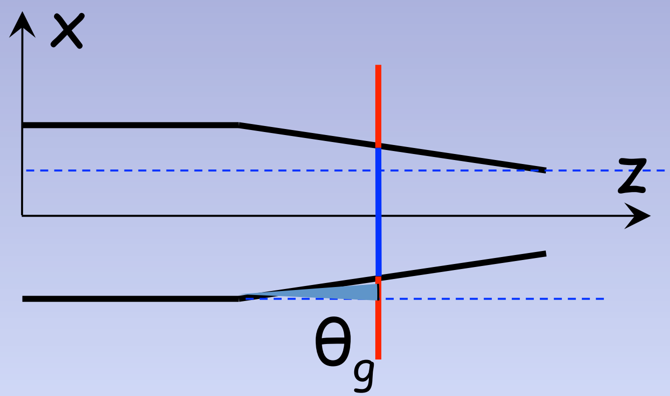

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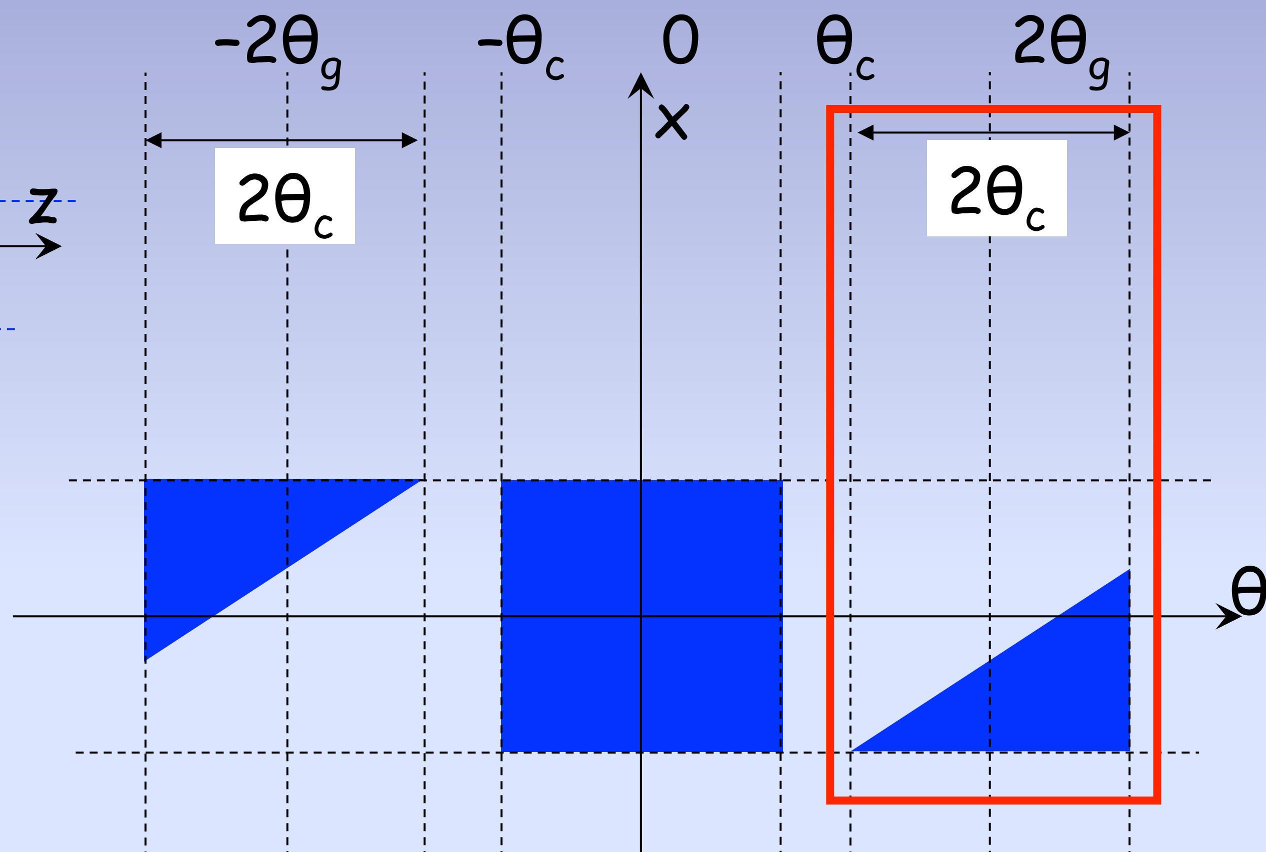

θ_c in the focusing section must be twice as large as θ_c in the straight section

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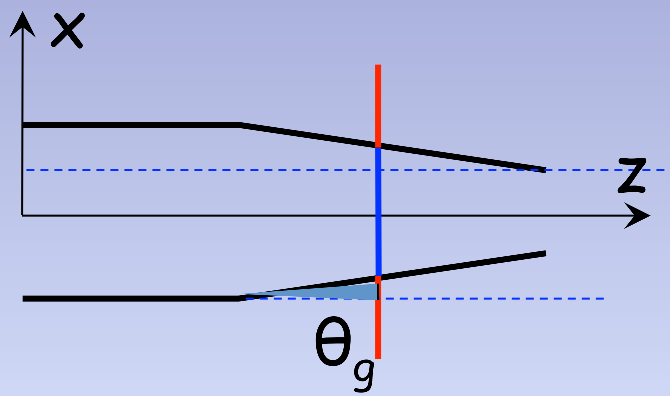

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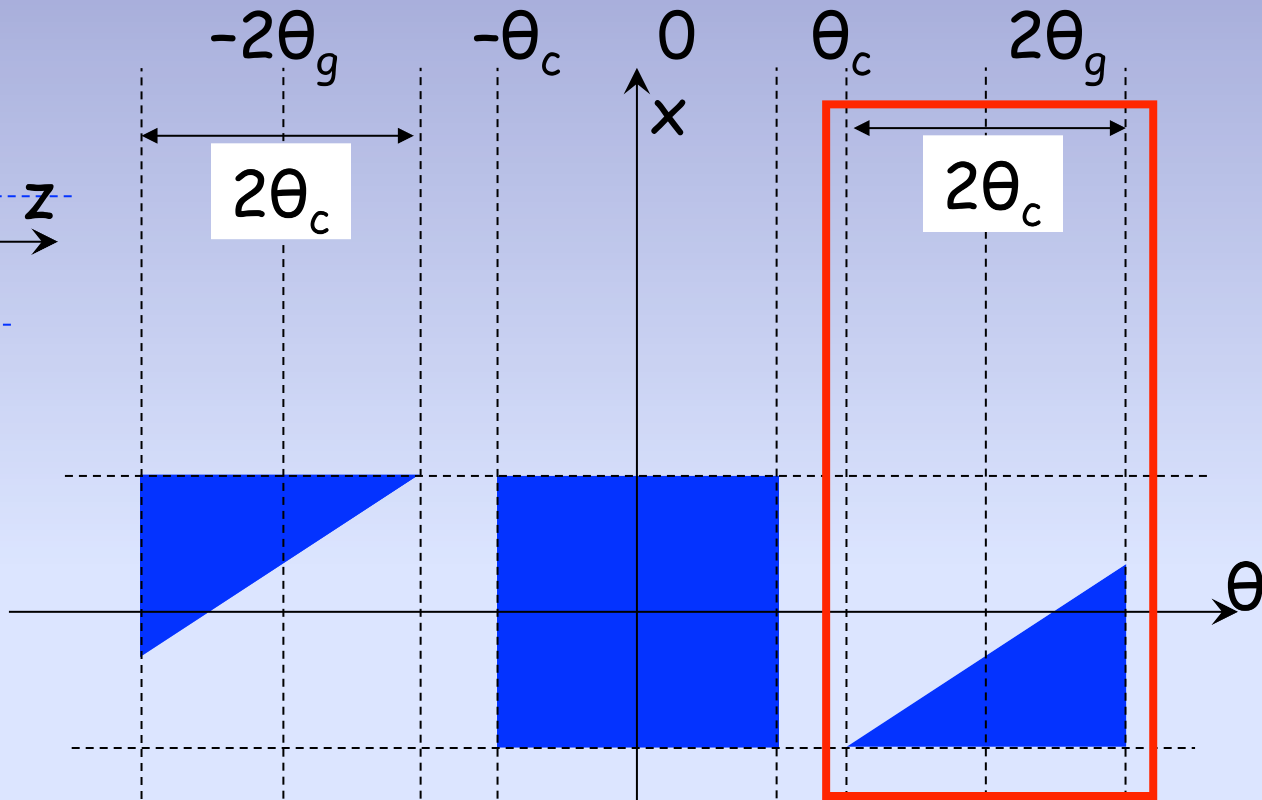
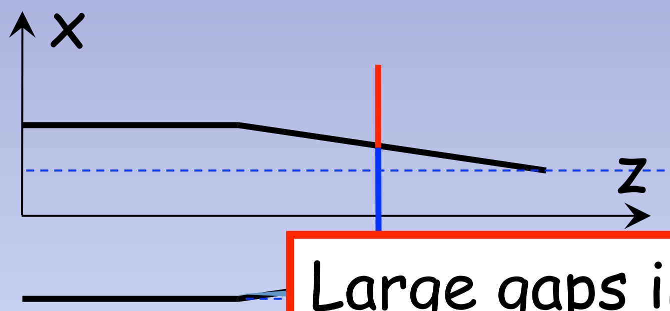
$\theta_c = 0.4°$



Rule of thumb: beam compression factor = ratio of θ_c in the focusing section to θ_c in the straight section

Case D - Focusing Guide

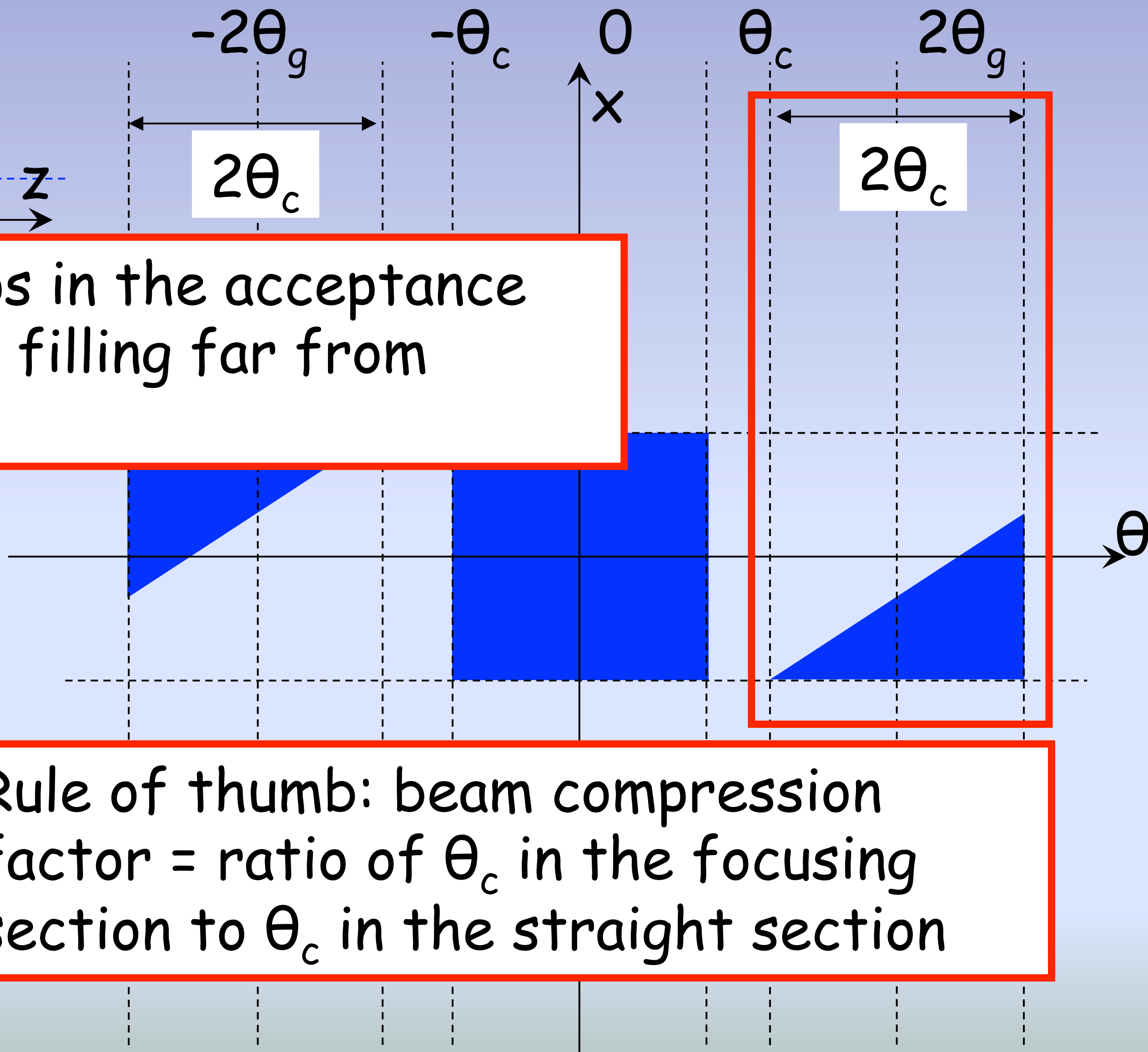


w_g = 10 - 5 cm

L_g = 2.5 m

θ_g = 0.6°

θ_c = 0.4°



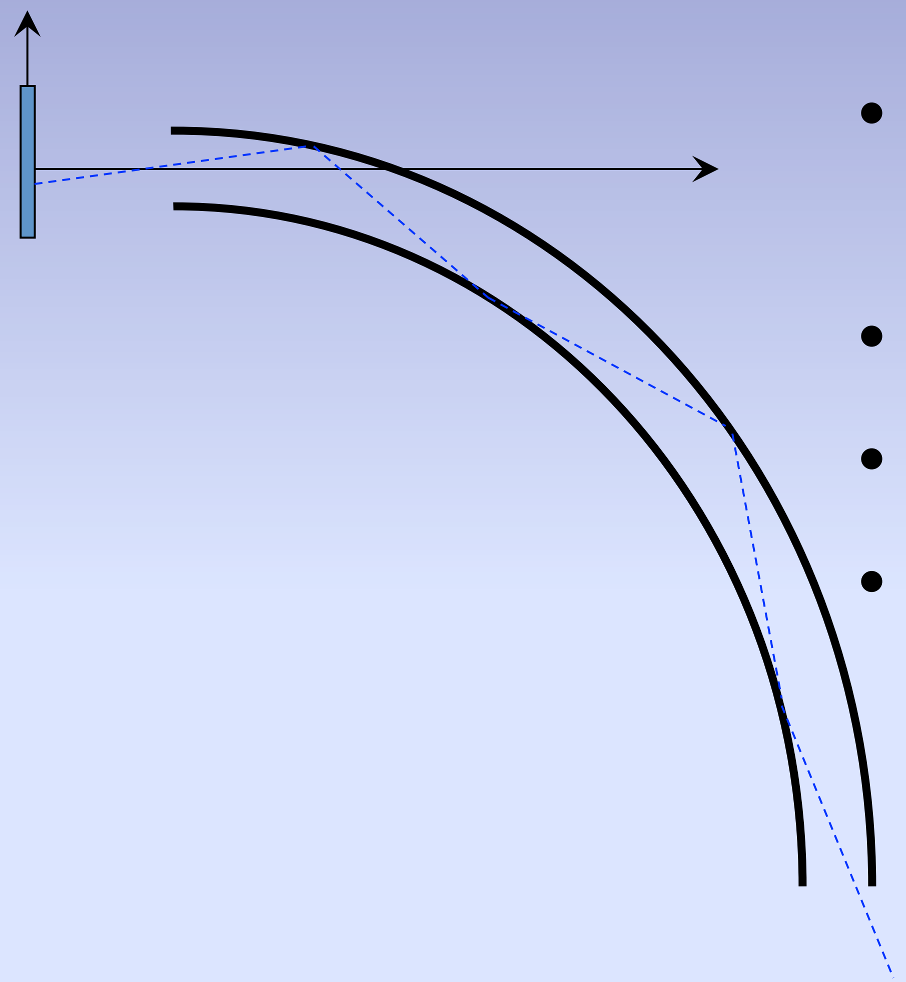
Large gaps in the acceptance diagram – filling far from optimal

Rule of thumb: beam compression factor = ratio of θ_c in the focusing section to θ_c in the straight section

Case D - Focusing Guide

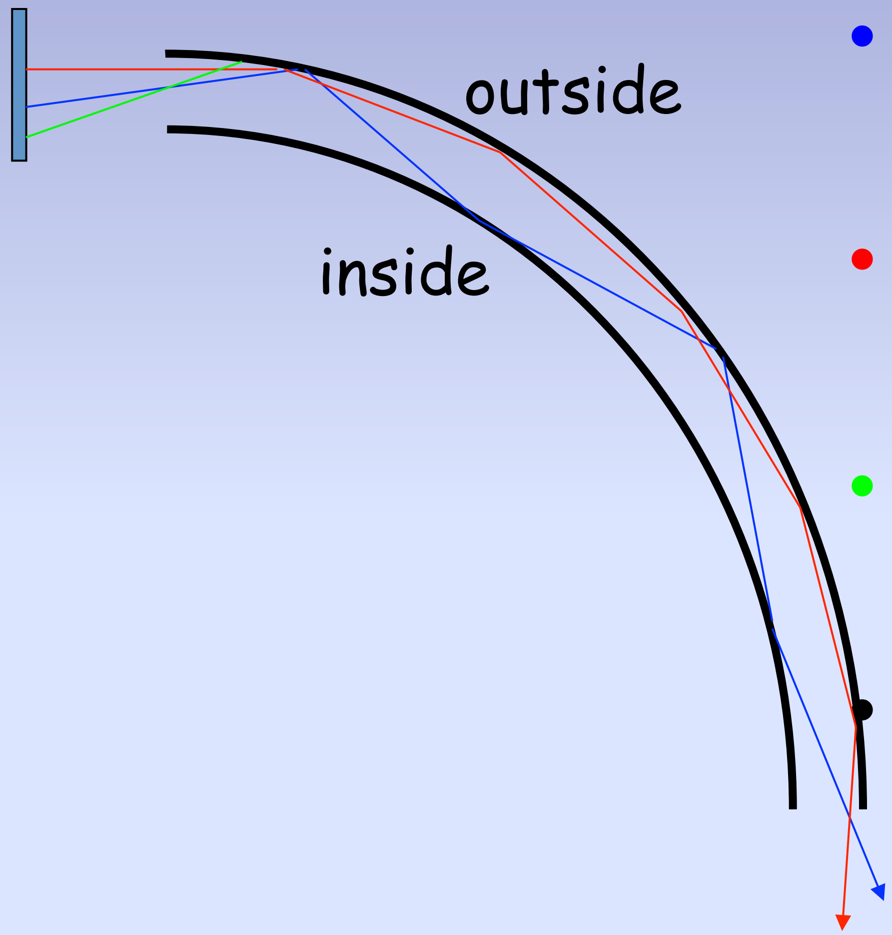
- Divergence quickly becomes very large
- Large critical angle is required in focusing section
- Rule of thumb:
- Focusing factor = Ratio of critical angles
- End up with “several beams”
- Liouville: focus in real space, defocus in divergence space

Curved Guides



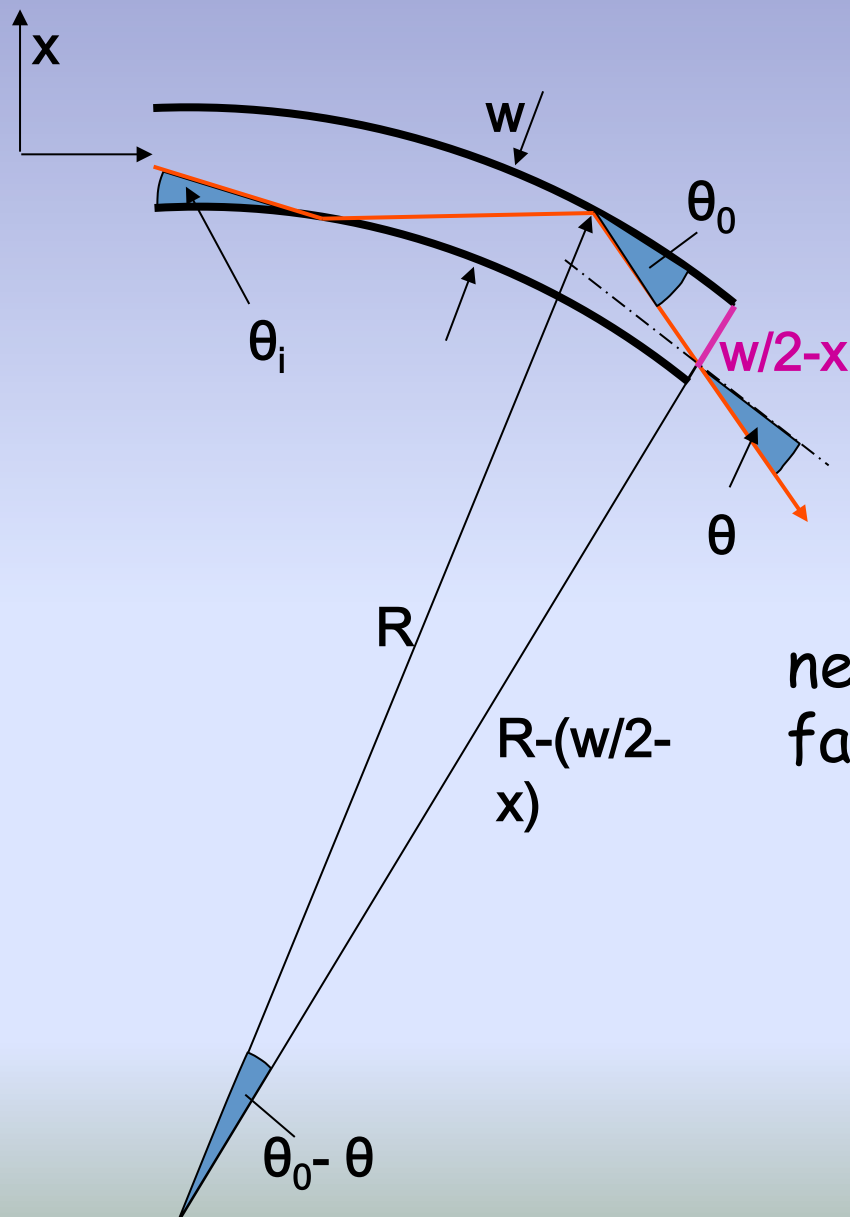
- Avoid direct line-of-sight
- Avoid gammas
- Avoid fast neutrons
- Reduce background

Curved Guides



- Blue – reflecting from both sides
- Red – garland reflections
- Green – exceeds critical angle
- Fewer neutrons along inside face - quantify

Curved Guides



$$\frac{R-(w/2-x)}{R}=\frac{\cos\theta_0}{\cos\theta}$$

For $\theta_0, \theta \ll 1$,

$$2x - w = R\left(\theta^2-\theta_0^2\right)$$

neutrons "just" reach inside face when $\theta = 0$ for $x = -w/2$:

$$\theta_0^* = \sqrt{2w/R}$$

Curved Guides

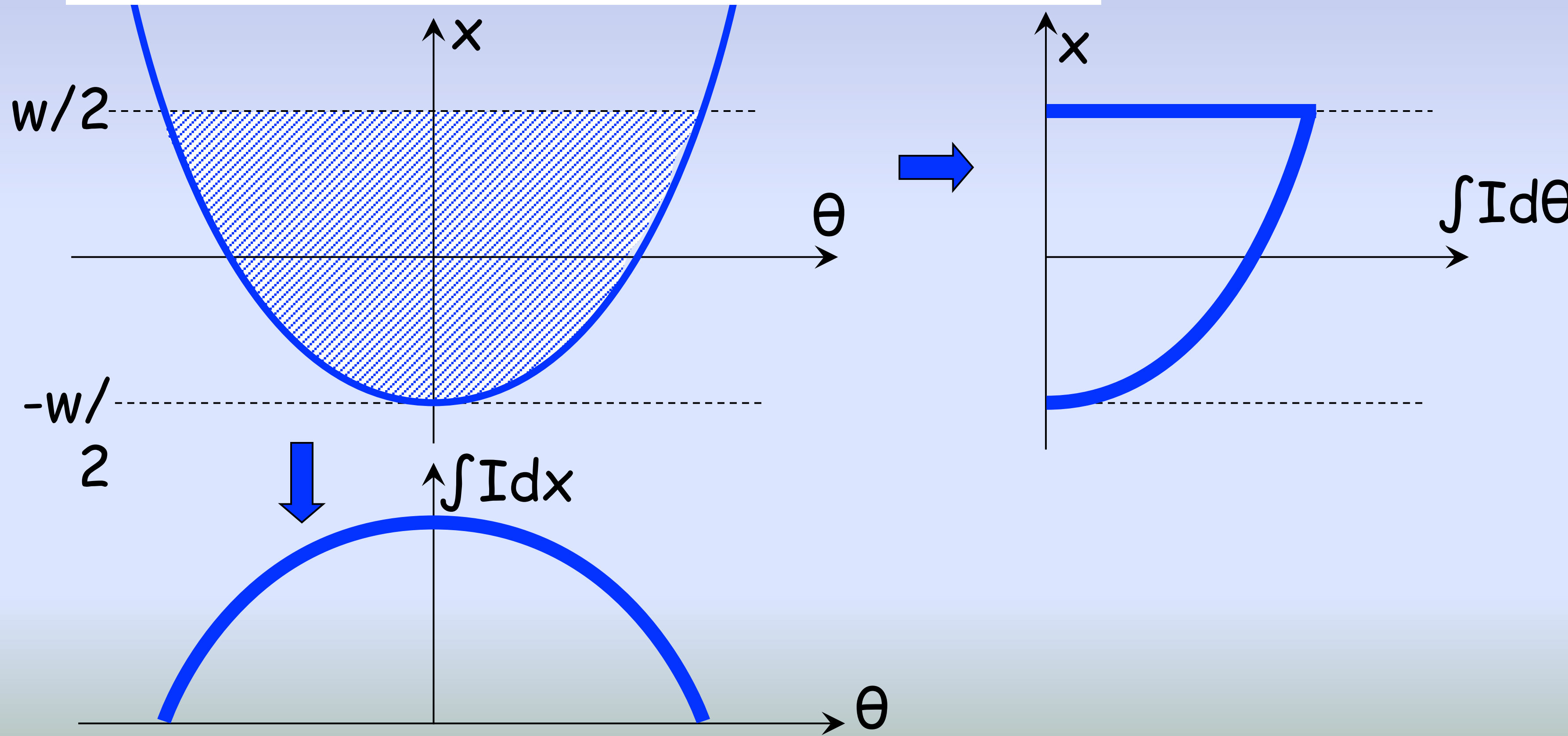
$$2x - w = R\left(\theta^2 - \theta_c^2\right)$$

$$\theta_0^* = \sqrt{2w/R}$$

$$\boxed{\begin{aligned} \lambda &= \lambda^* \\ \theta_c &= \theta_0^* \end{aligned}}$$

for natural Ni,

$\theta_c = \lambda[\text{Å}]\times 0.1°$



Curved Guides

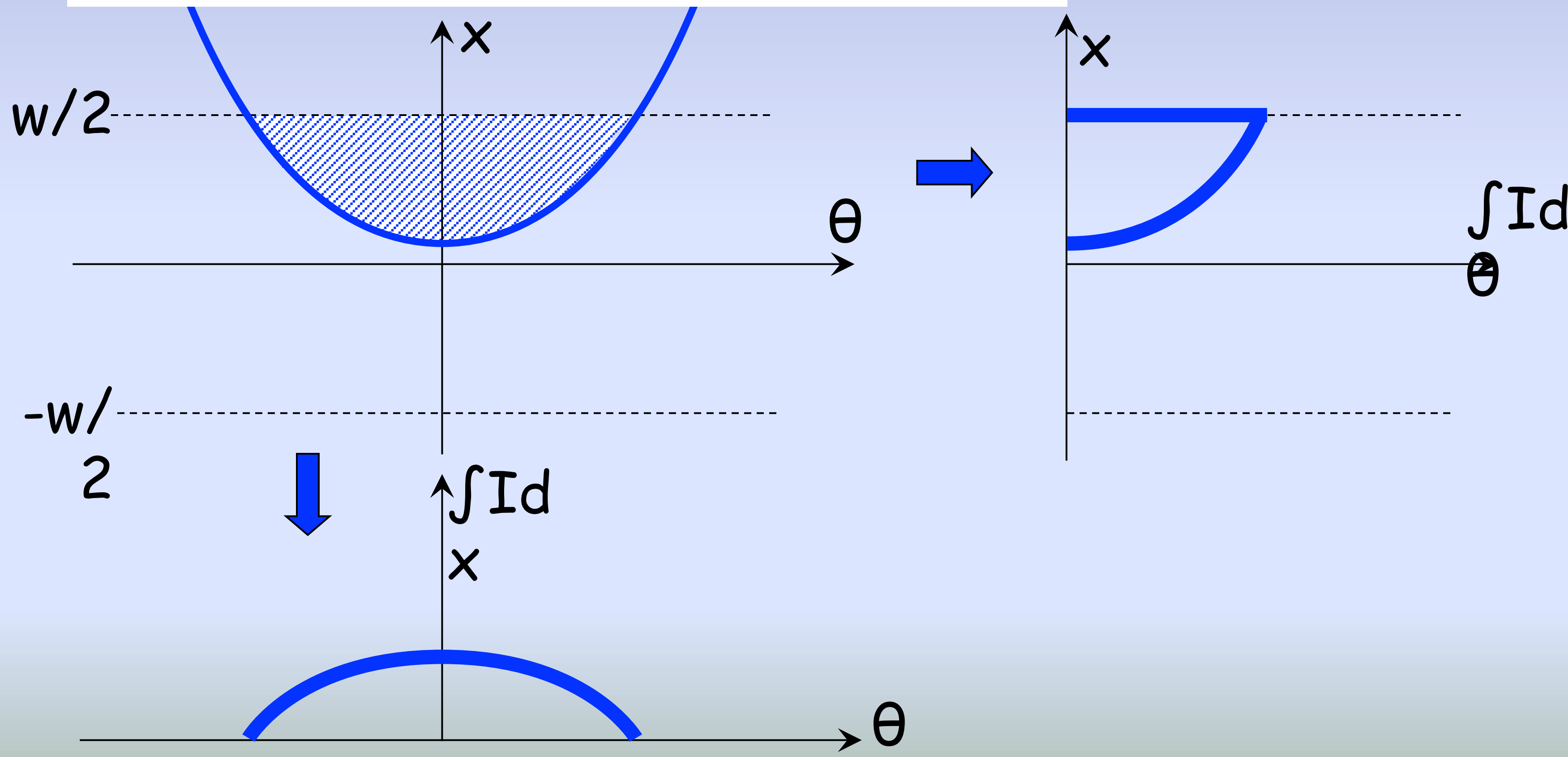
$$2x - w = R\left(\theta^2 - \theta_c^2\right)$$

$$\theta_0^* = \sqrt{2w/R}$$

$$\boxed{\lambda < \lambda^*}$$

for natural Ni,

$$\theta_c = \lambda[\text{Å}]\times 0.1°$$



Curved Guides

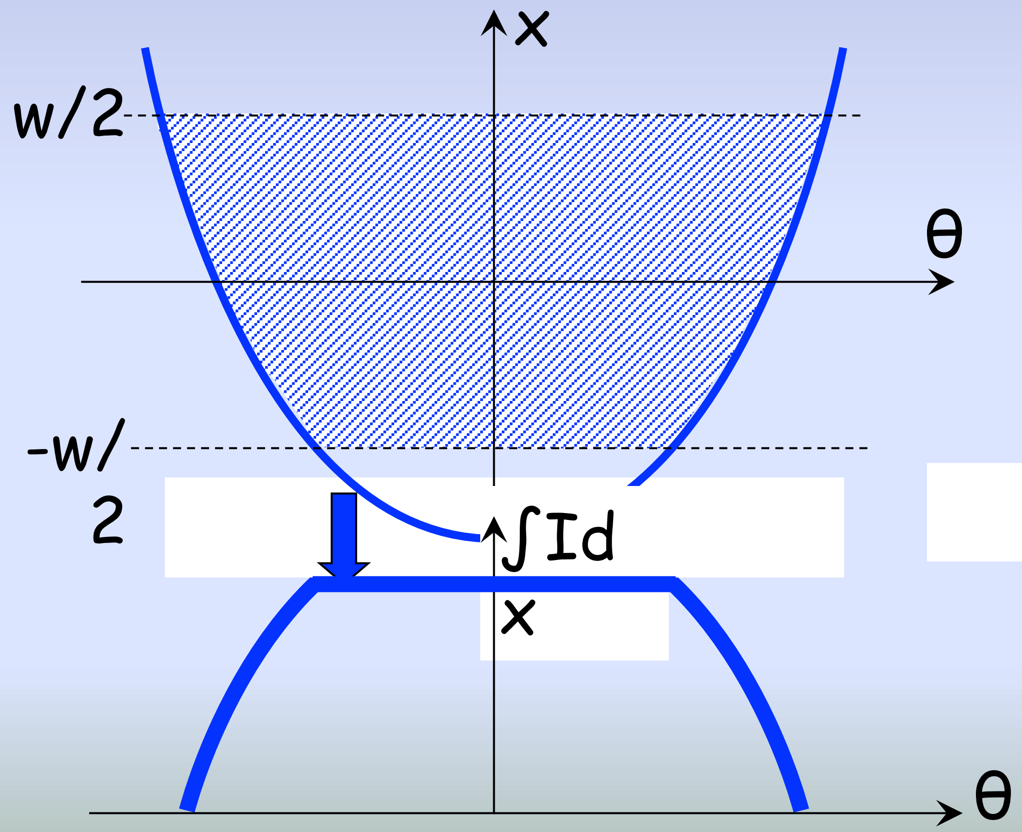
$$2x - w = R\left(\theta^2 - \theta_c^2\right)$$

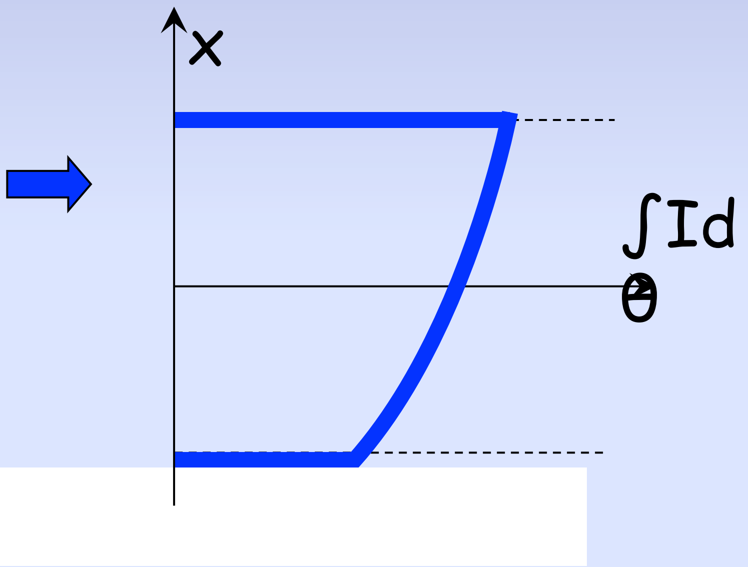
$$\theta_0^* = \sqrt{2w/R}$$

$$\boxed{\lambda > \lambda^*}$$

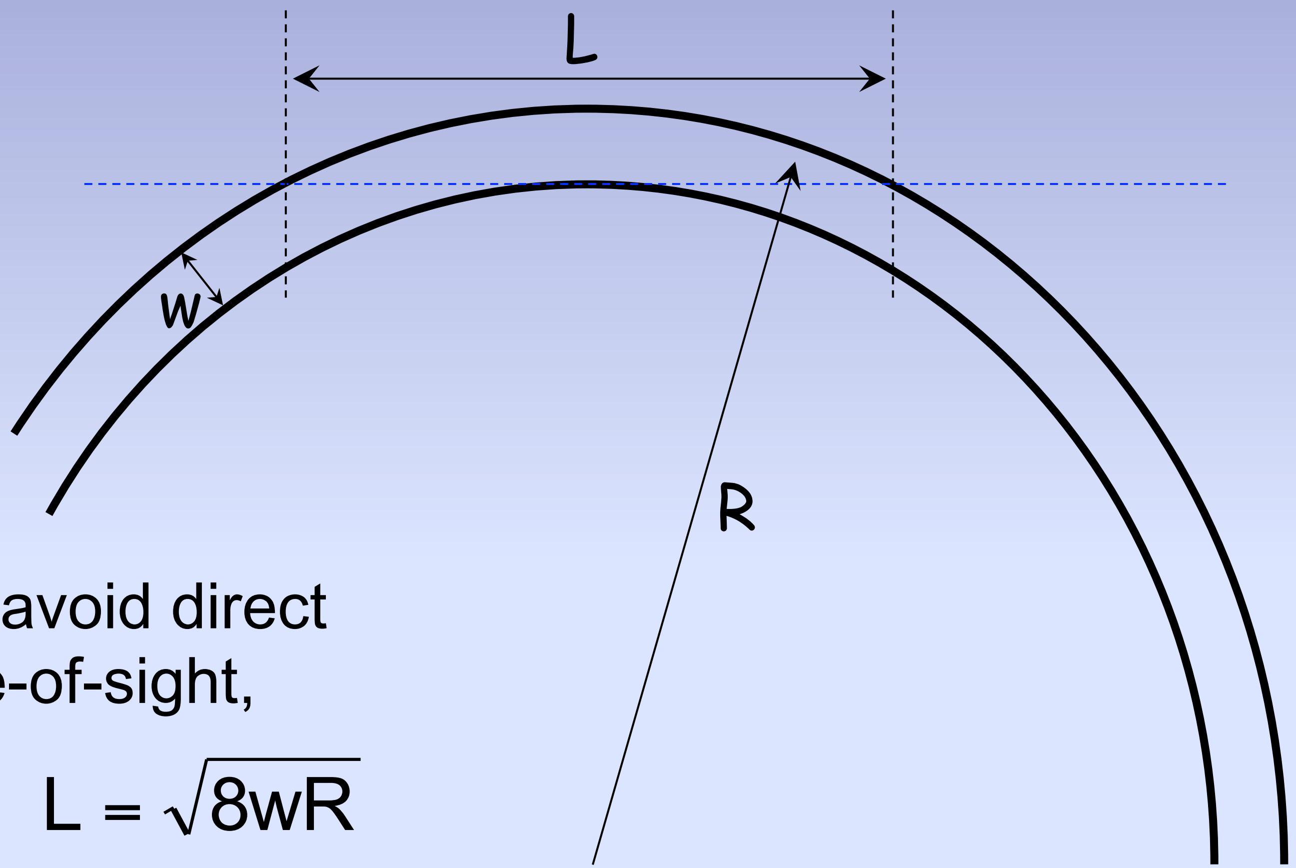
for natural Ni,

$\theta_c = \lambda[\text{Å}]\times 0.1°$





Curved Guides



To avoid direct line-of-sight,

$$L = \sqrt{8wR}$$

General rules

- Propagating in real space produces a shear of the phase space diagram but does not change the area of phase space diagram nor does it increase the phase space density.
- Guide dimension cuts the phase space distribution, and the mirror flips that section by 180°.
- Never flip a section of phase space onto another as this would violate the Liouville theorem.

The End