

Neutron Optics Technologies

3rd April 2016

Ken Andersen

Announcements

- Social Dinner Monday 4/4 at 20:30 at ULISSE
 - Couscous and other Sicilian specialities
- Excursion Tuesday 5/4 to SEGESTA
 - 13:00 Bus Departure
 - 13:45 Arrival at Segesta
 - 14:00 Lunch
 - 15:30 Visit to Archeological Site
 - 17:30 Bus Departure
- Join SoNS facebook group

Slow Neutrons vs Light

	light	neutrons
λ	$< \mu\text{m}$	$< \text{nm}$
E	$> \text{eV}$	$> \text{meV}$
n	$1 \rightarrow 4$	$0.99997 \rightarrow 1.00001$
θ_c	90°	1°
B	$10^{18} \text{ p/cm}^2/\text{ster/s}$ (60W lightbulb)	$10^{14} \text{ n/cm}^2/\text{ster/s}$ (60MW reactor)
spin	1	$\frac{1}{2}$
interaction	electromagnetic	strong force, magnetic
charge	0	0

Neutron Optics

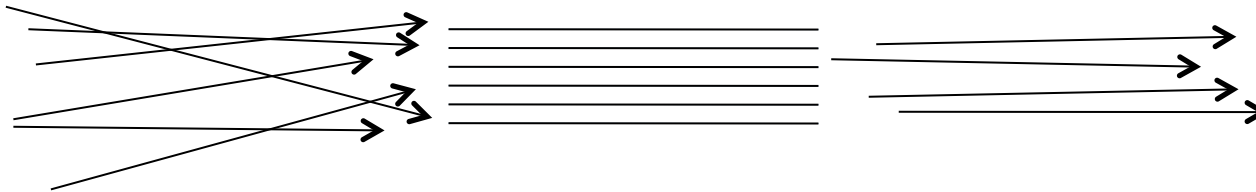
- Absorption
- Reflection
 - from surfaces: Snell's law
- Diffraction
 - from multilayers and crystals: Bragg's law
- Refraction
 - from materials and magnetic fields

Collimation

Reducing divergence

- improving resolution
- reducing background
- reducing flux

Soller collimator



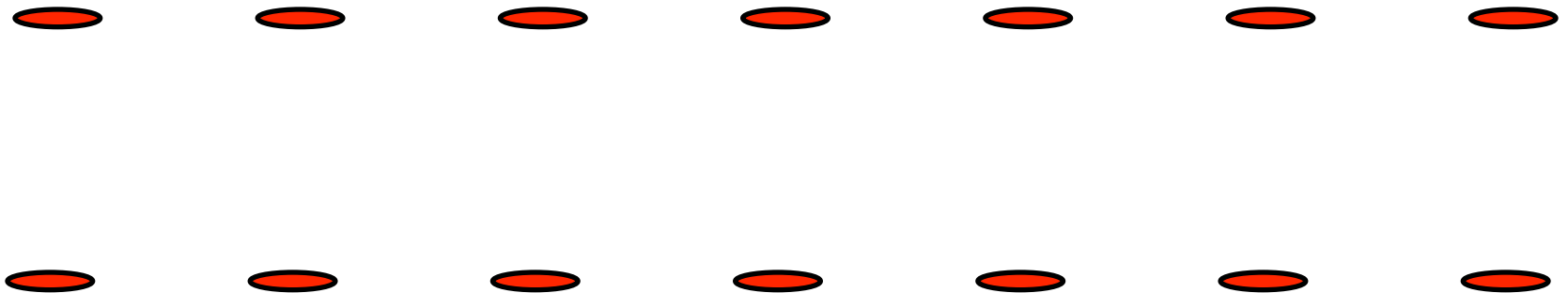
Pin-holes separated by distance



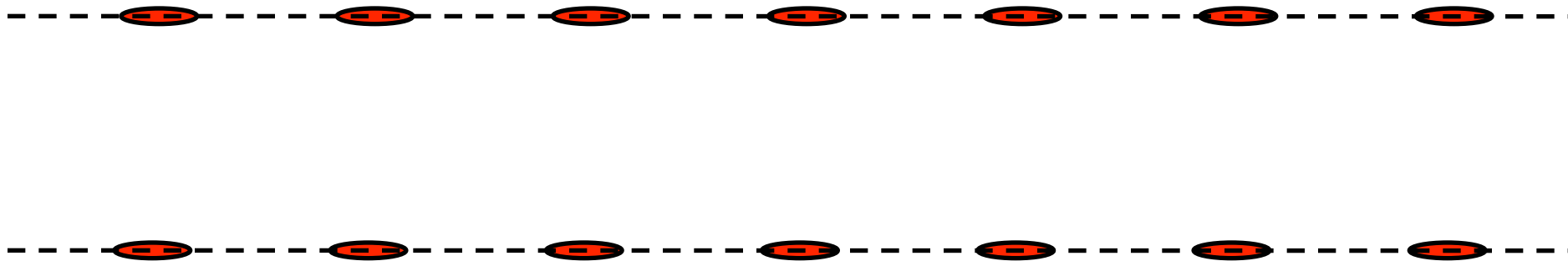
Neutron Optics

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 - from surfaces: Snell's law
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 - from multilayers and crystals: Bragg's law
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 - from materials and magnetic fields

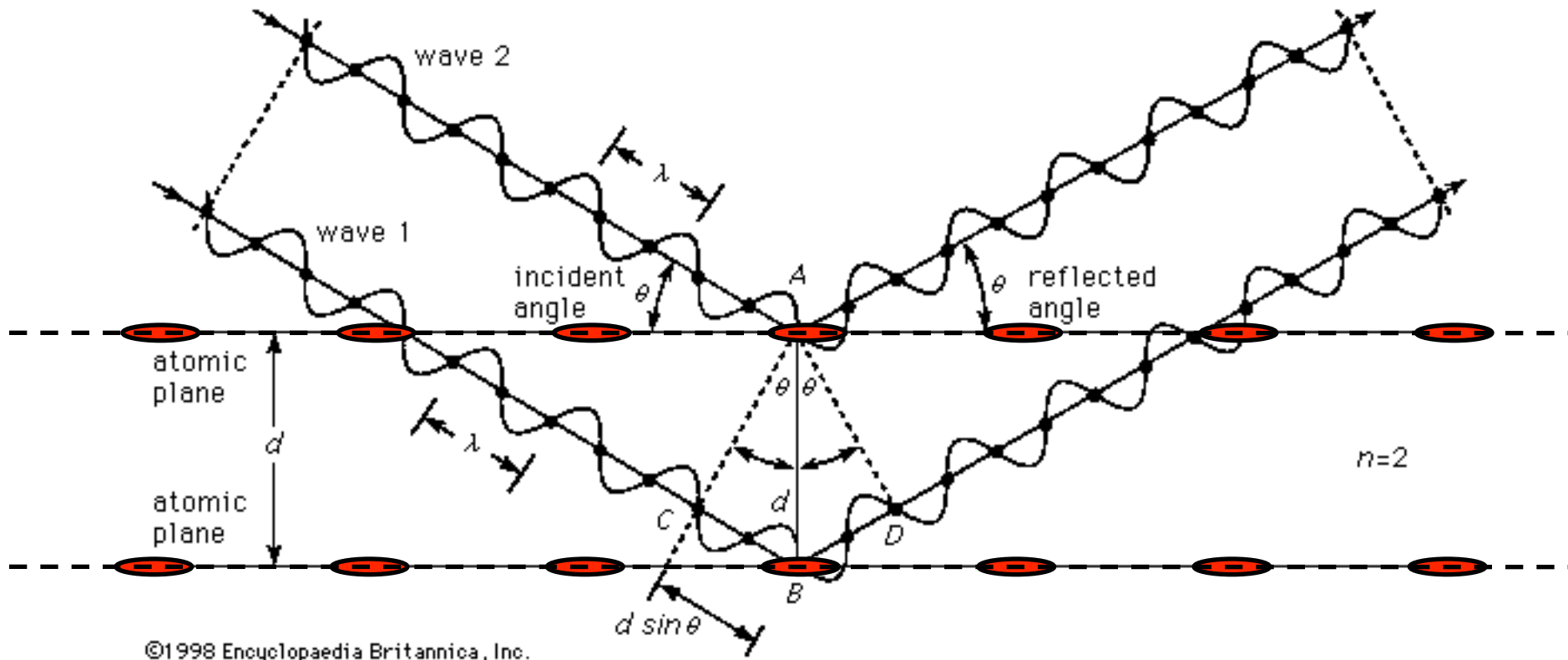
Diffraction: Bragg's Law



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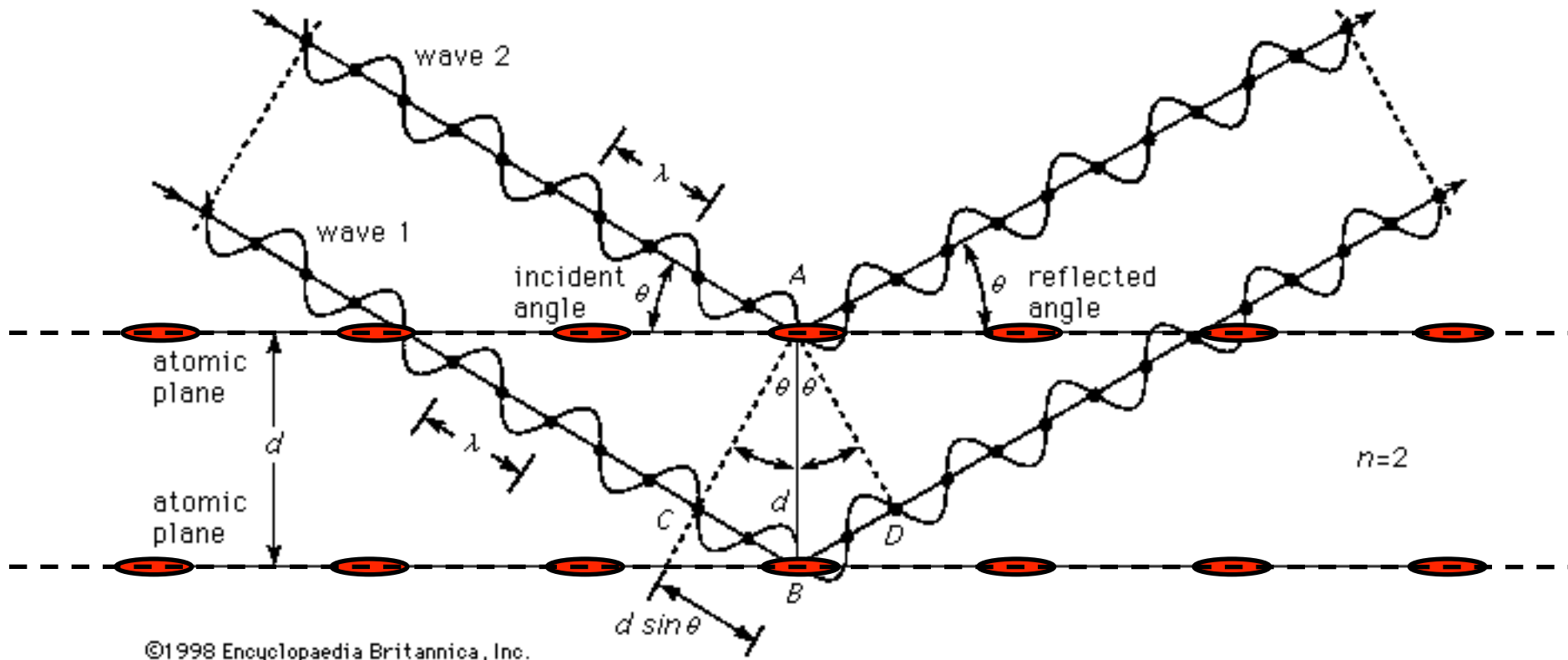


Diffraction: Bragg's Law



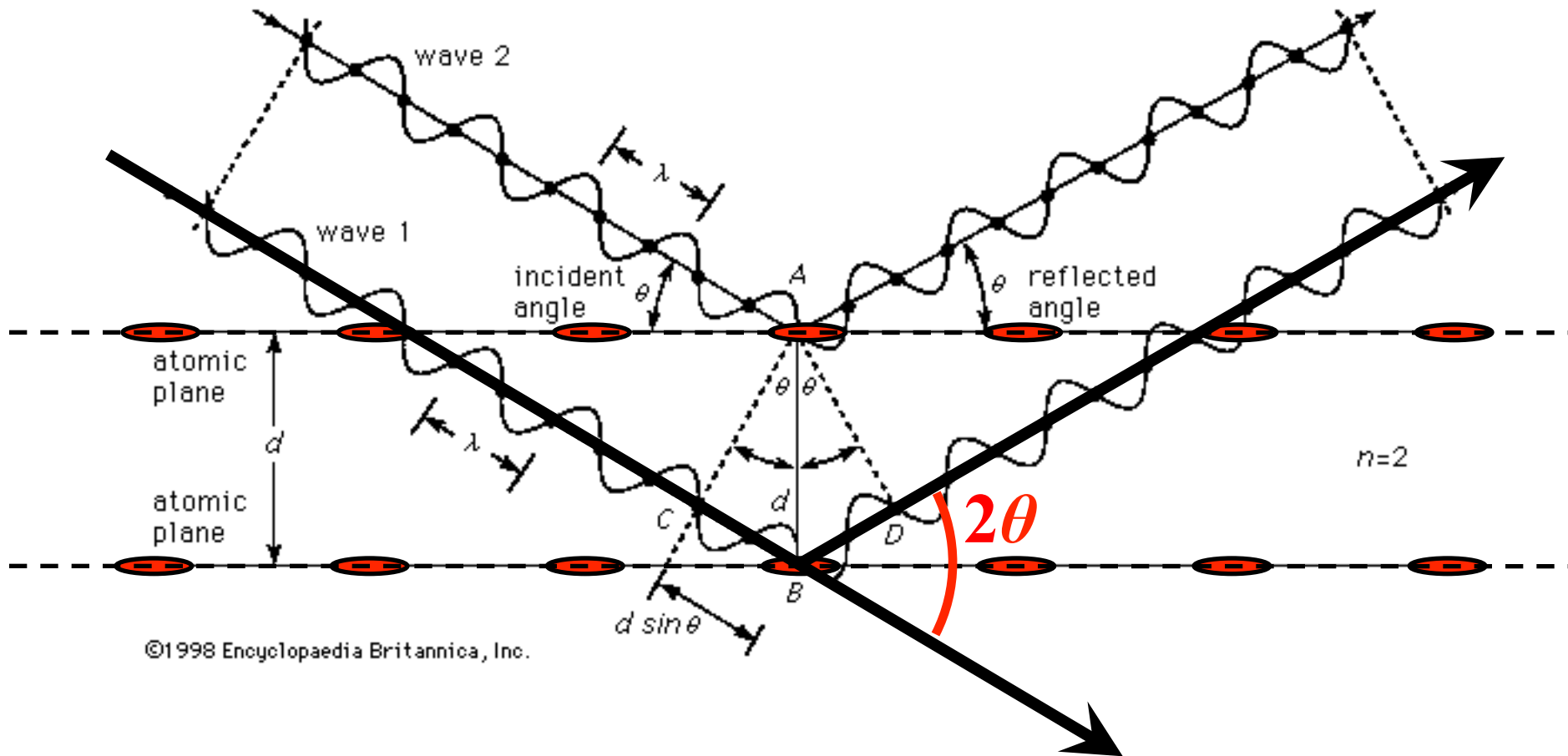
Diffraction: Bragg's Law

$$\lambda = 2d \sin \theta$$



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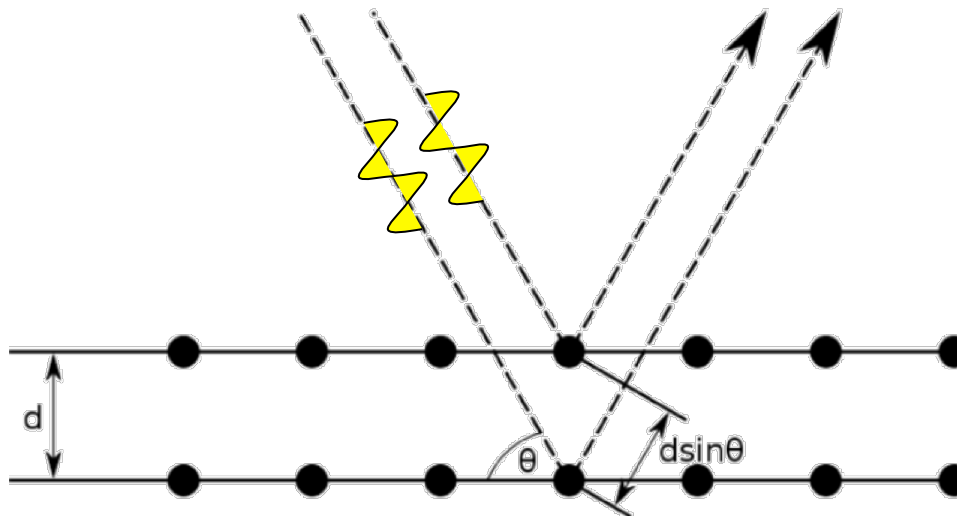


Diffraction: Bragg's Law

Wavevector:

$$k = \frac{2\pi}{\lambda} \quad p = \hbar k$$

$$\lambda = 2d \sin \theta$$

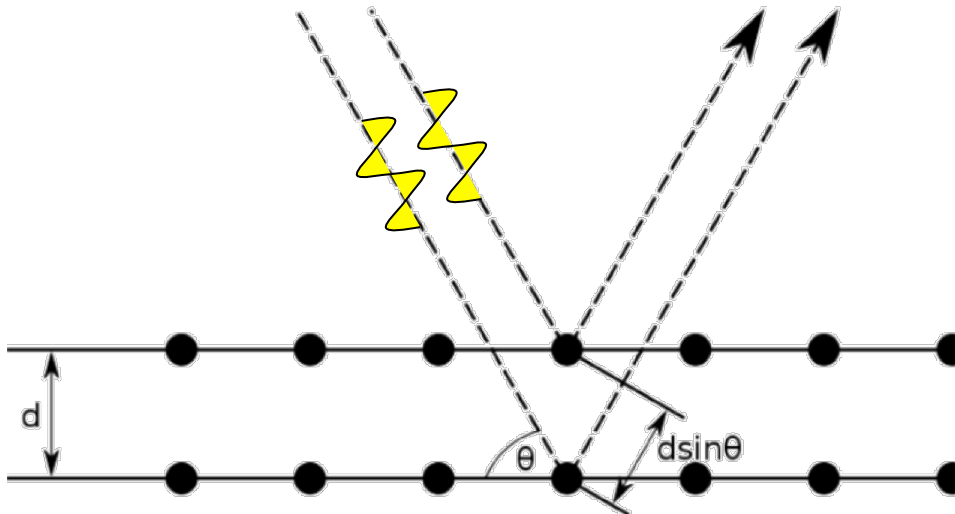


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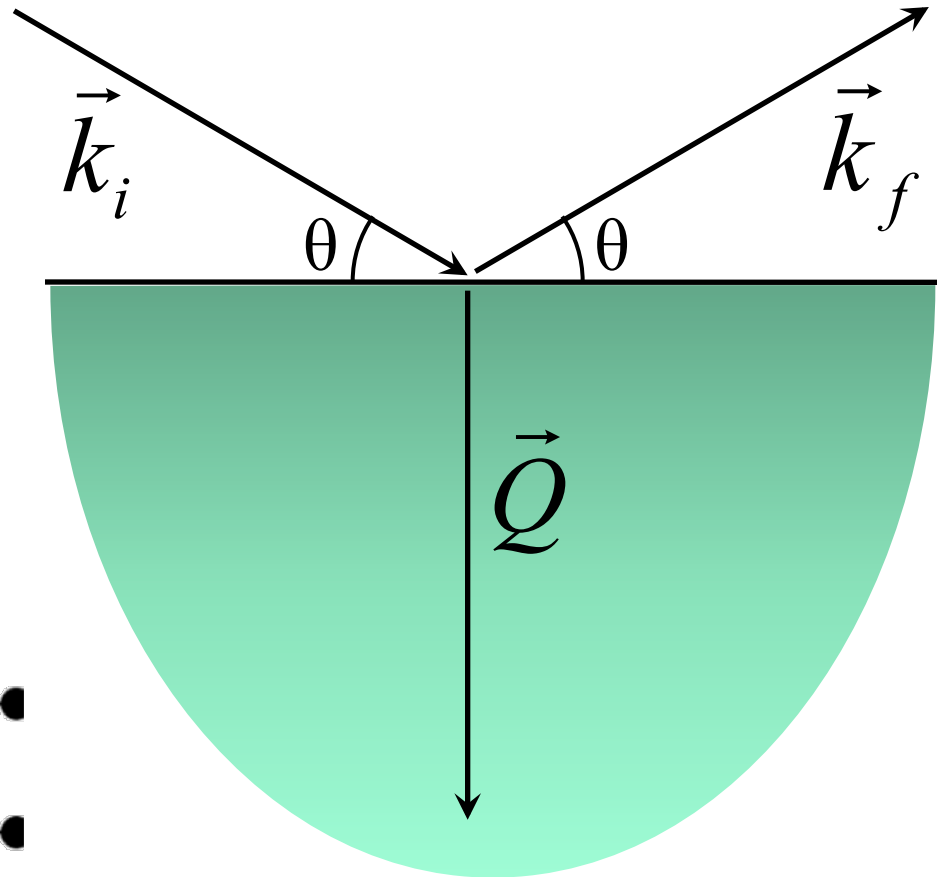
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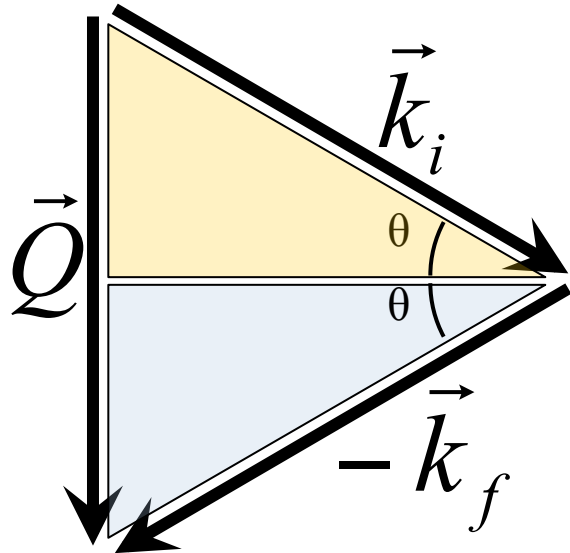
$$|\vec{k}_i| = |\vec{k}_f| = k$$



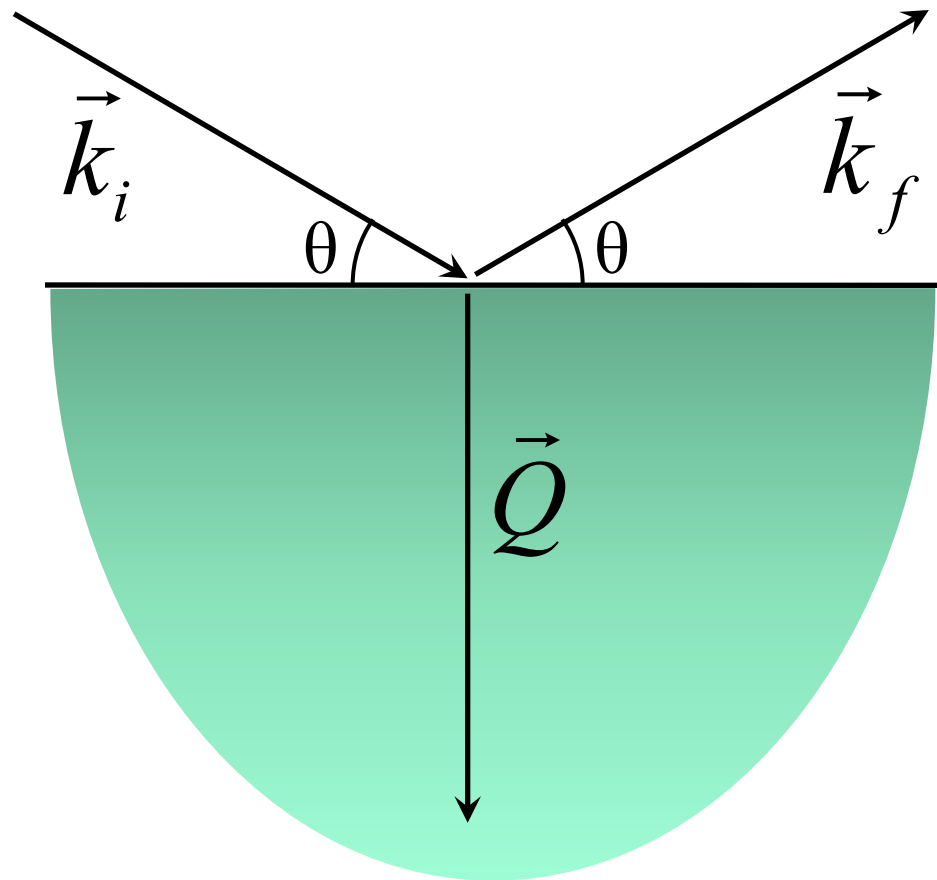
Diffraction: Bragg's Law

$$\vec{k}_i = \vec{k}_f + \vec{Q}$$

$$\Rightarrow \vec{Q} = \vec{k}_i - \vec{k}_f$$



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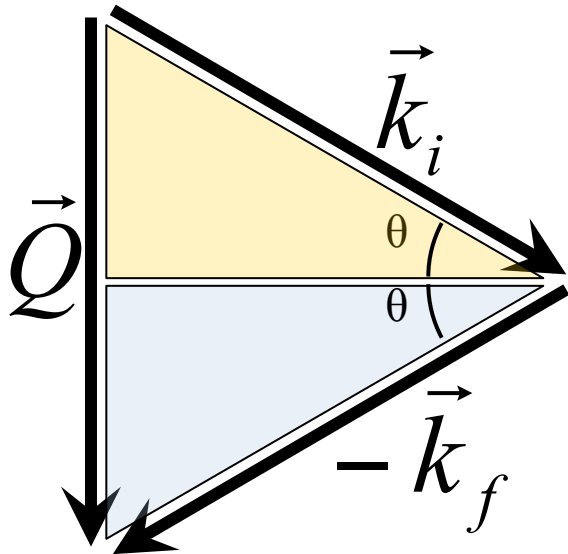


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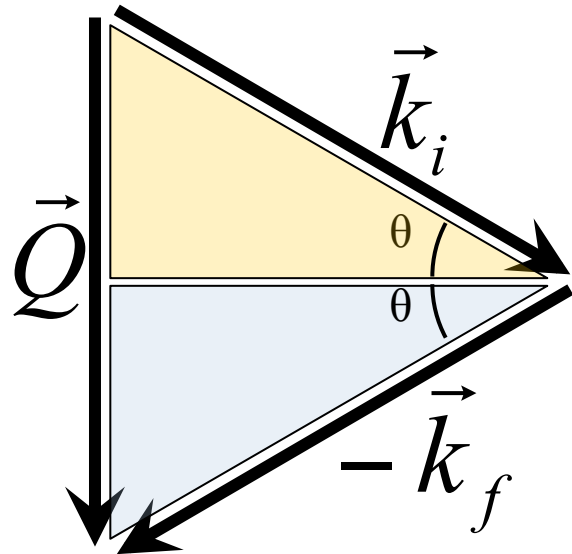
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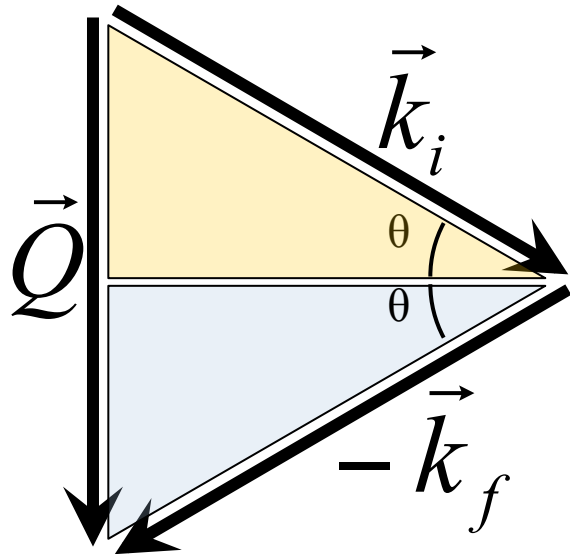
$$\lambda = 2d \sin \theta$$

$$k = \frac{2\pi}{\lambda}$$

Diffraction: Bragg's Law

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$$Q = 2k \sin \theta$$

$$\lambda = 2d \sin \theta$$

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Bragg's Law:
$$Q = \frac{2\pi}{d}$$

Diffraction: Bragg's Law

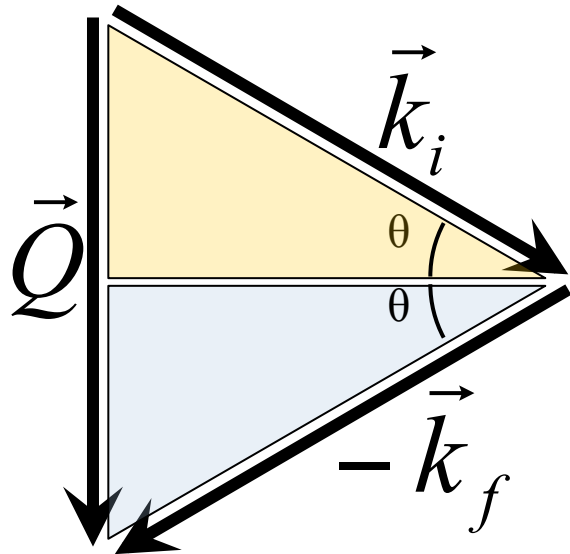
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$$Q = 2k \sin \theta$$

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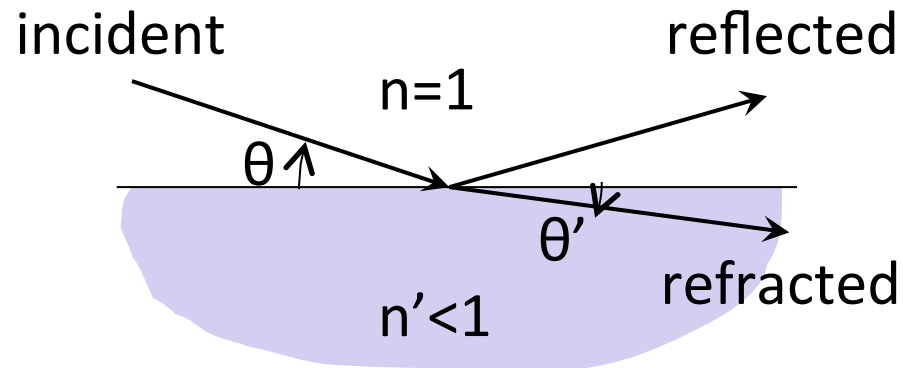


Bragg's Law:

$$Q = \frac{2\pi}{d}$$

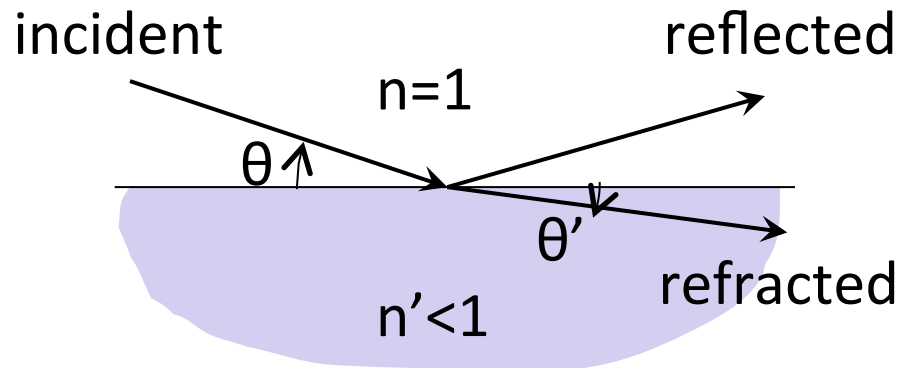
Conversion: $Q = 4\pi \sin \theta / \lambda$

Reflection: Snell's Law



$$\frac{\cos \theta}{\cos \theta'} = \frac{v_1}{v_2} = \frac{n'}{n} = n'$$

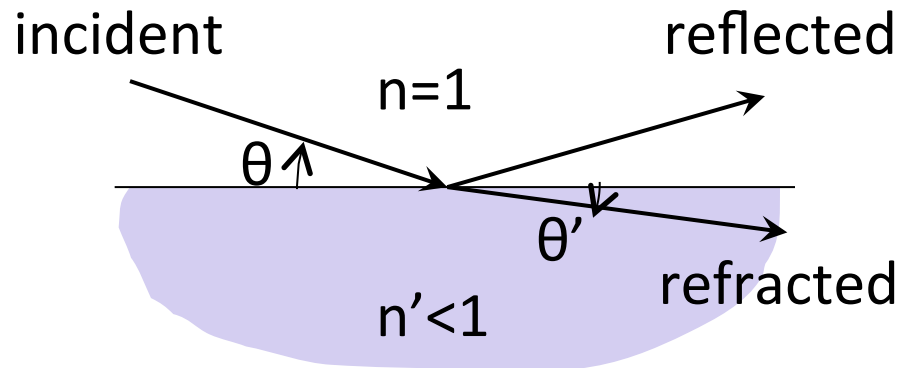
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$\theta' = 0$: critical angle of total reflection θ_c

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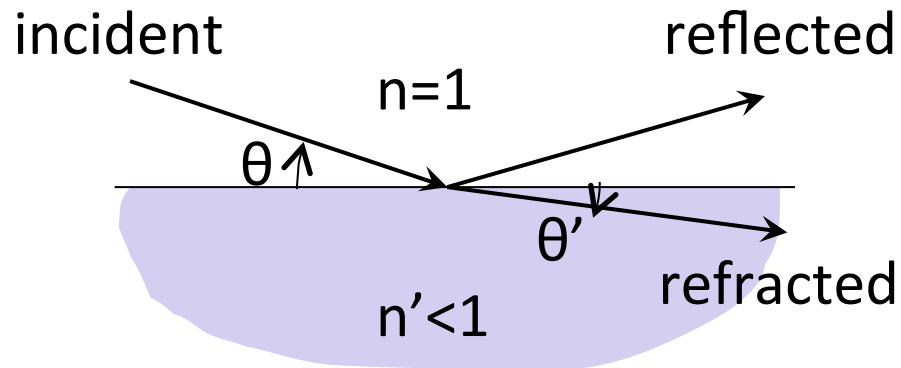


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$\theta' = 0$: critical angle of total reflection θ_c

$$\left. \begin{aligned} \cos \theta_c &= n'/n = n' \\ n' &= 1 - \frac{N\lambda^2 b}{2\pi} \\ \cos \theta_c &\approx 1 - \theta_c^2/2 \end{aligned} \right\} \Rightarrow \theta_c = \lambda \sqrt{Nb/\pi}$$

Reflection: Snell's Law



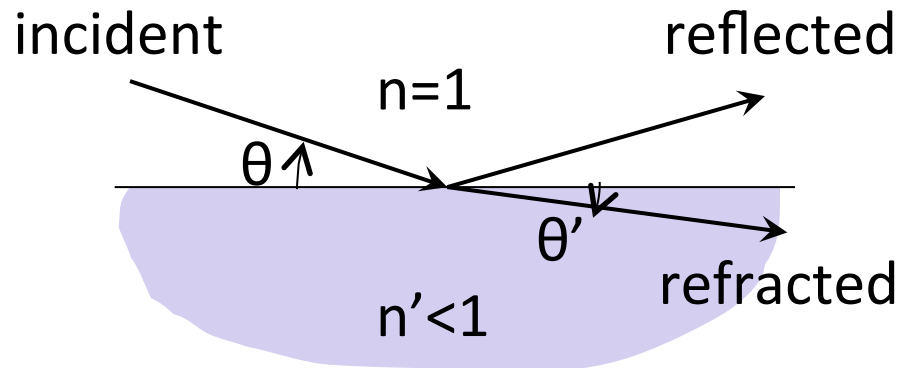
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for natural Ni,
 $\theta_c = \lambda[\text{\AA}] \times 0.1^\circ$
 $Q_c = 0.0218 \text{\AA}^{-1}$

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Definition:

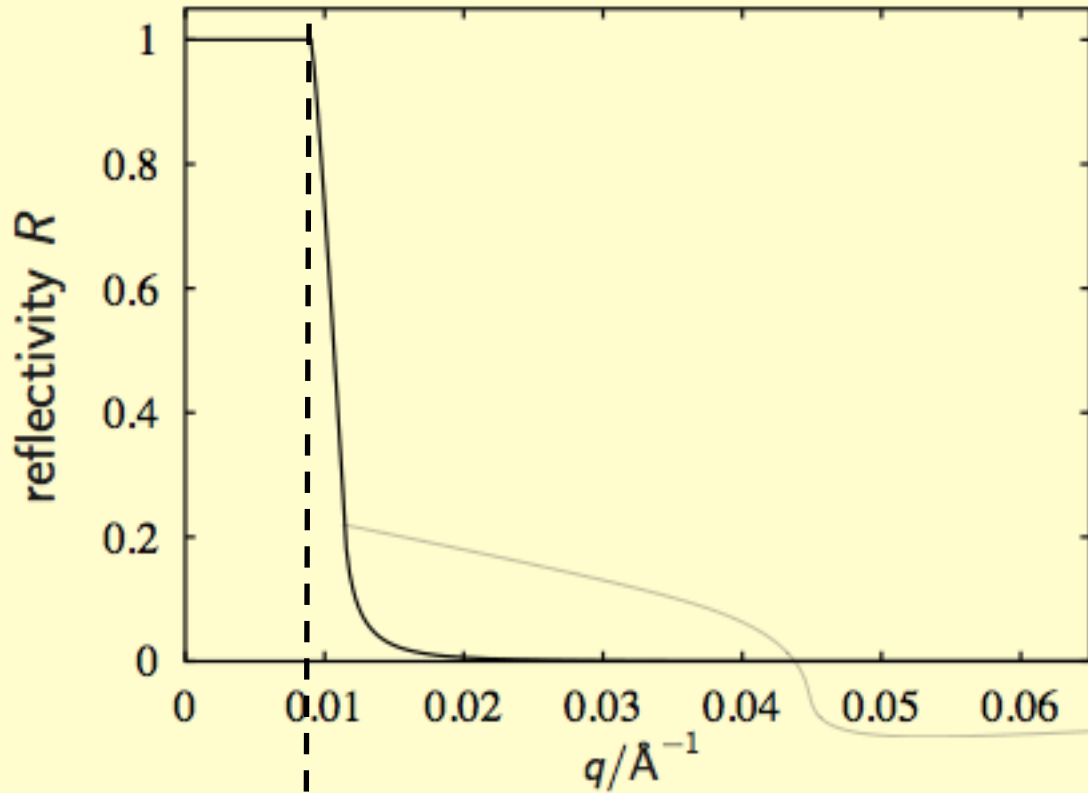
$$Q = 4\pi \sin \theta / \lambda$$

for natural Ni,

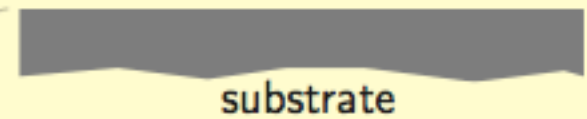
$$\theta_c = \lambda[\text{\AA}] \times 0.1^\circ$$

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Neutron Supermirrors

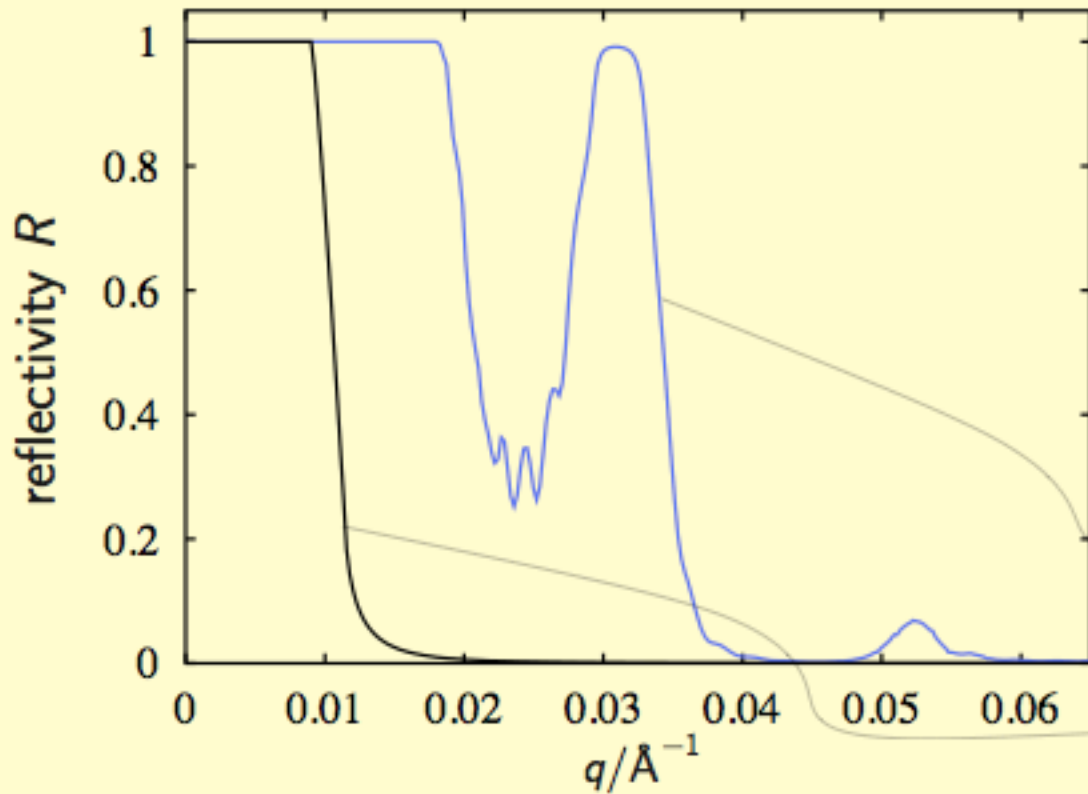


Q_c as given by Snell's Law

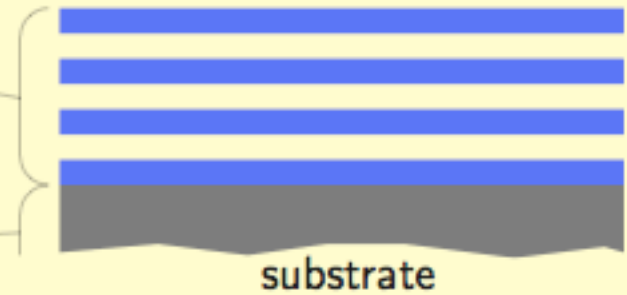


Courtesy of J. Stahn, PSI

Neutron Supermirrors

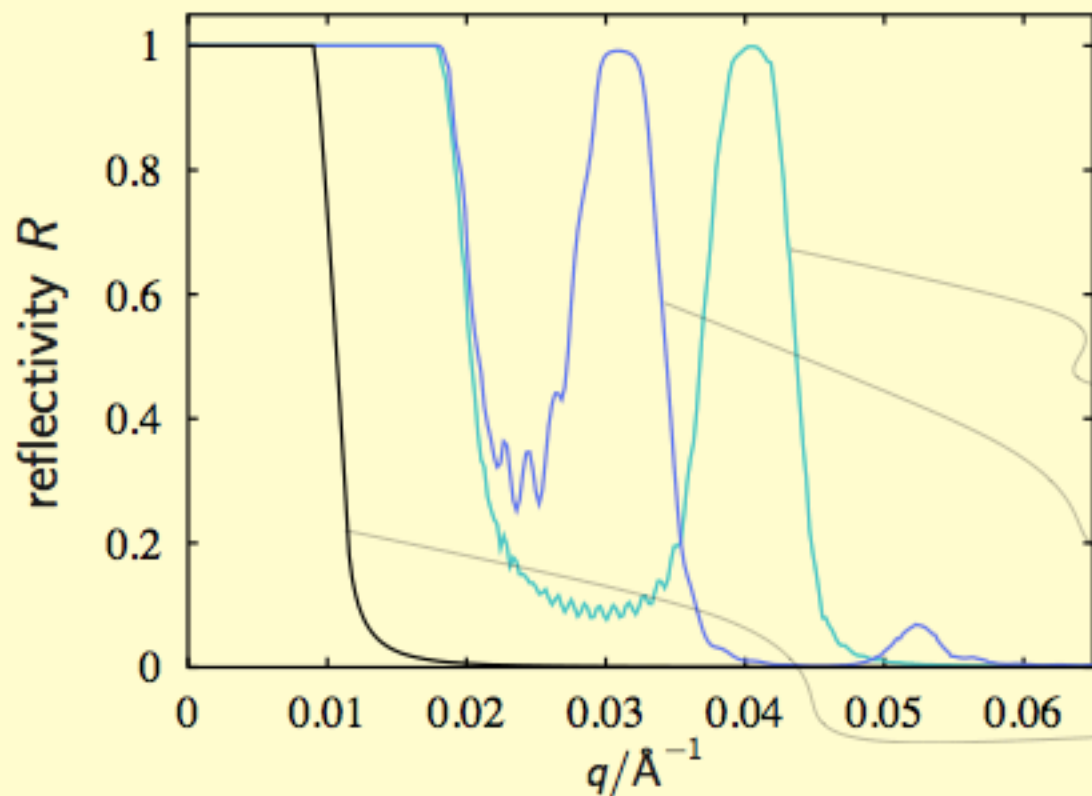


sketch of a multilayer

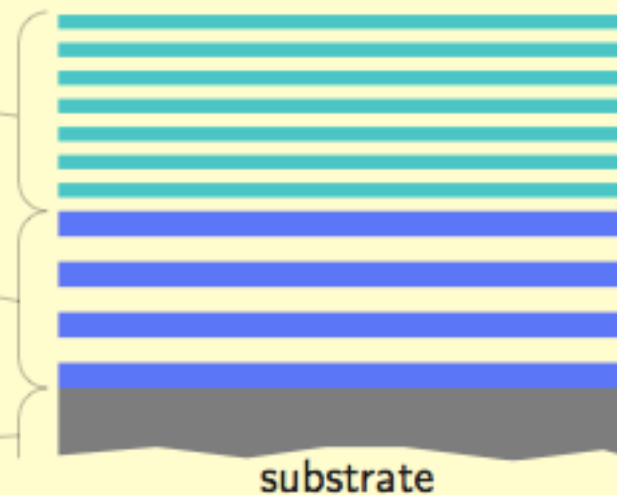


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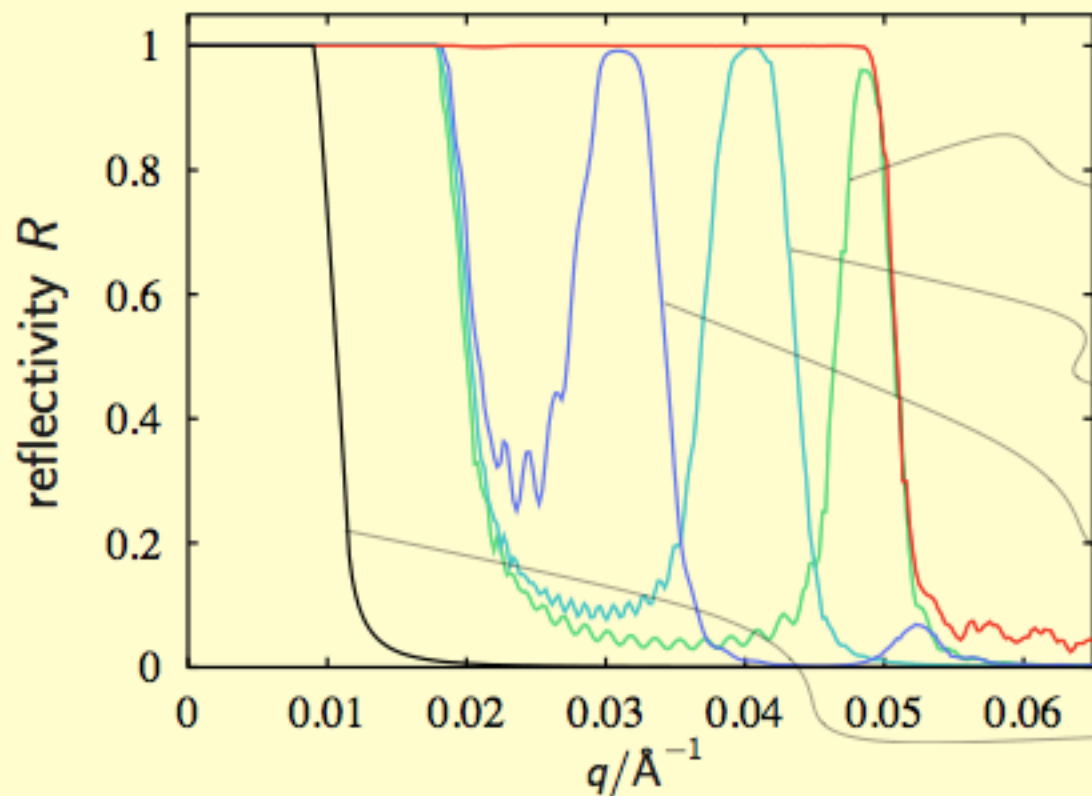


sketch of a multilayer stack

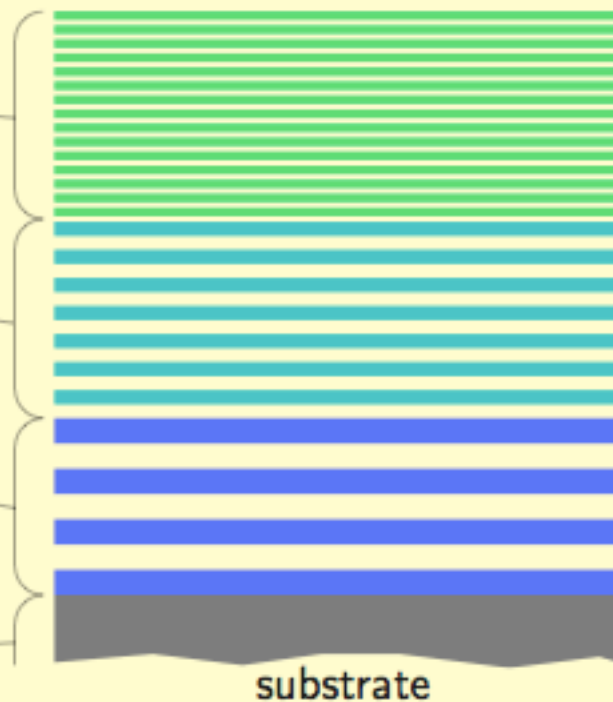


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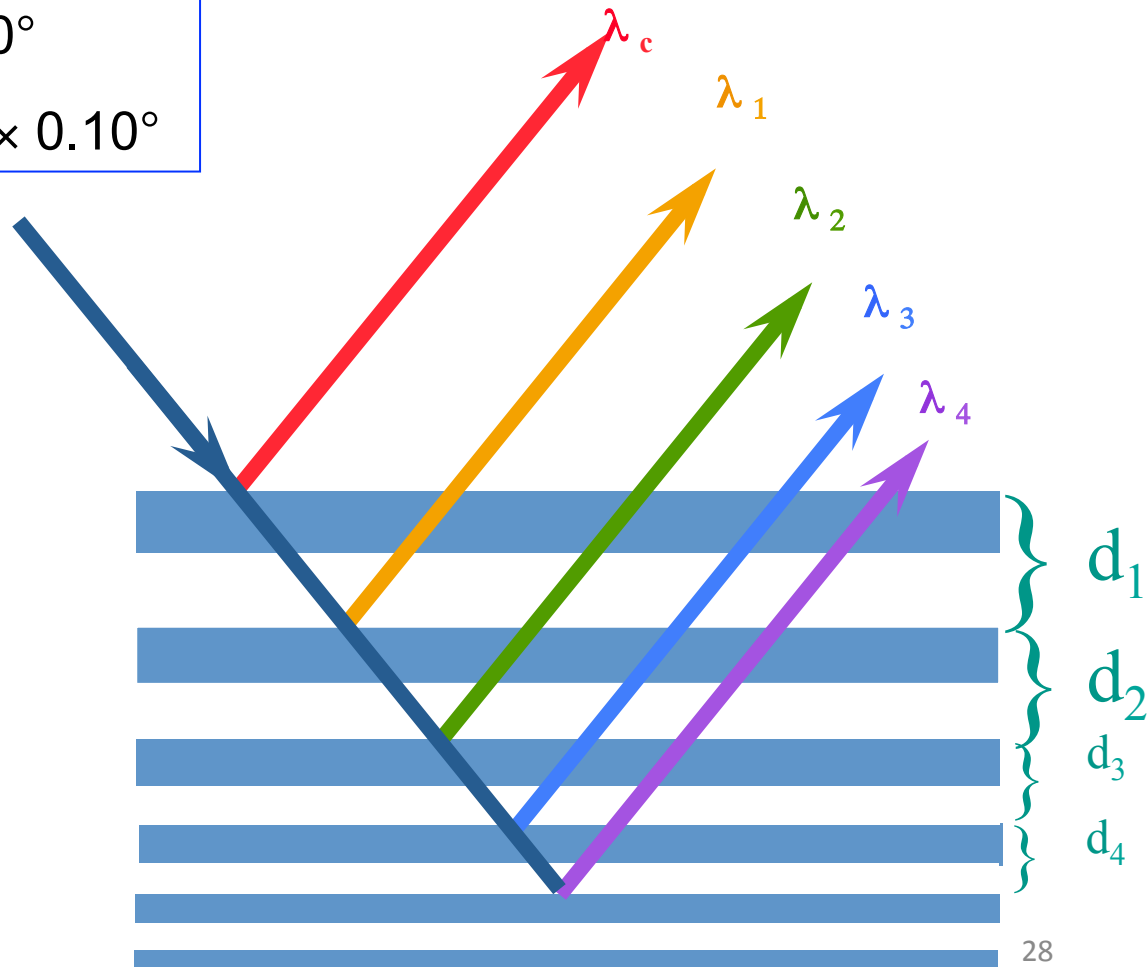


Courtesy of J. Stahn, PSI

Neutron Supermirrors

Reflection: $\theta_c(\text{Ni}) = \lambda[\text{\AA}] \times 0.10^\circ$

Multilayer: $\theta_c(\text{SM}) = m \times \lambda[\text{\AA}] \times 0.10^\circ$

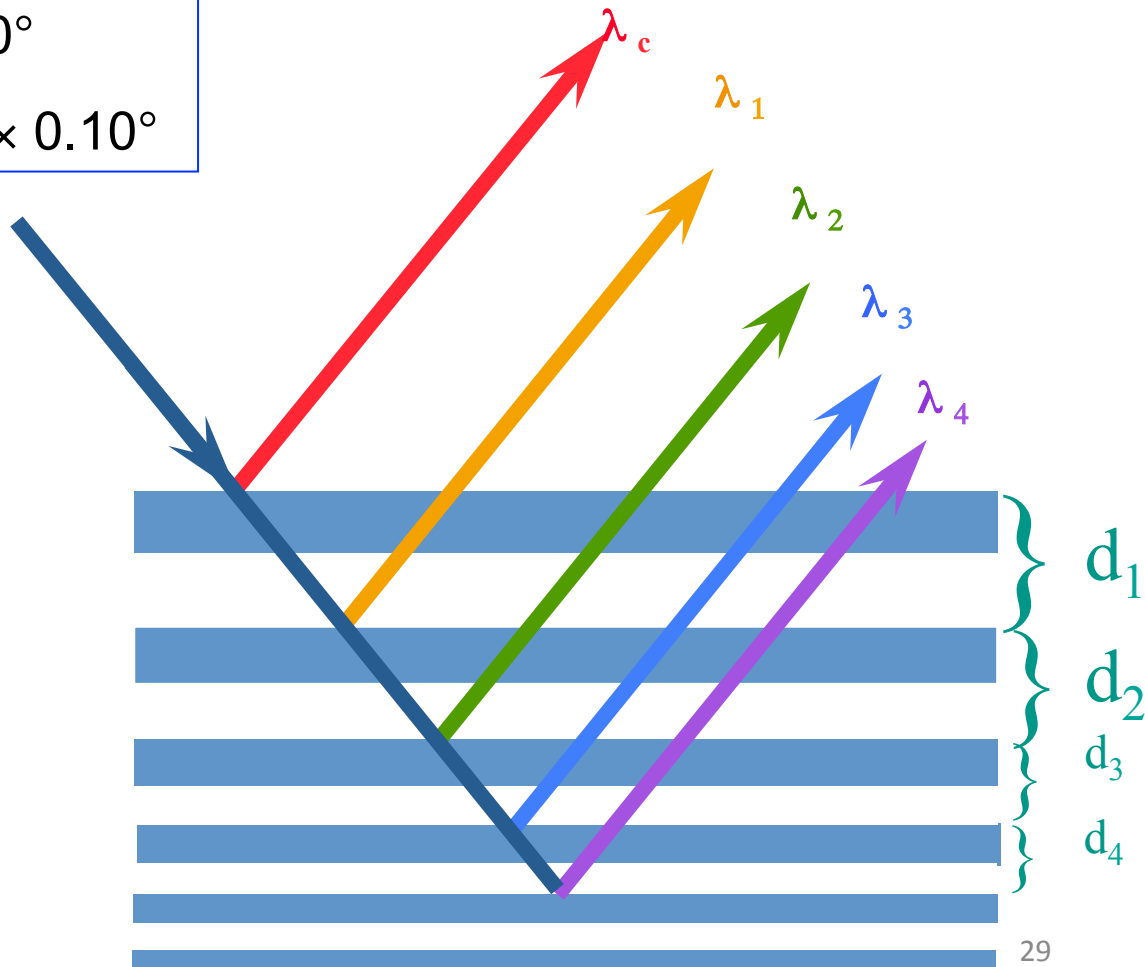


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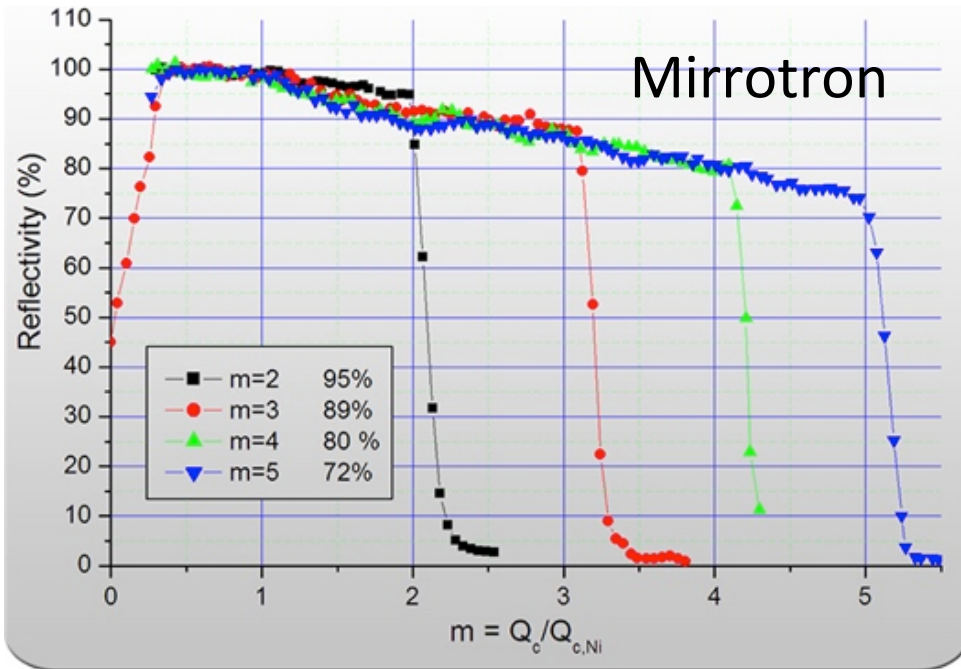
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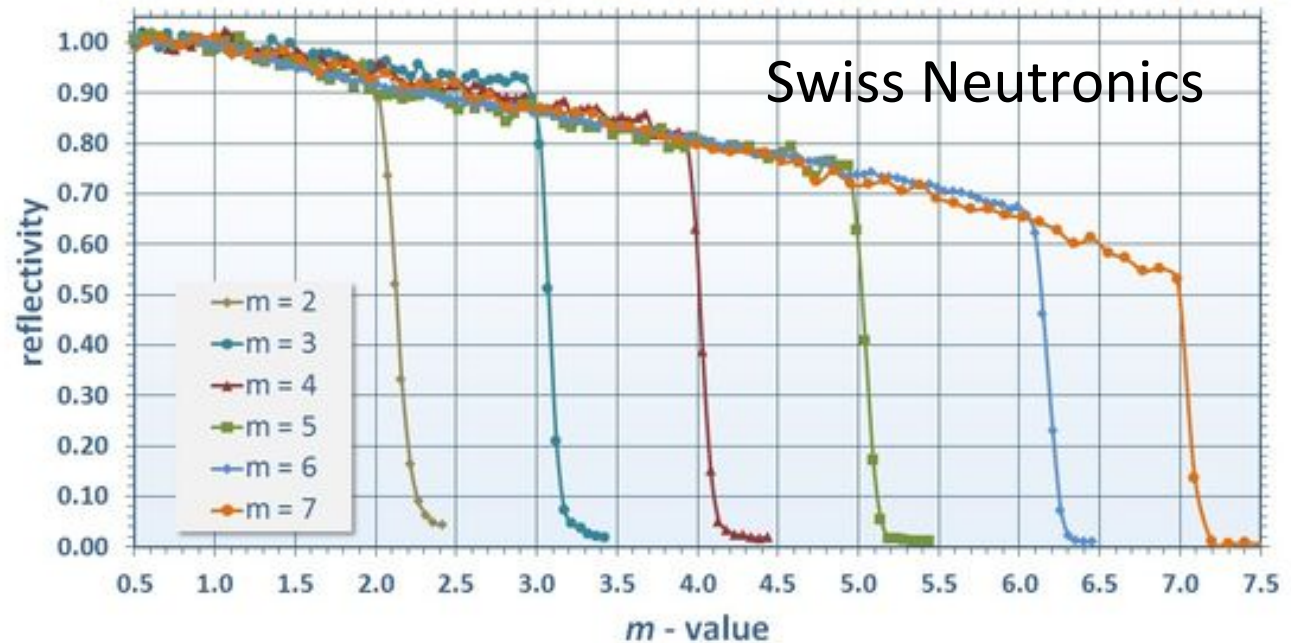
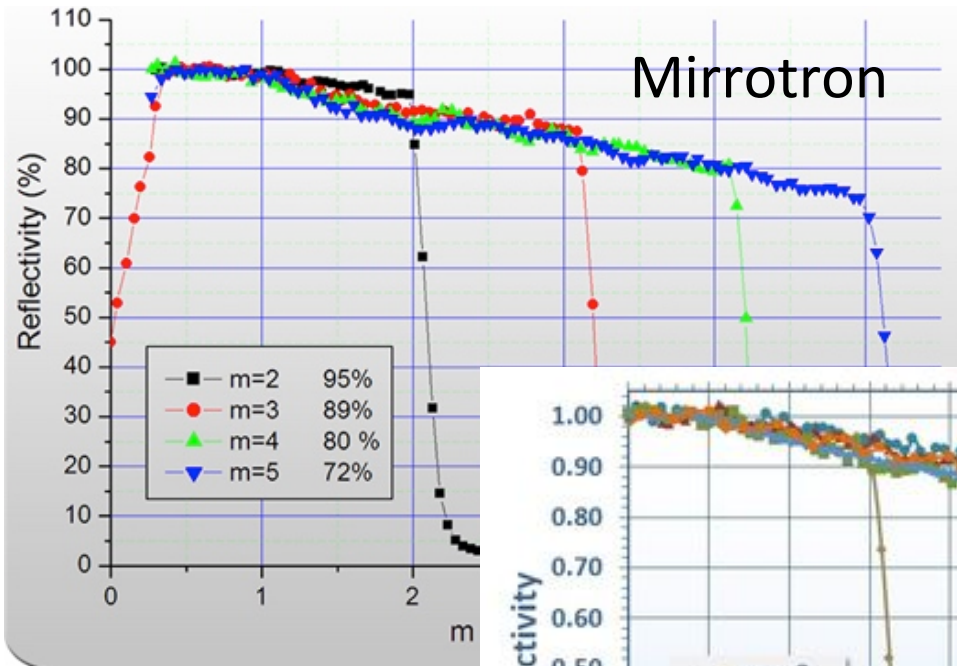
“m-number”
Supermirror critical angle



State-of-the-art Supermirrors



State-of-the-art Supermirrors



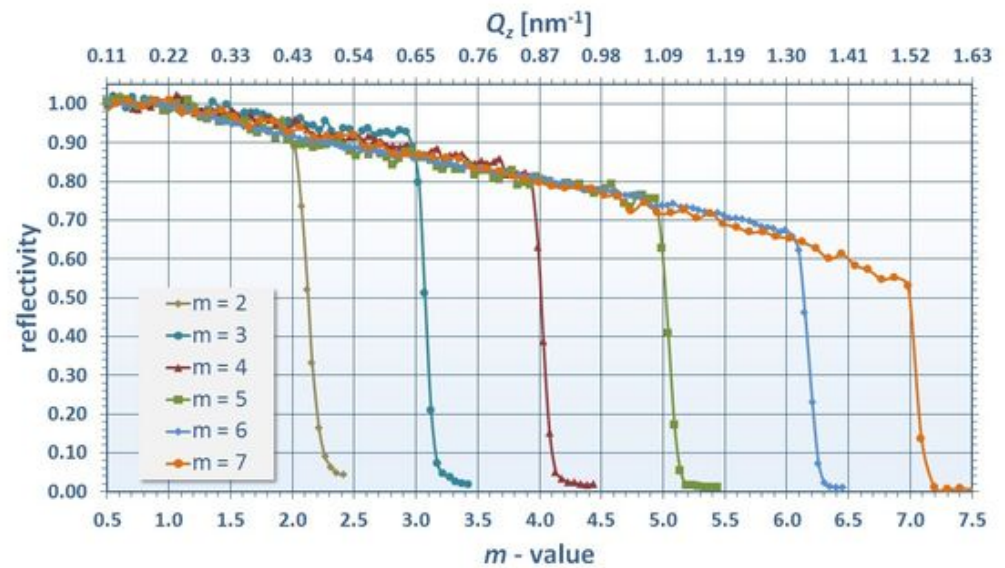
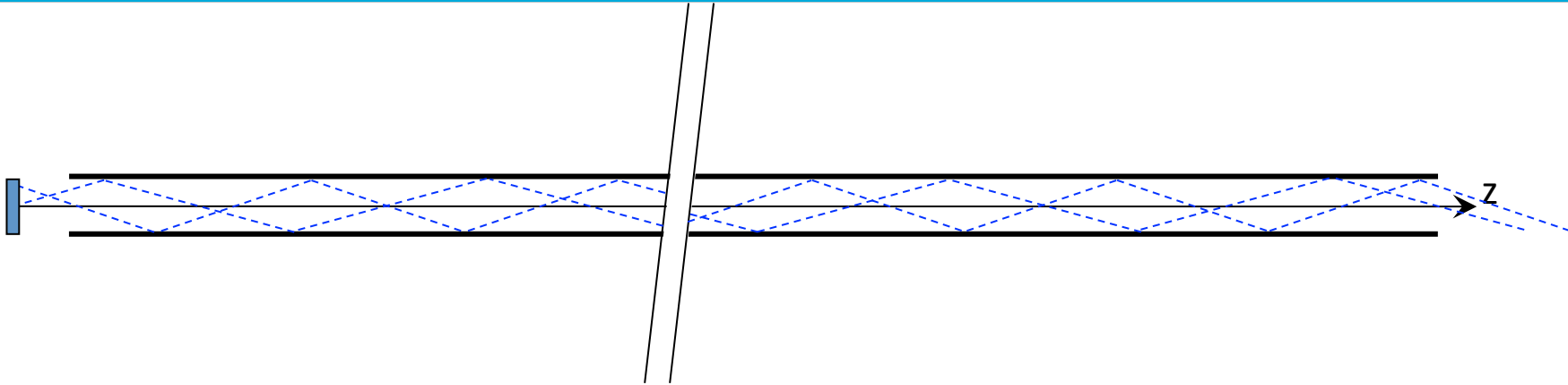
Neutron Guides



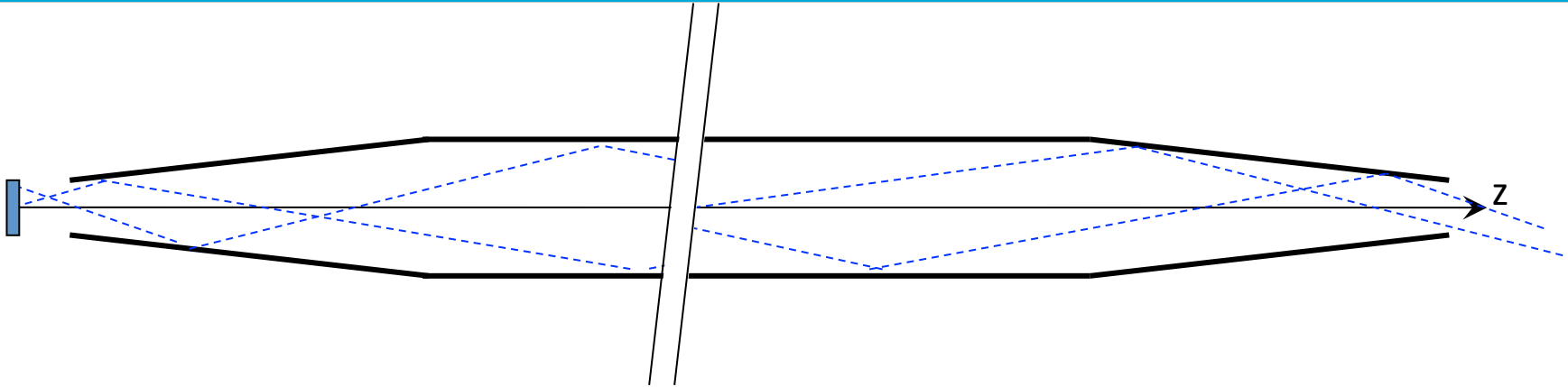
What are guides used for?

- Transport divergence
 - large m-numbers needed for short wavelengths
 - ballistic geometry required for supermirror guides
- Create space
 - build instruments far from neutron source
- Improve TOF resolution
- Reduce background
 - transport only “good” neutrons
- Focusing
 - increased divergence: increased flux

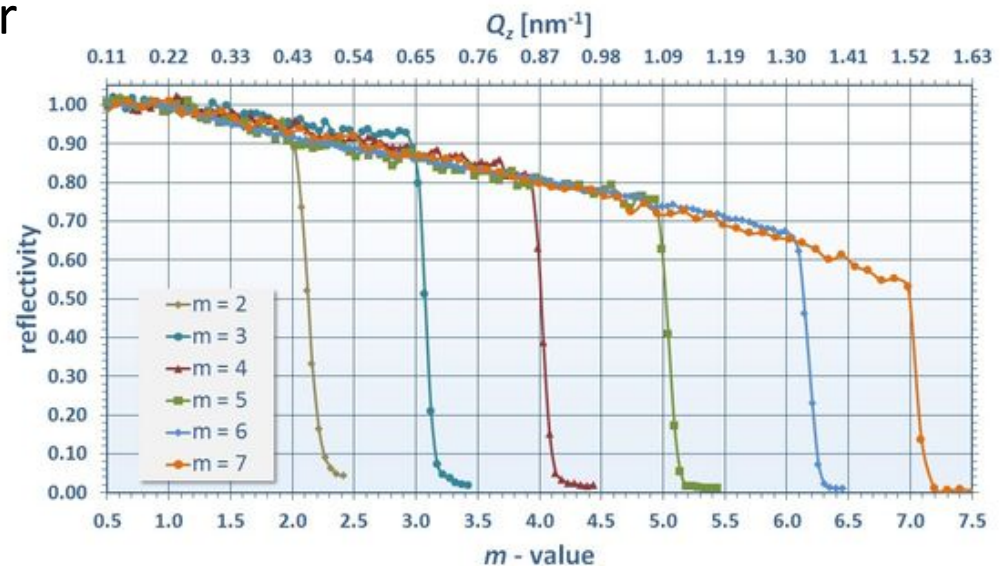
Ballistic guides



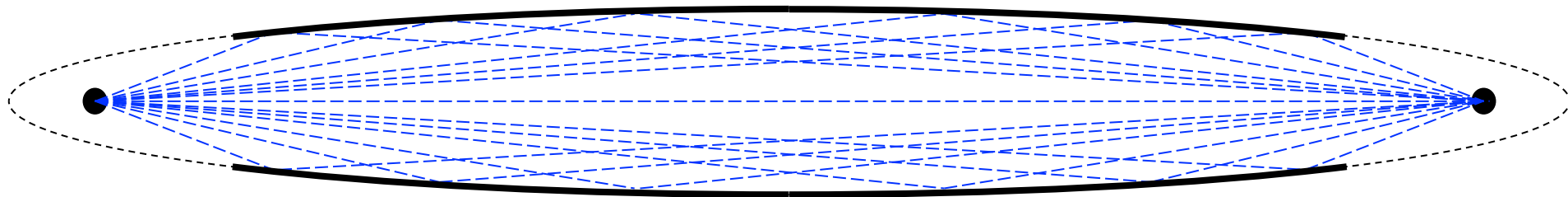
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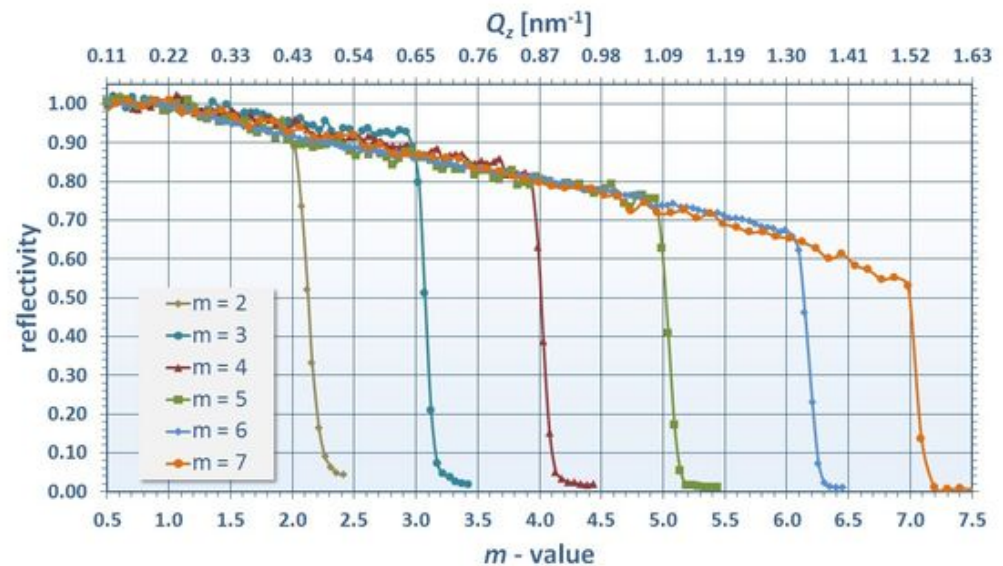
- Used to transport neutrons over long distances
- Minimise number of reflections
- Minimise reflection angles
- Increase width slowly to decrease divergence
adiabaticity - reversible



Ballistic guides

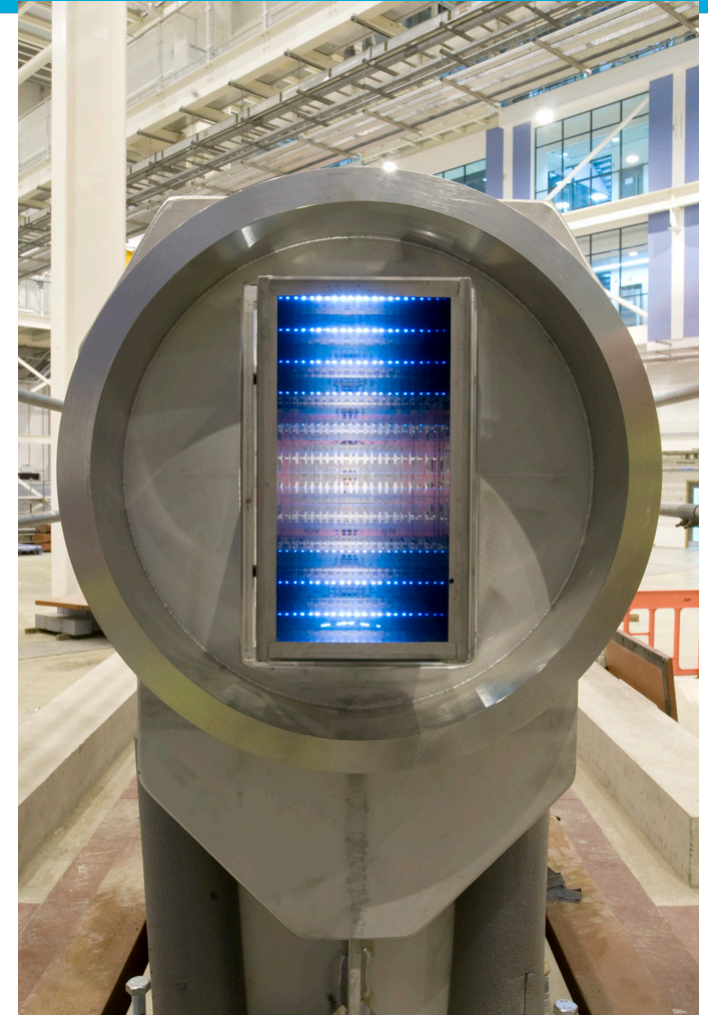


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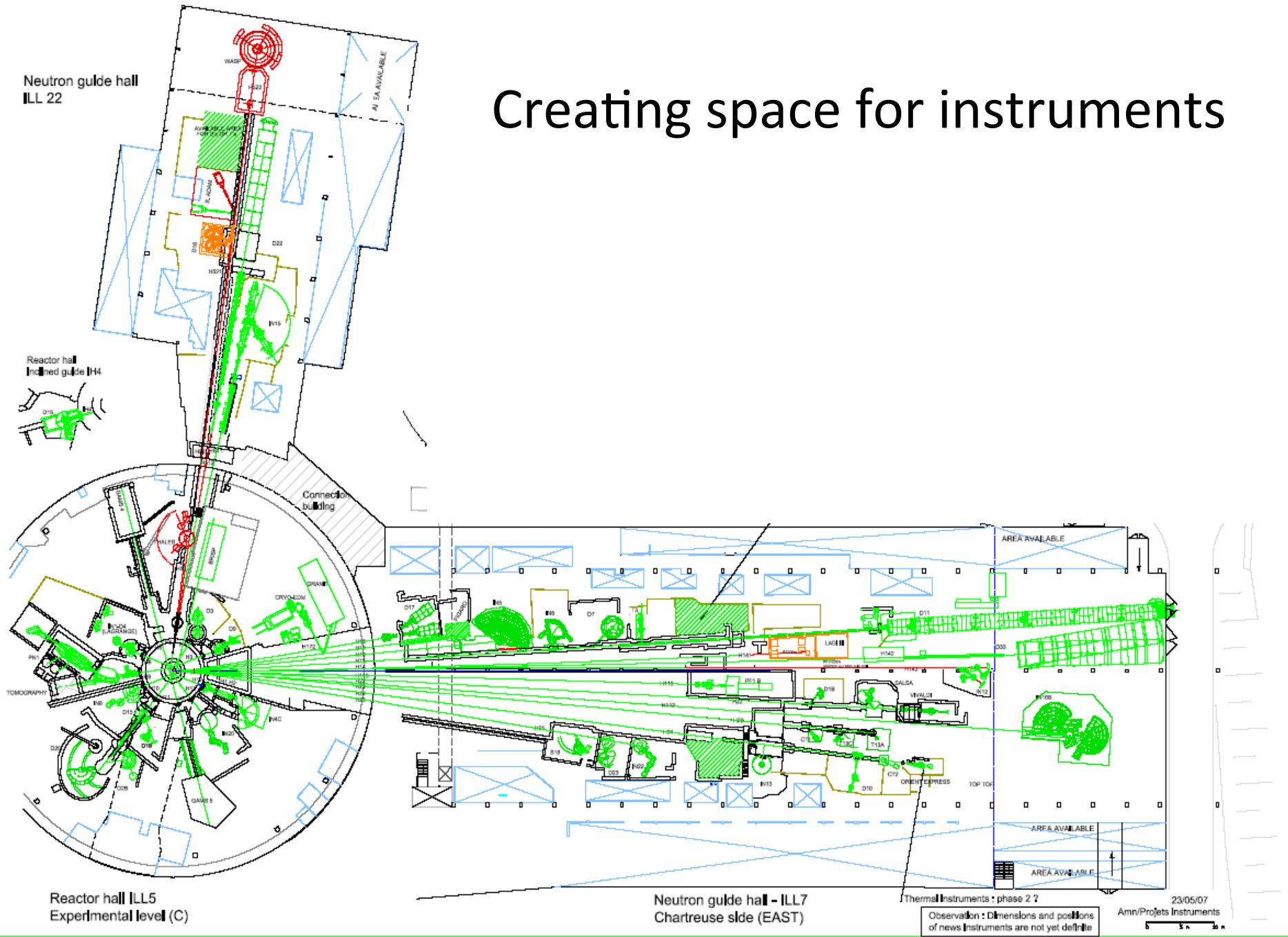


Ballistic guides

WISH @ ISIS: ballistic (elliptical) guide

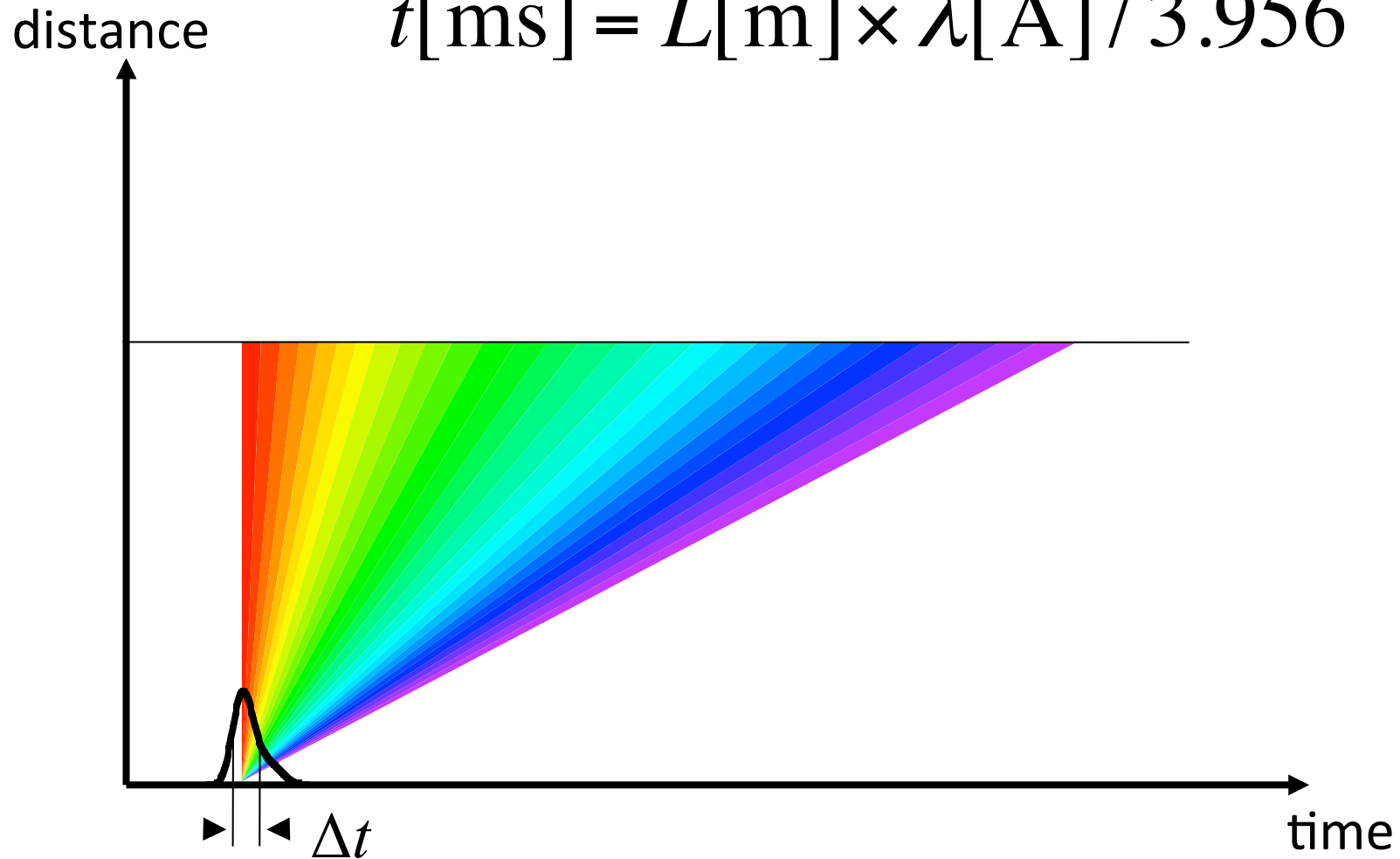


Creating space for instruments



Time-of-flight (TOF) resolution

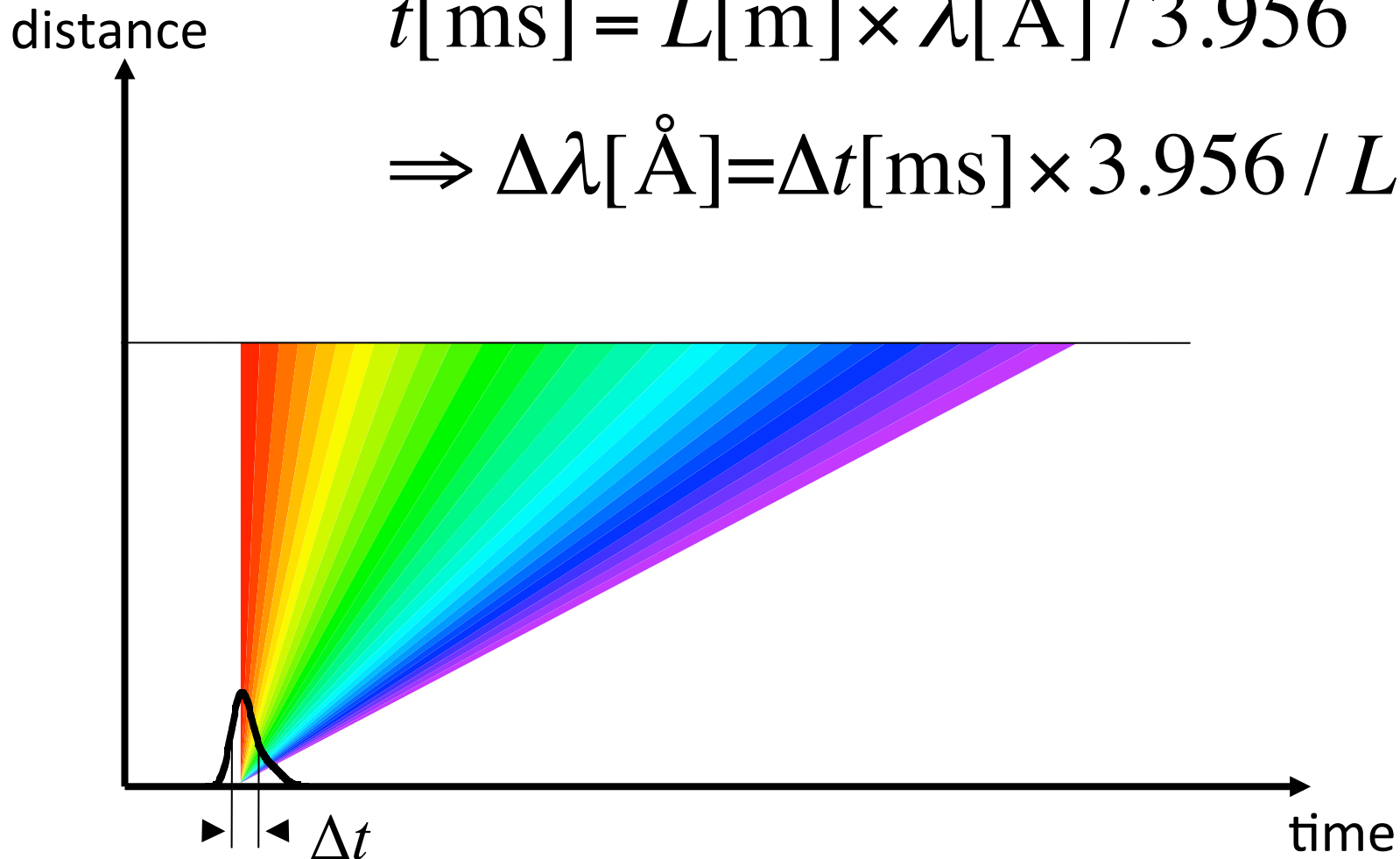
$$t[\text{ms}] = L[\text{m}] \times \lambda[\text{\AA}] / 3.956$$



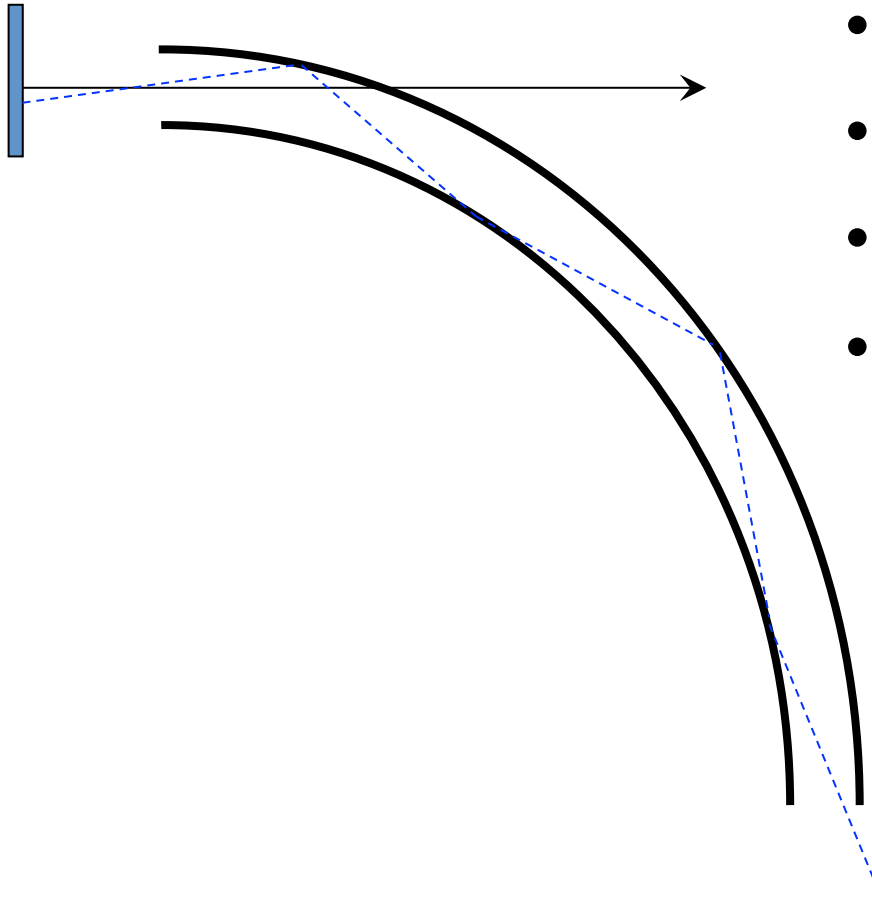
Time-of-flight (TOF) resolution

$$t[\text{ms}] = L[\text{m}] \times \lambda[\text{\AA}] / 3.956$$

$$\Rightarrow \Delta\lambda[\text{\AA}] = \Delta t[\text{ms}] \times 3.956 / L[\text{m}]$$

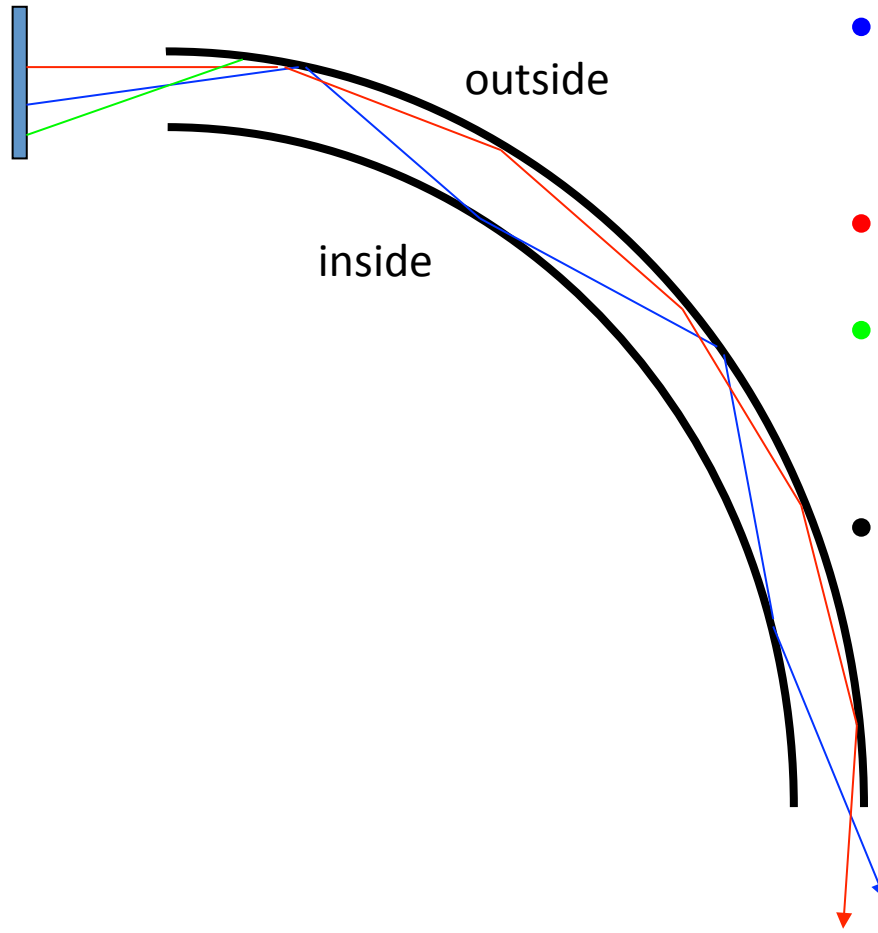


Curved Guides



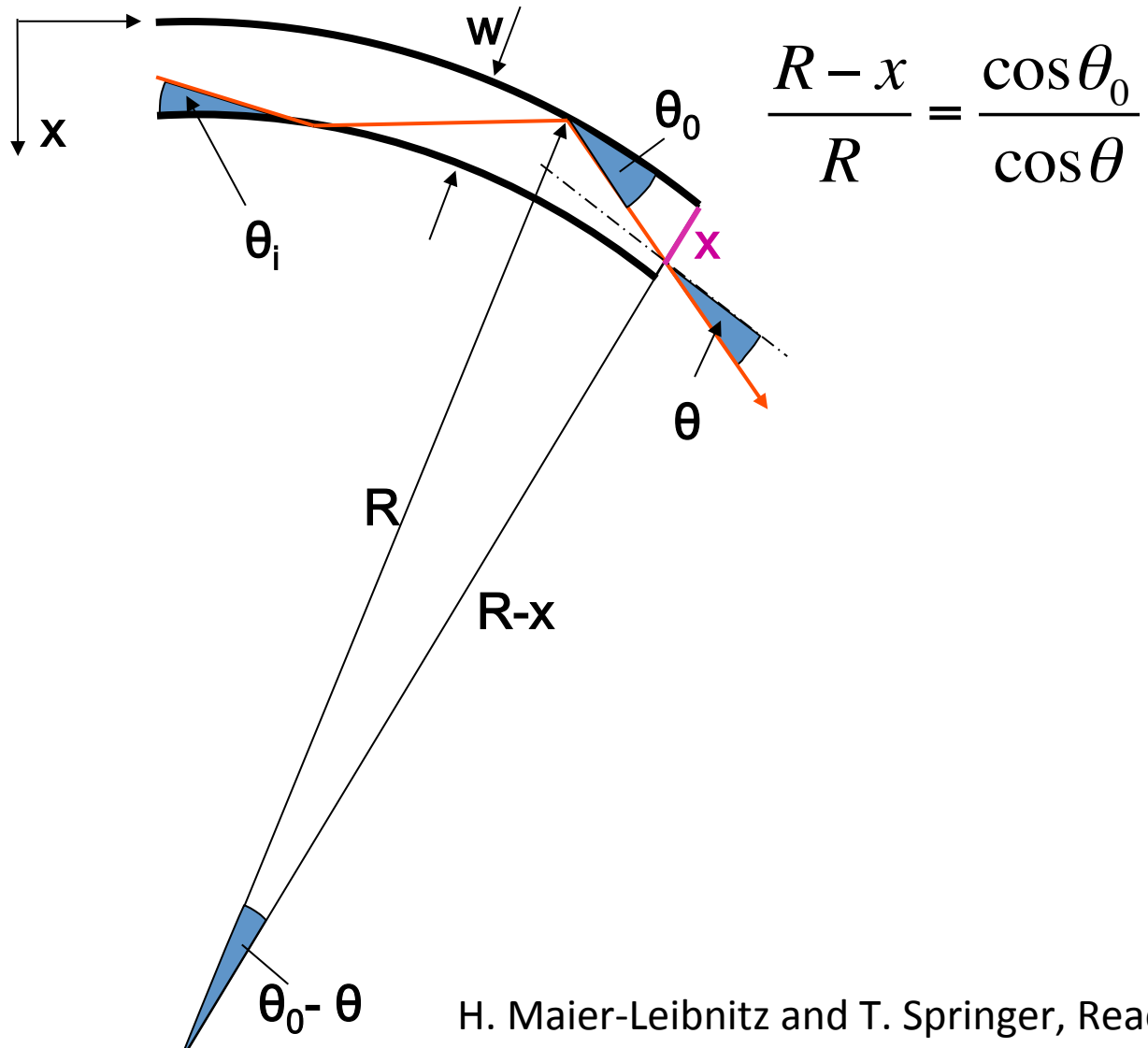
- Avoid direct line-of-sight
- Avoid gammas
- Avoid fast neutrons
- Reduce background

Curved Guides

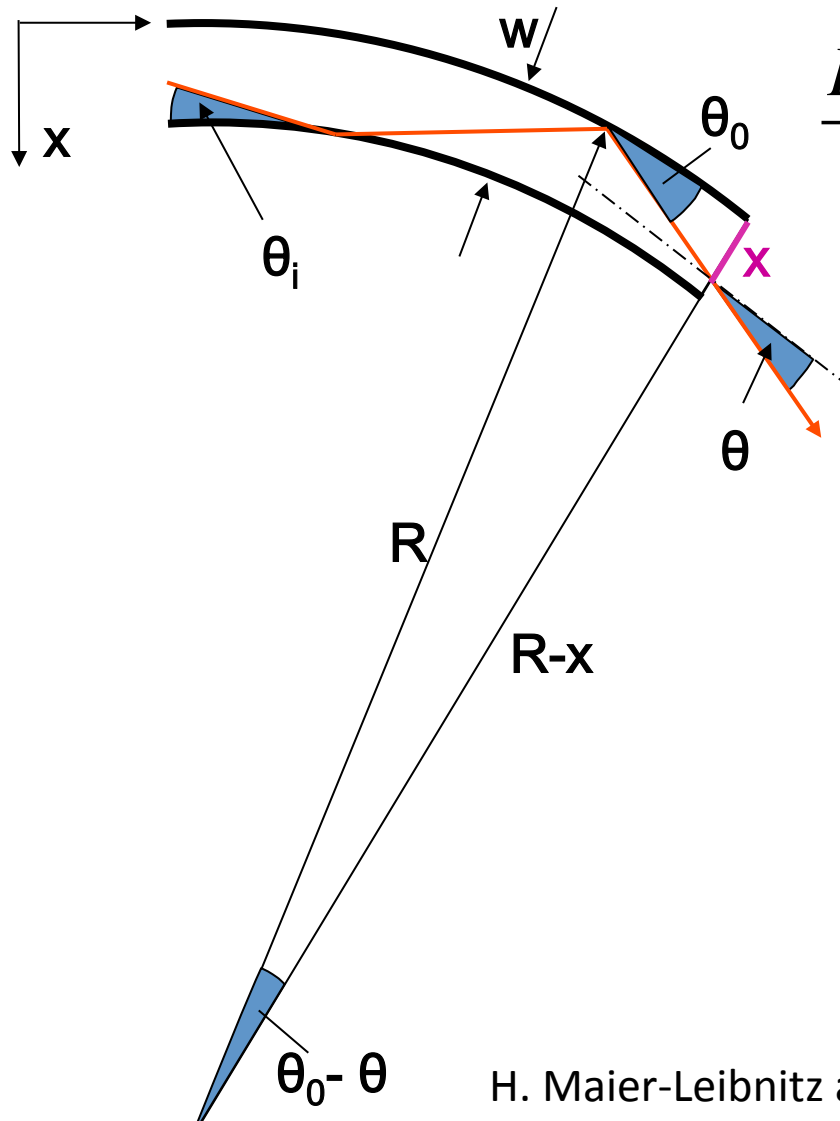


- **Blue** – reflecting from both sides
- **Red** – garland reflections
- **Green** – exceeds critical angle
- Fewer neutrons along inside face - quantify

Curved Guides



Curved Guides

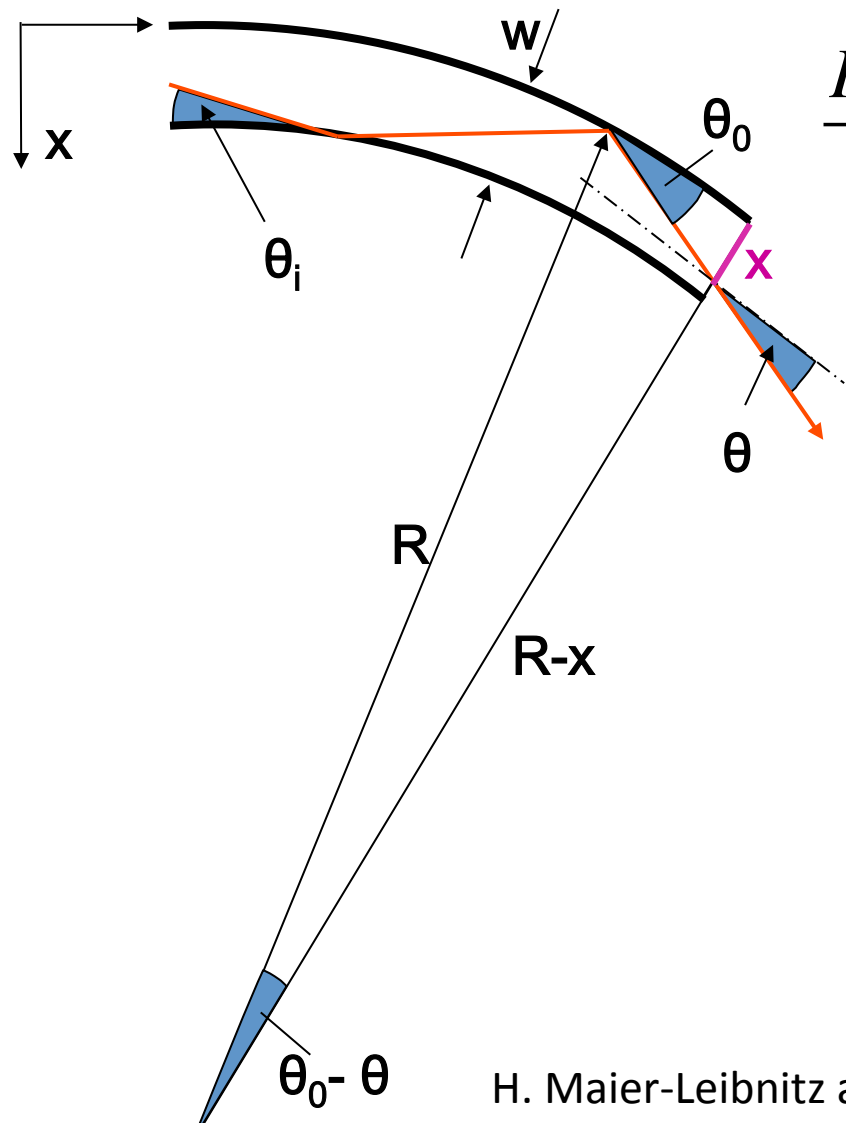


$$\frac{R-x}{R} = \frac{\cos \theta_0}{\cos \theta}$$

For $\theta_0, \theta \ll 1$,

$$x = -\frac{1}{2} R(\theta^2 - \theta_0^2)$$

Curved Guides



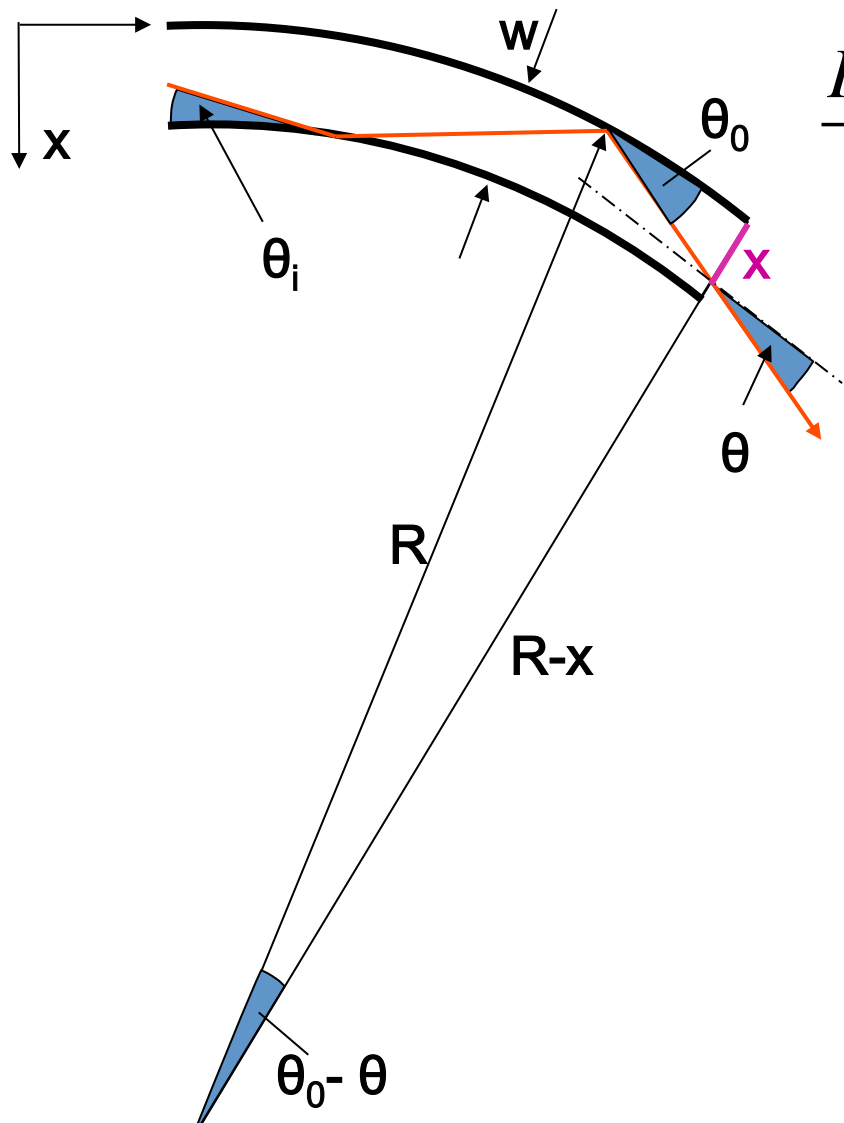
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For $\theta_0, \theta \ll 1$,
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neutrons “just” reach inside face
 when $\theta = 0$ for $x = w$:

$$\theta_0^* = \sqrt{2w / R}$$

Curved Guides



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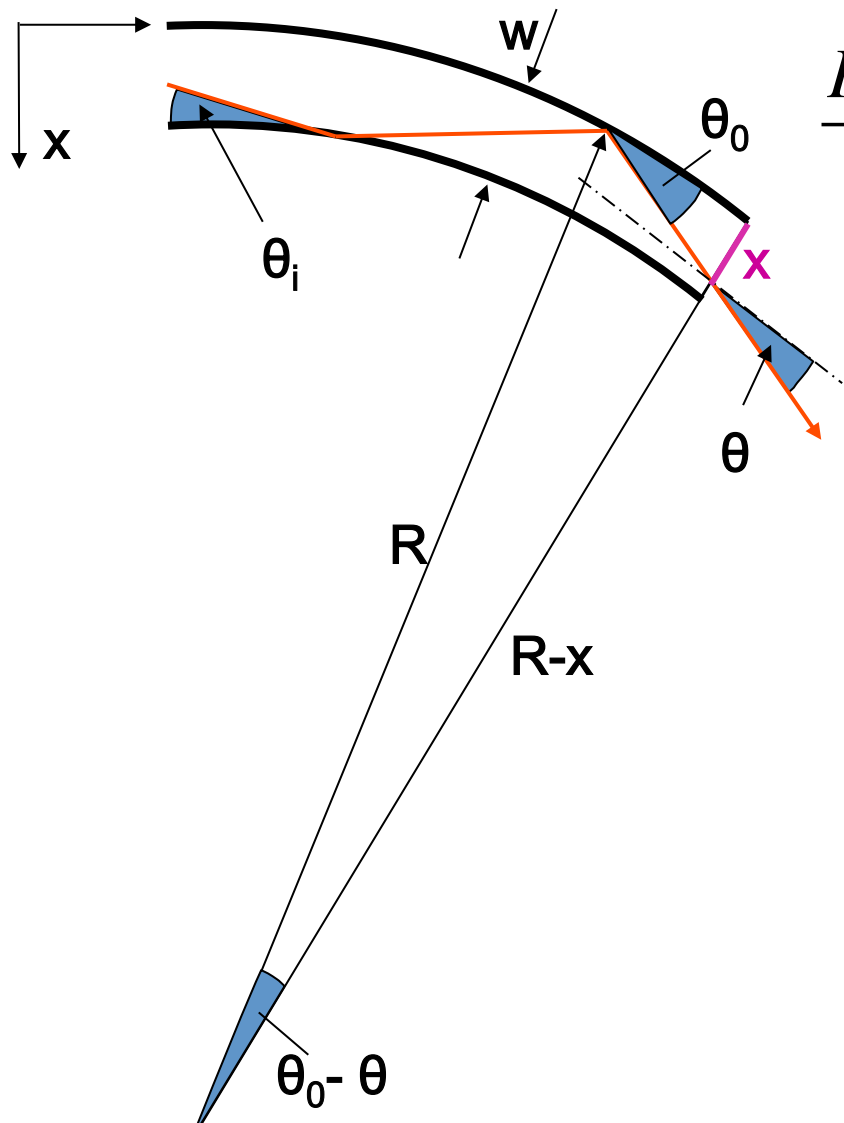
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$$\theta_c = m \times \lambda[\text{\AA}] \times 0.1^\circ$$

Curved Guides



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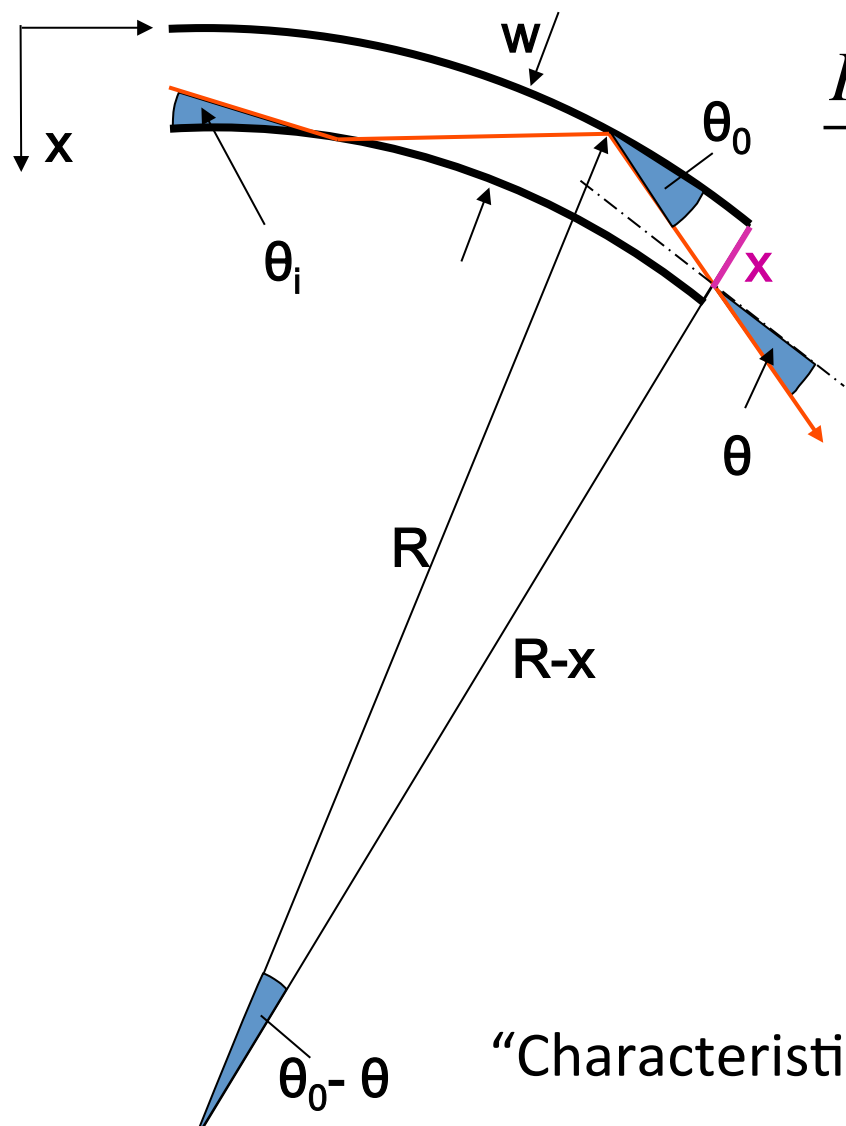
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$$\theta_c = \theta_0^* \Rightarrow \lambda^* = \frac{575}{m} \sqrt{2w/R}$$

Curved Guides



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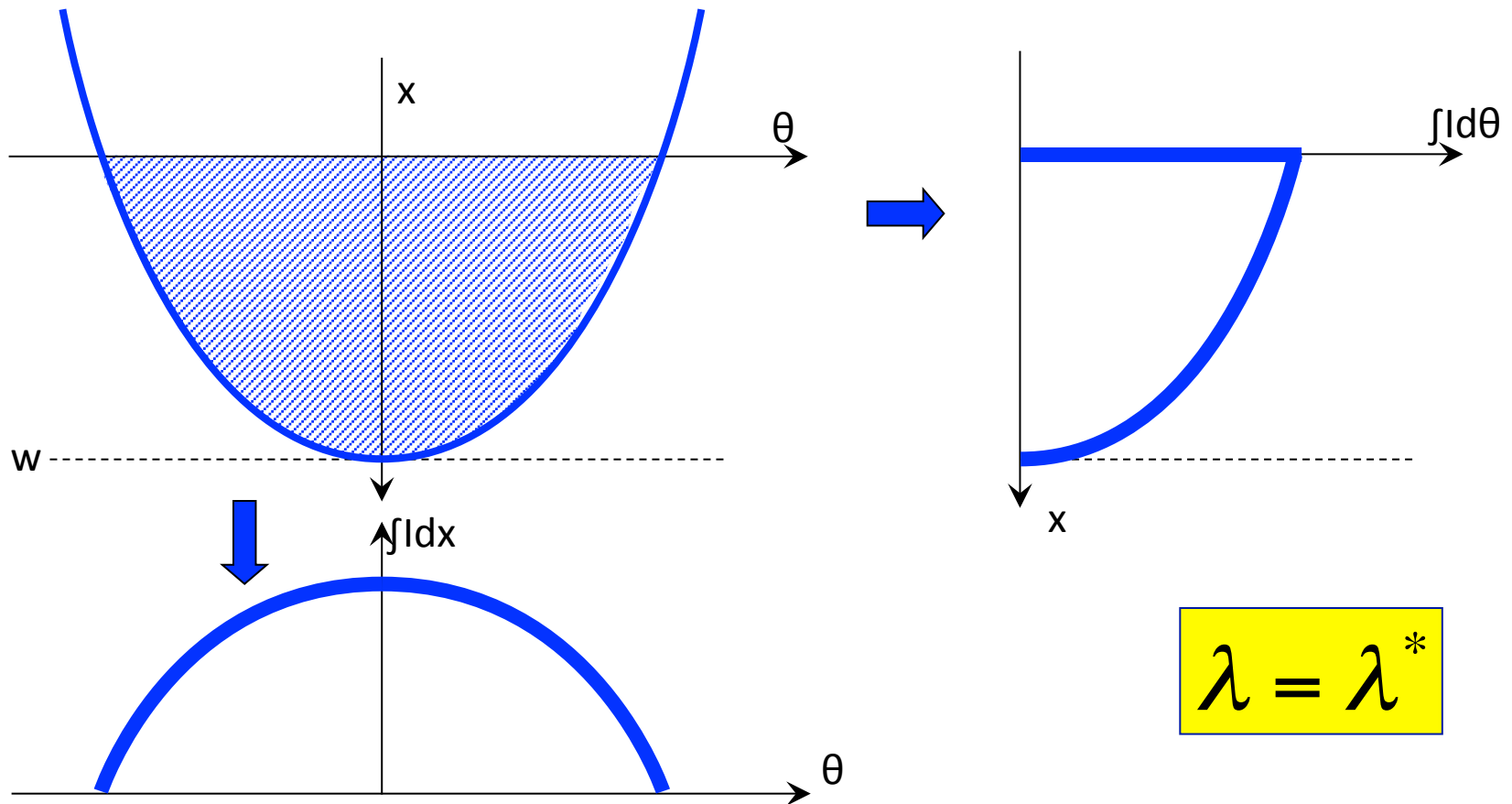
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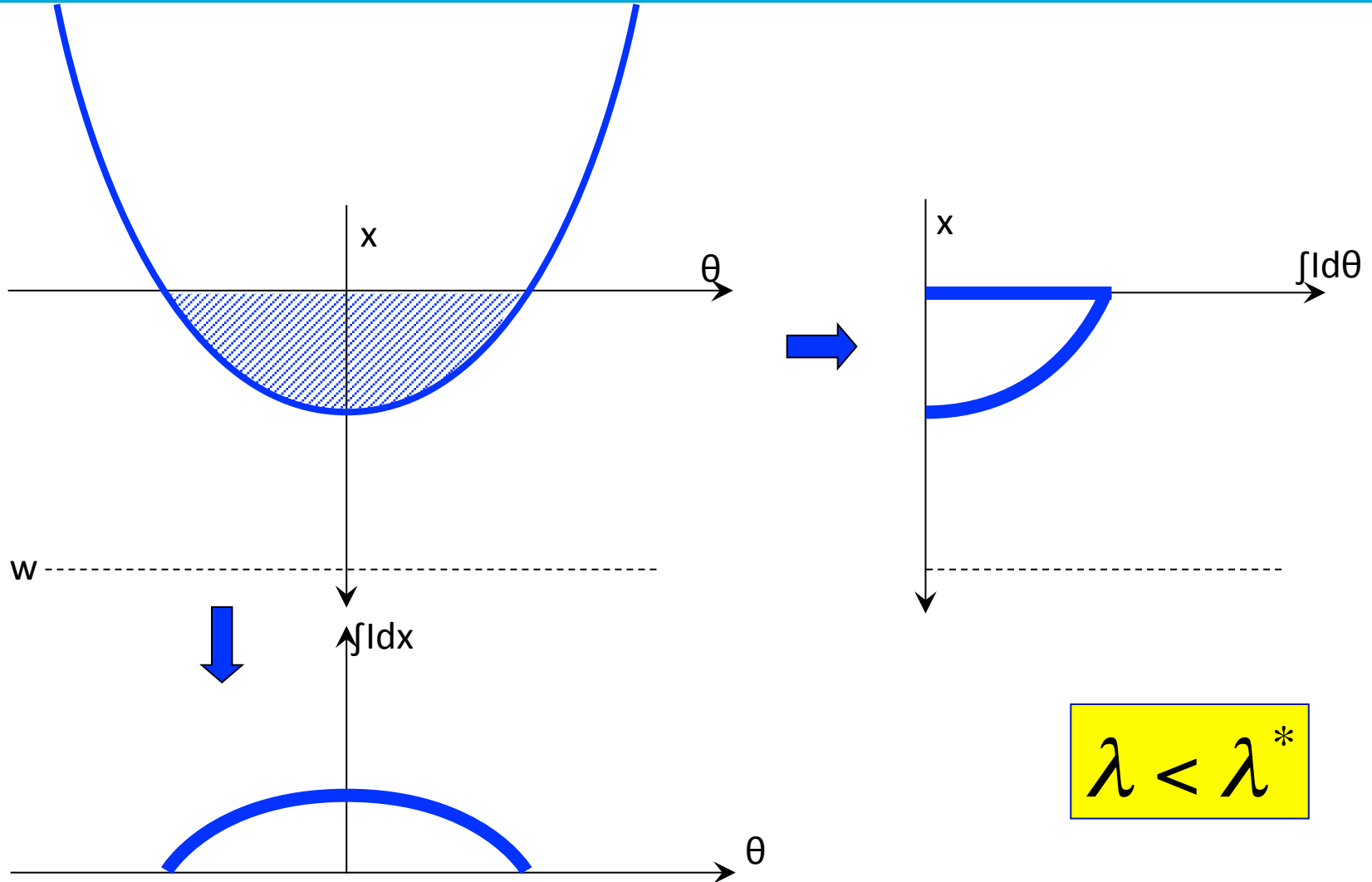
“Characteristic wavelength”

Curved Guides

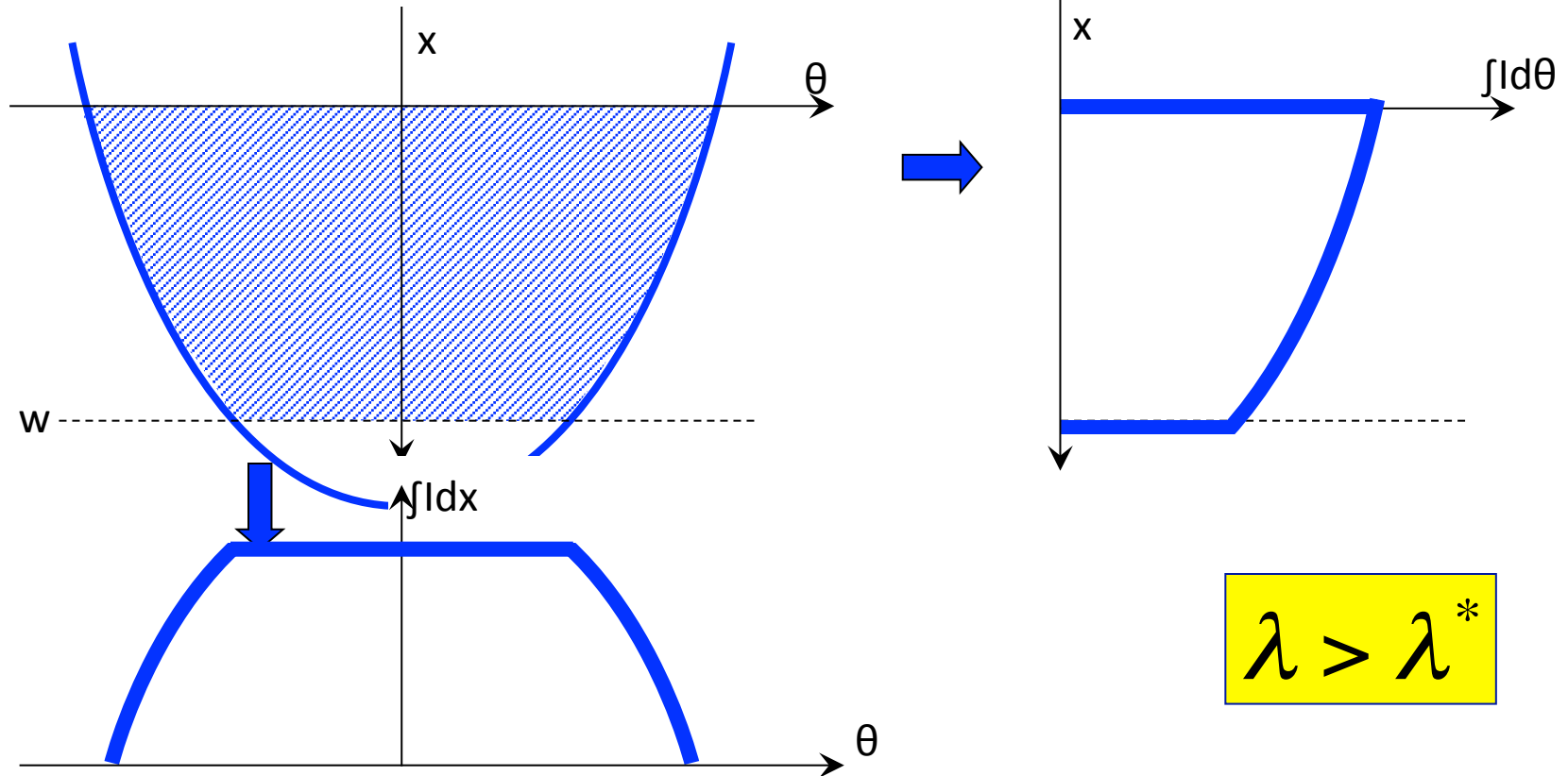


$$\lambda = \lambda^*$$

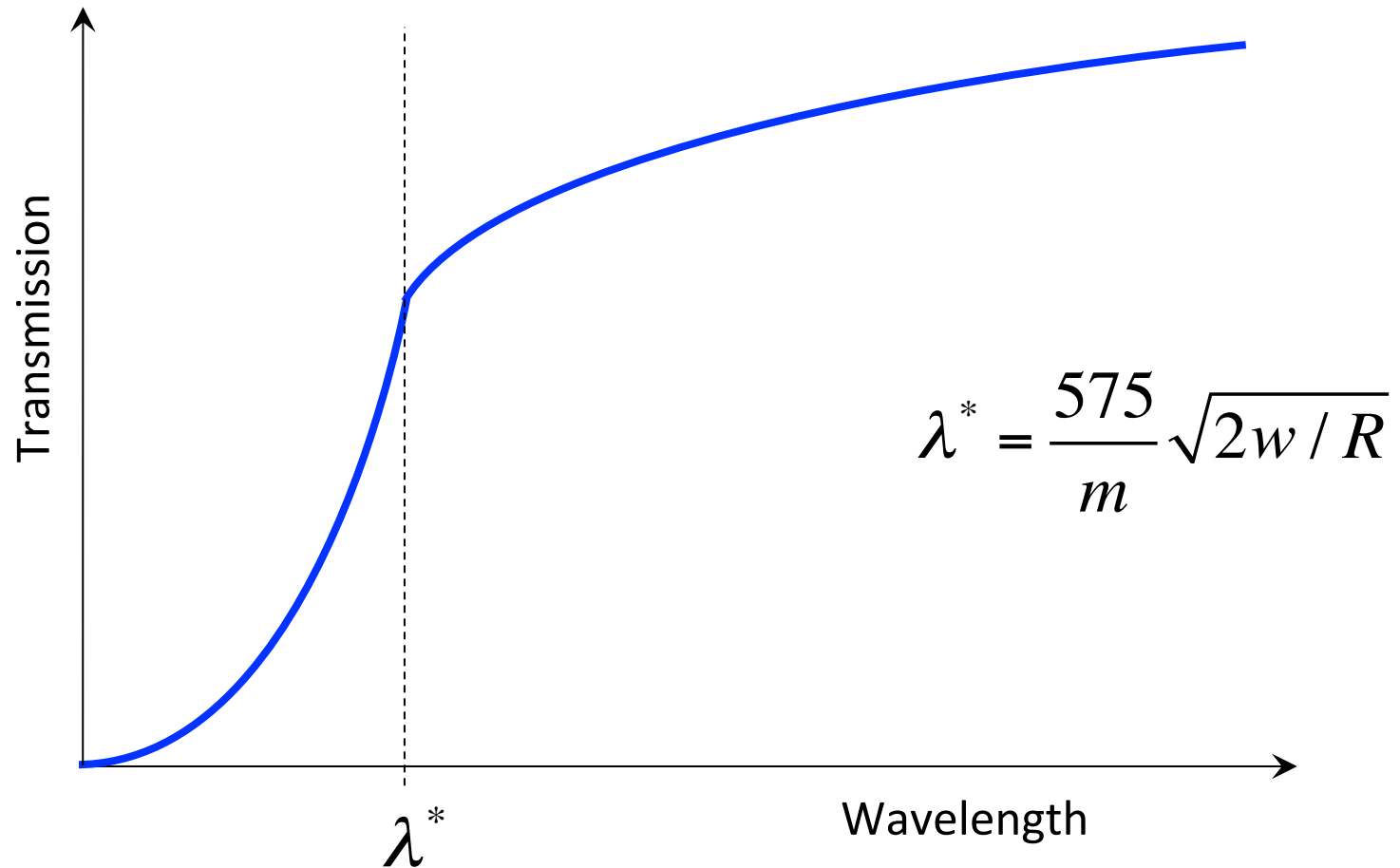
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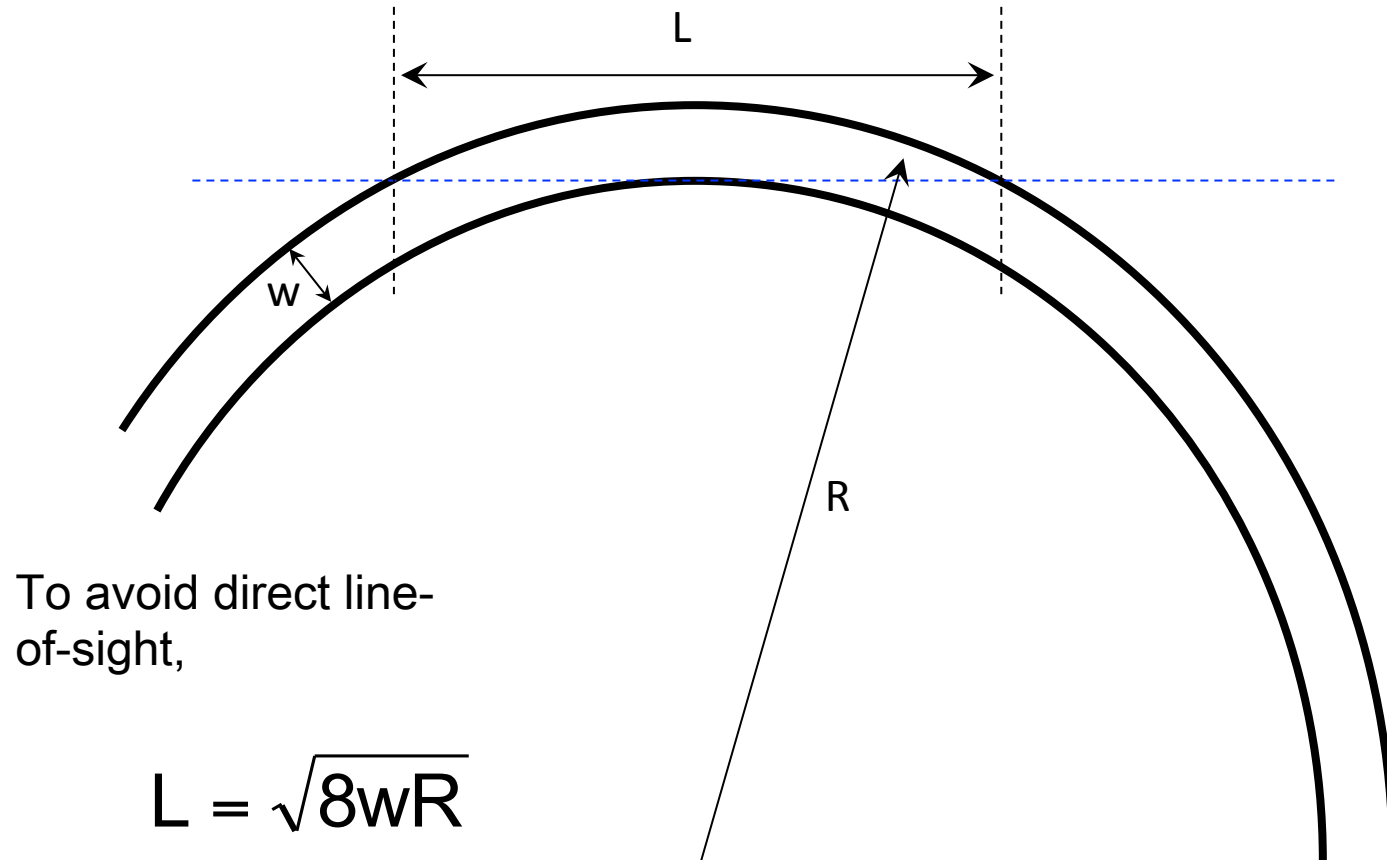
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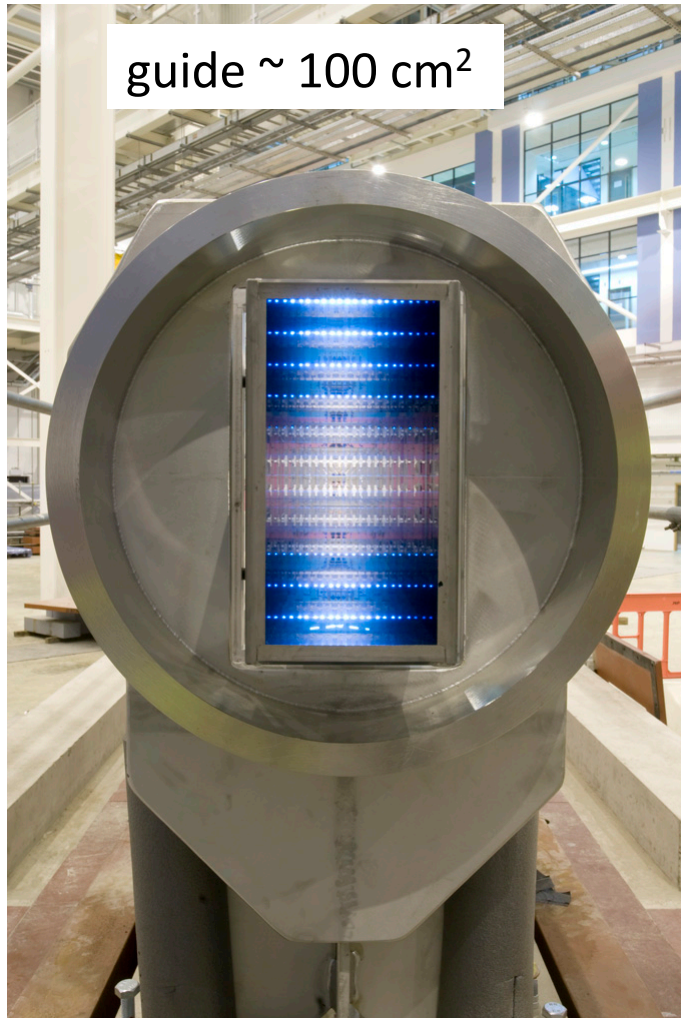
Curved Guides



Curved Guides



Focusing

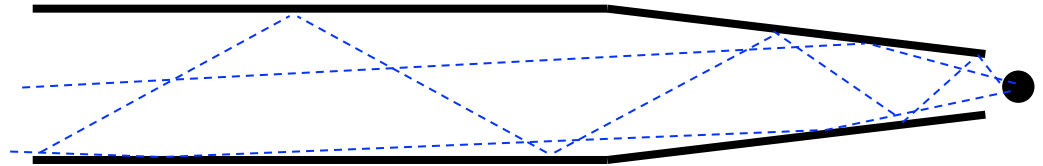


samples $< 1 \text{ cm}^2$

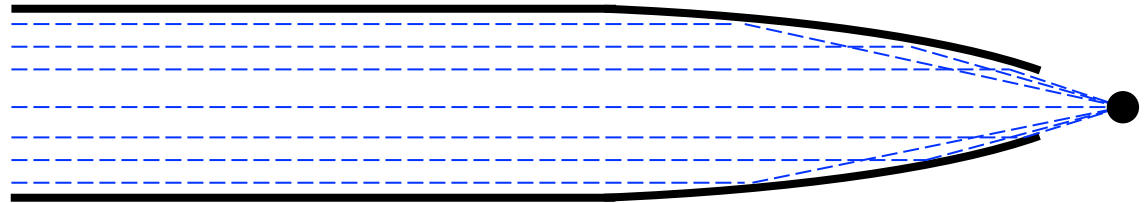


Focusing

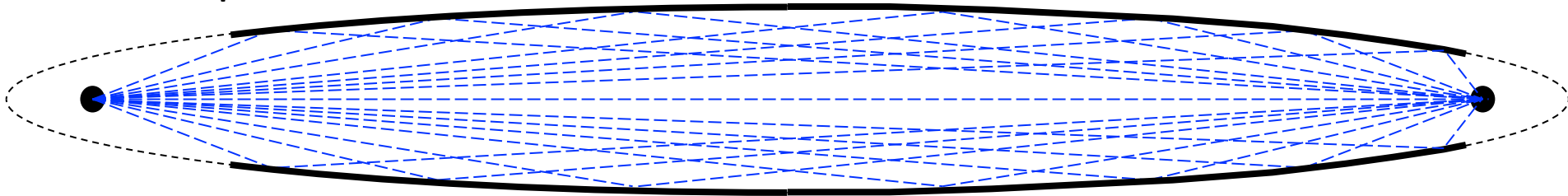
Linear tapering



Parabolic

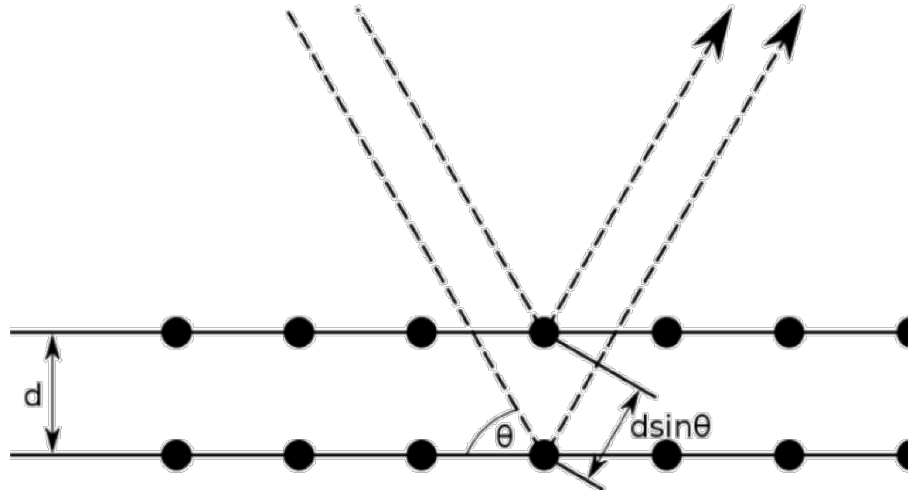


Elliptic

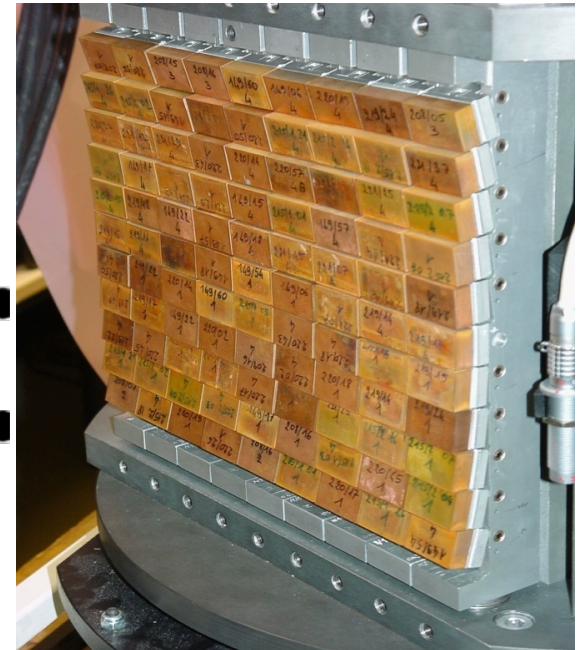


Crystal Monochromators

Graphite 002



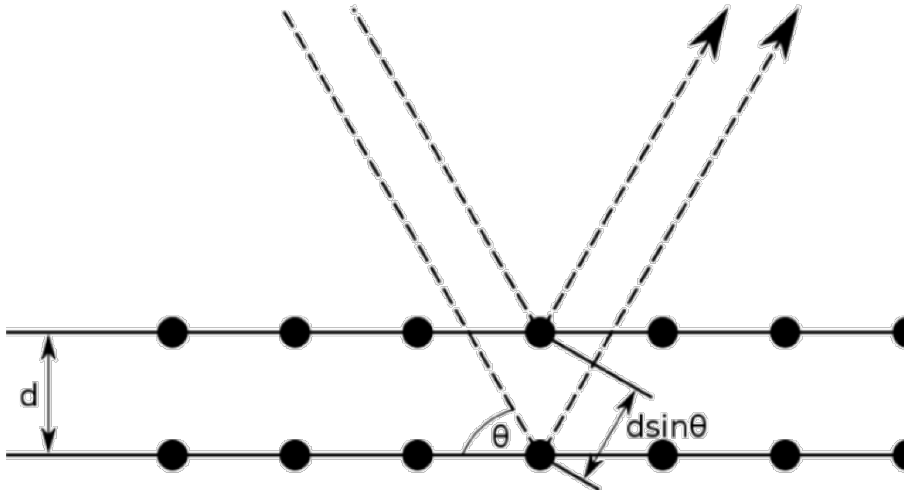
Copper 200



	d-spacing
Germanium 333	1.089 Å
Copper 200	1.807 Å
Silicon 111	3.135 Å
Graphite 002	3.355 Å

Crystal Monochromators

Graphite 002

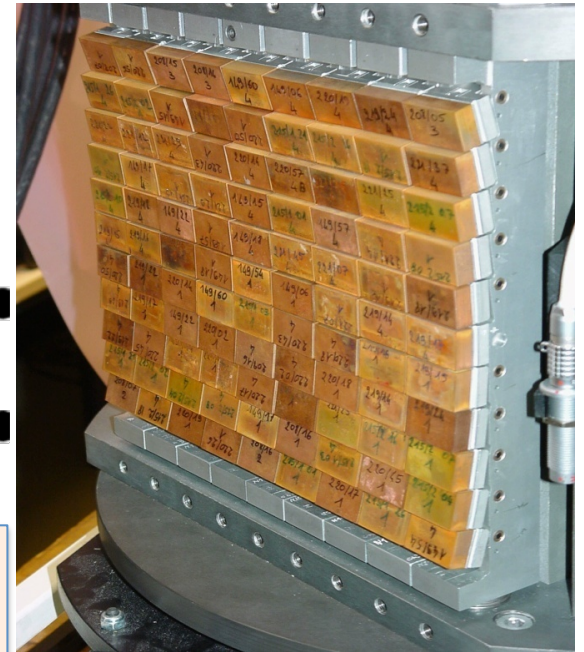


$$\lambda = 2d \sin \theta$$

$$\Rightarrow \Delta\lambda/\lambda = \Delta d/d + \cot \theta \Delta\theta$$

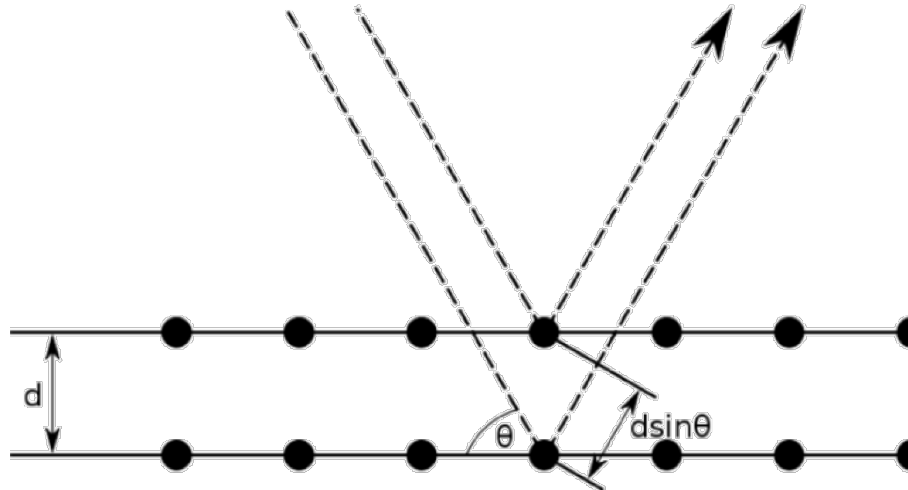
$$\Delta\lambda = \frac{\partial\lambda}{\partial d} \Delta d + \frac{\partial\lambda}{\partial\theta} \Delta\theta$$

Copper 200



Crystal Monochromators

Graphite 002

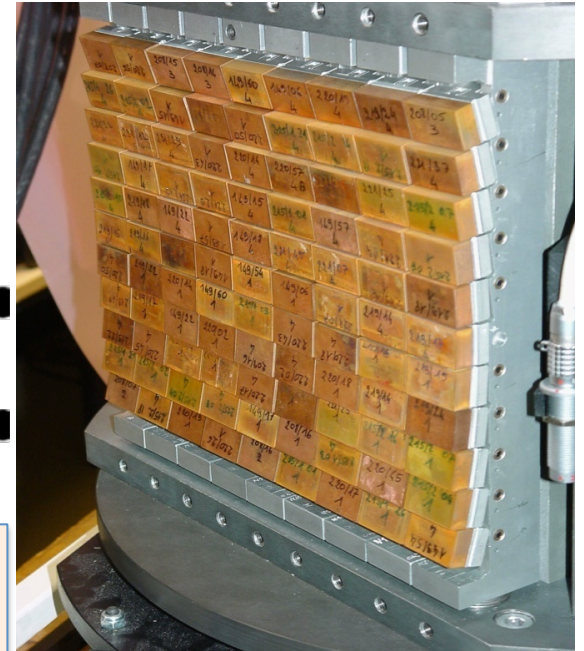


$$\lambda = 2d \sin \theta$$

$$\Rightarrow \Delta\lambda/\lambda = \cancel{\Delta d/d} + \cot \theta \Delta\theta$$

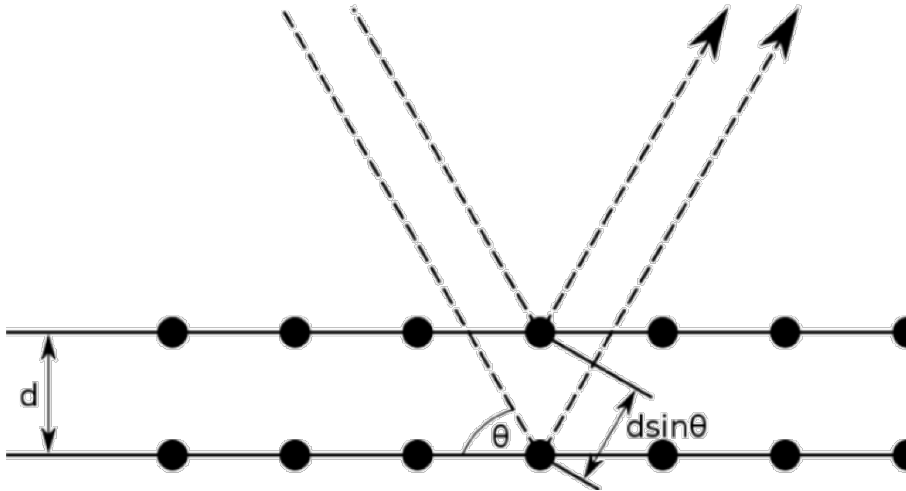
$$\Delta\lambda = \frac{\partial\lambda}{\partial d} \Delta d + \frac{\partial\lambda}{\partial\theta} \Delta\theta$$

Copper 200



Crystal Monochromators

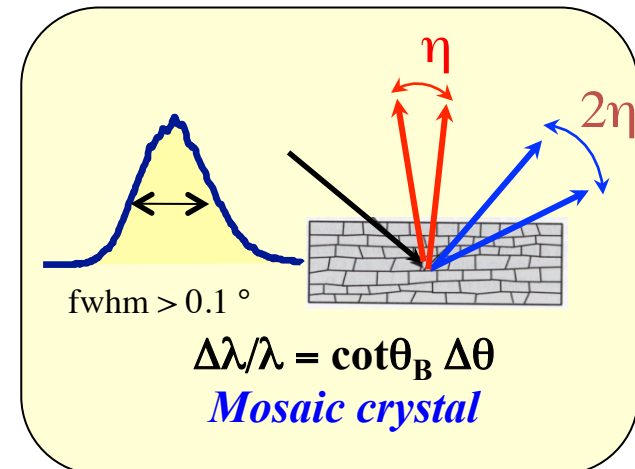
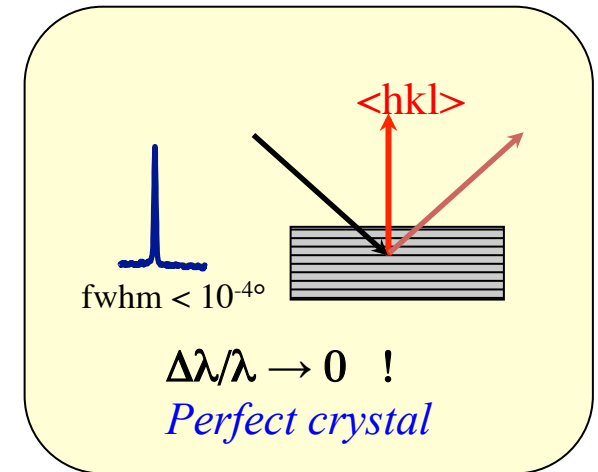
Graphite 002



$$\lambda = 2d \sin \theta$$

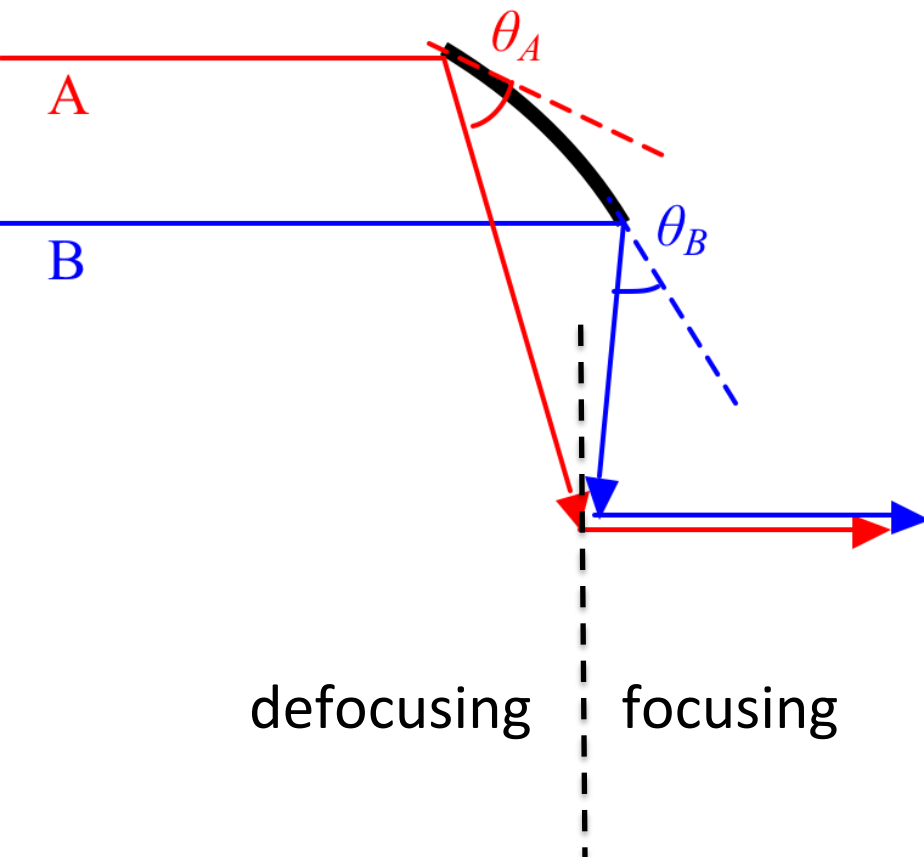
$$\Rightarrow \Delta\lambda/\lambda = \cancel{\Delta d/d} + \cot \theta \Delta\theta$$

$$\Delta\lambda = \frac{\partial \lambda}{\partial d} \Delta d + \frac{\partial \lambda}{\partial \theta} \Delta\theta$$



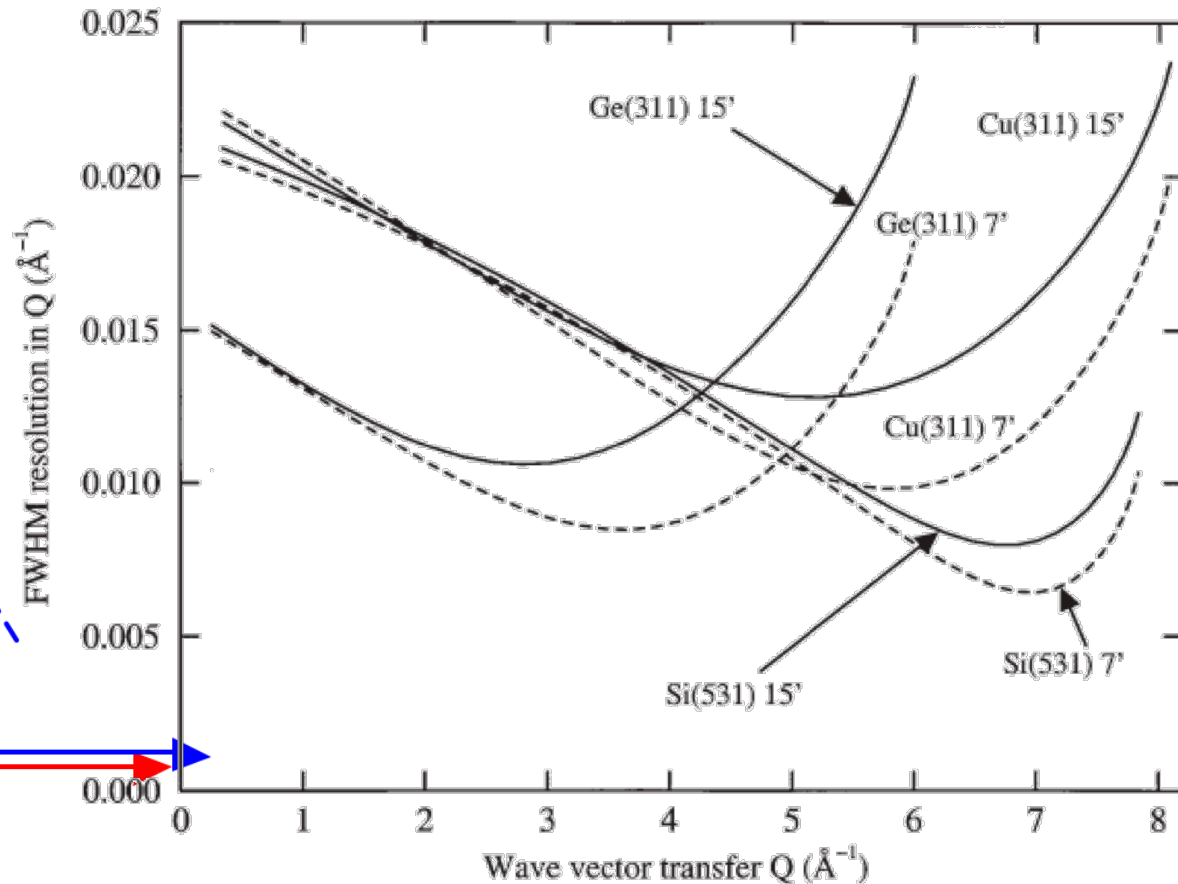
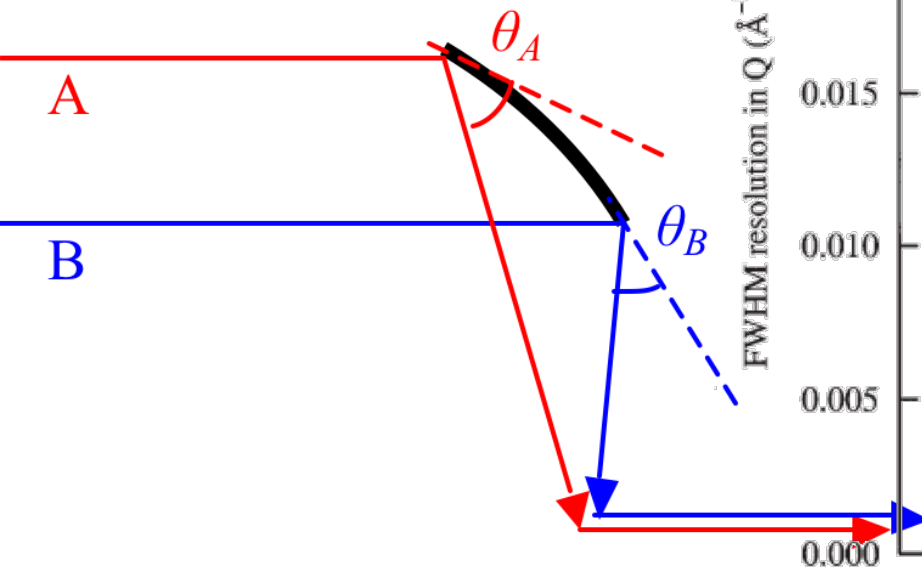
Focusing Monochromators

Q-focusing



Focusing Monochromators

Q-focusing



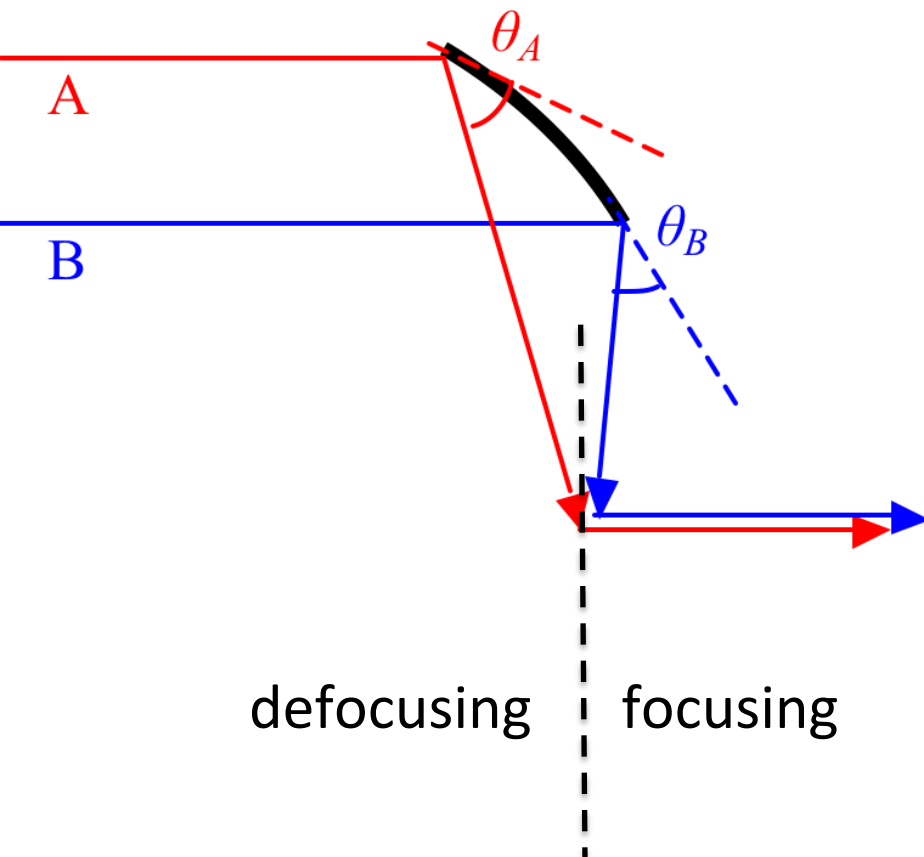
Powder Diffractometer Optimization:

G. Caglioti, A. Paoletti, F.P. Ricci, Nucl. Instr. Meth 3, 223 (1958)

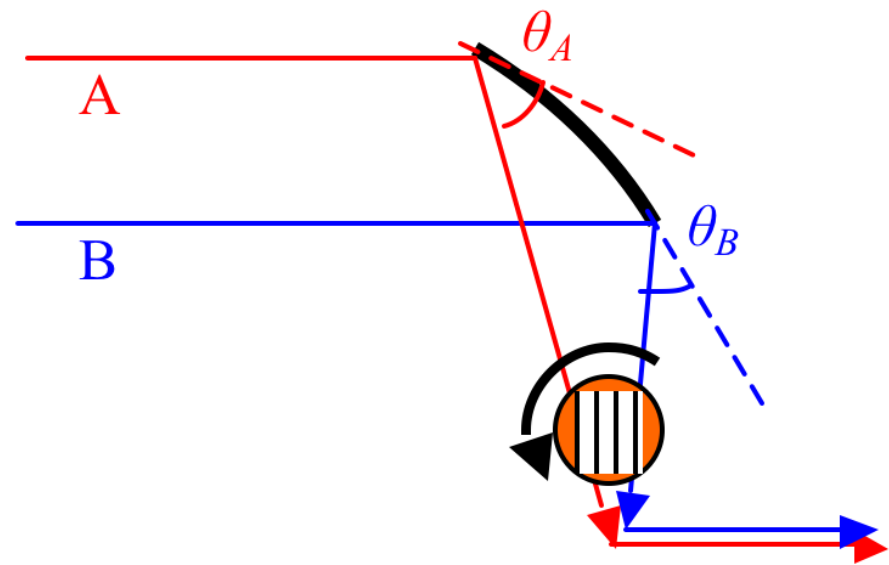
A.W. Hewat, Nucl. Instr. Meth. 127, 361 (1975)

Focusing Monochromators

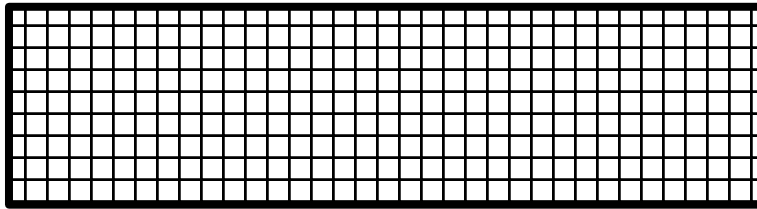
Q-focusing



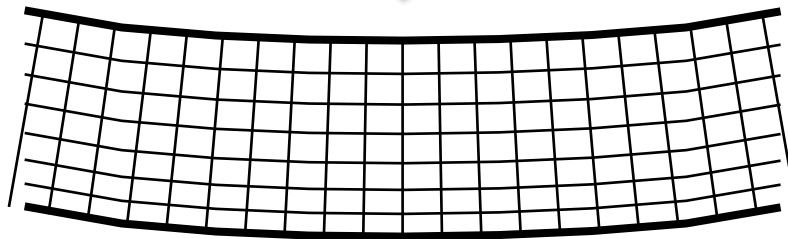
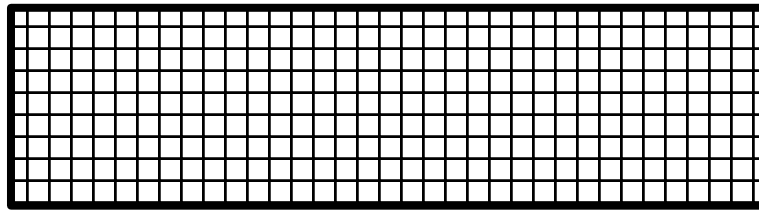
Time-focusing



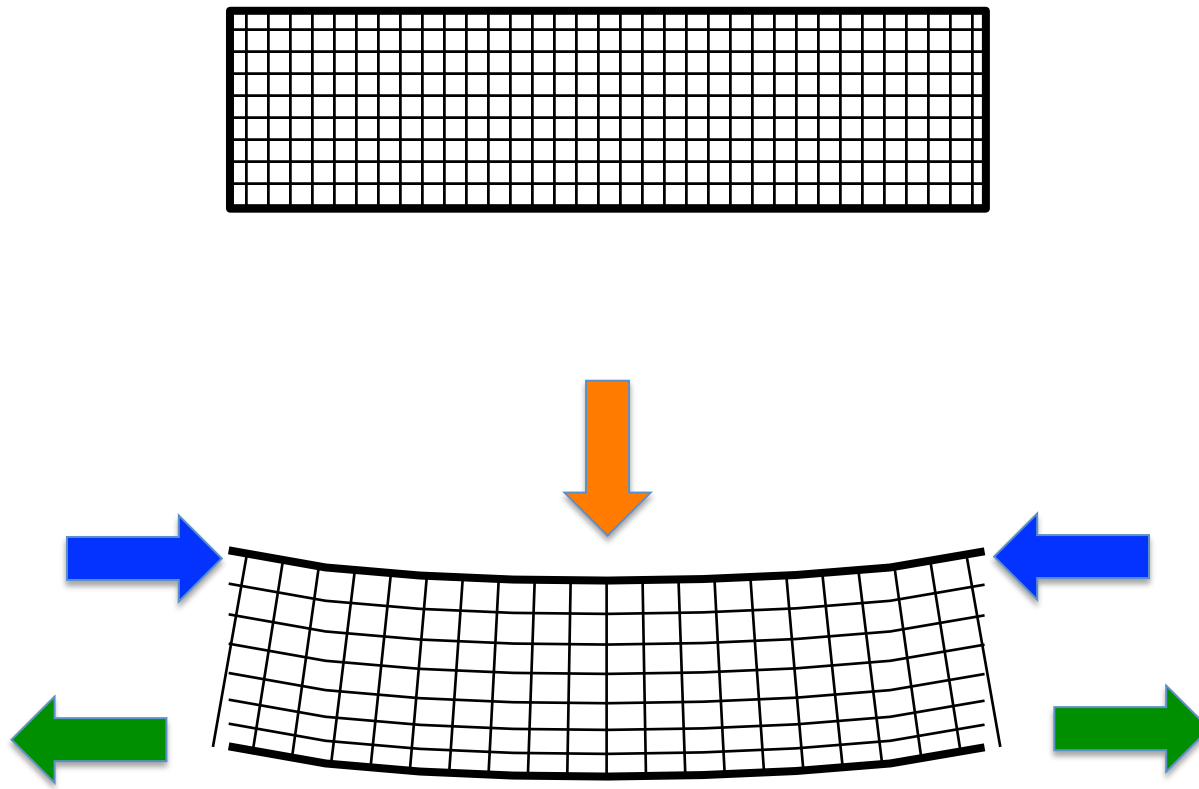
Bent Perfect Crystal Monochromators



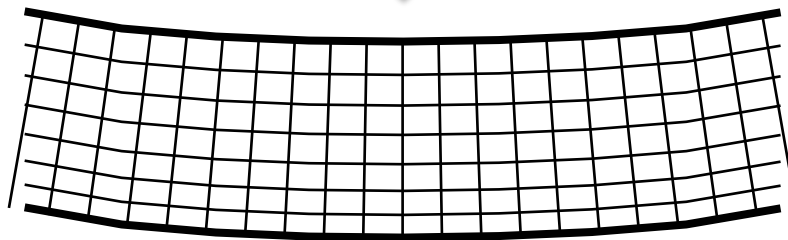
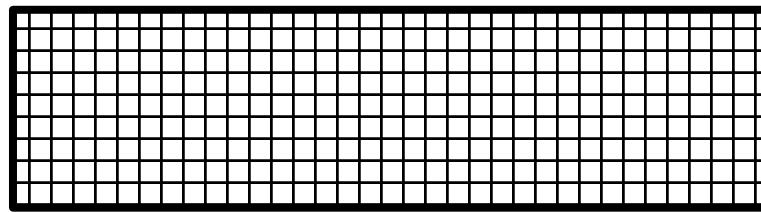
Bent Perfect Crystal Monochromators



Bent Perfect Crystal Monochromators



Bent Perfect Crystal Monochromators

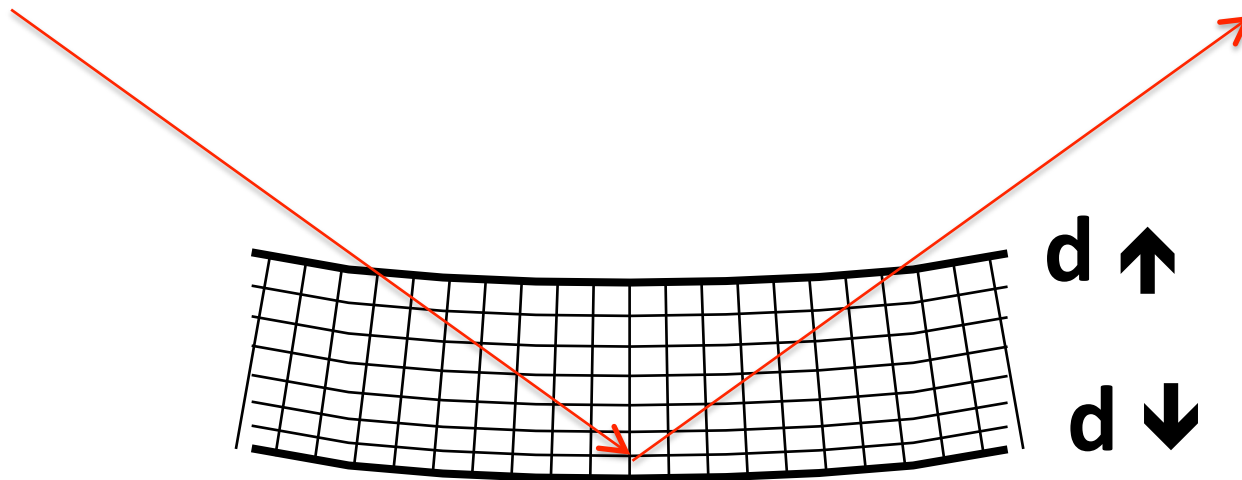


$d \uparrow$

$d \downarrow$

Bent Perfect Crystal Monochromators

1. d-spacing varies with depth
2. orientation varies with depth



Summary

- Bragg's Law
- Neutron Guides
 - how they work
 - what they do
- Crystal Monochromators
- tools for calculating how they work ...

Thank you!

