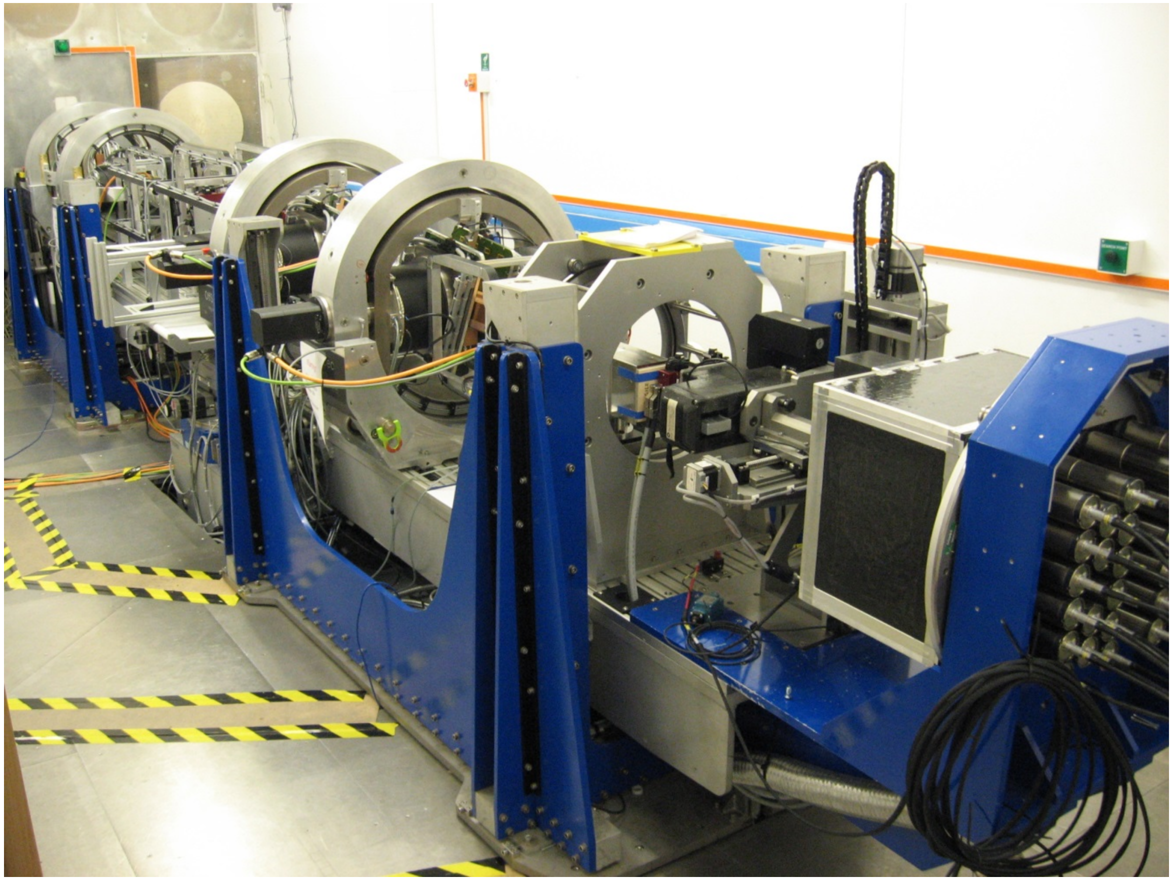
Fundamental Science with Neutron Spin Precession

Ad van Well

OffSpec @ ISIS



Reflectometer with spin-echo option
meant for SERGIS and SESANS, but ……

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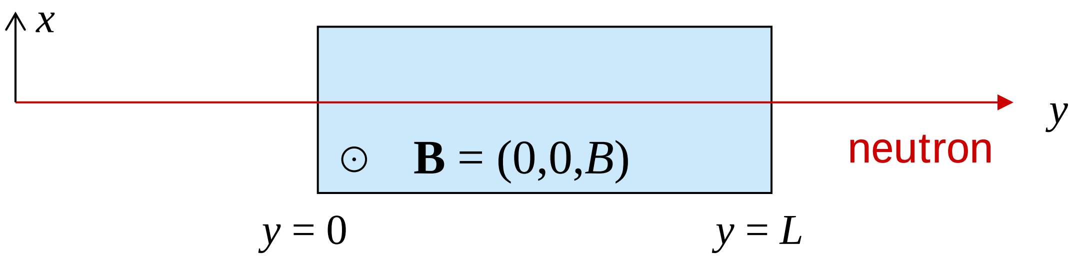
Interaction neutron with magnetic field

quantum-mechanical description



Assume $\mathbf{B}$ in z – direction ⟺ quantization axis

The neutron wave function is superposition of plus and minus state, in spinor notation:

$$\Psi = \begin{bmatrix} \psi^{+} \\ \psi^{-} \end{bmatrix},$$

where ψ^{+} and ψ^{-} represent the plus (spin parallel to $\mathbf{B}$)

and minus (spin anti-parallel to $\mathbf{B}$) state.

Interaction neutron with magnetic field

quantum-mechanical description

The spin of the neutron is expressed by the Pauli spin operators

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Separating the spin-dependent part from the wave function we may write

$$\Psi = \begin{bmatrix} \psi^+ \\ \psi^- \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} \varphi = \chi\varphi,$$

with $|a|^2$ and $|b|^2$ the probabilities that a measurement of the spin will show

to be plus or minus, hence $|a|^2 + |b|^2 = 1$

We define spin-up and spin down by $|+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \chi_+$ $\quad |-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \chi_-$

and $\langle +| = (1,0) = \chi_+^\dagger$ $\quad \langle -| = (0,1) = \chi_-^\dagger$

then $\chi = a|+\rangle + b|-\rangle$

$$\chi^\dagger = a^* \langle +| + b^* \langle -|$$

Interaction neutron with magnetic field

quantum-mechanical description

The expectation value of the Pauli spin operator

$$p_i = \langle \hat{\sigma}_i \rangle = \frac{\Psi^* \hat{\sigma}_i \Psi}{\Psi^* \Psi} = \chi^\dagger \hat{\sigma}_i \chi = \begin{pmatrix} a^* & b^* \end{pmatrix} \hat{\sigma}_i \begin{pmatrix} a \\ b \end{pmatrix}$$

leading to

$$p_x = ab^* + a^* b$$
$$p_y = i\left(a^* b - ab^*\right)$$
$$p_z = aa^* - bb^*$$

special cases

$$p_x = 1 \quad \begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad p_x = -1 \quad \begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i \\ -i \end{pmatrix}$$

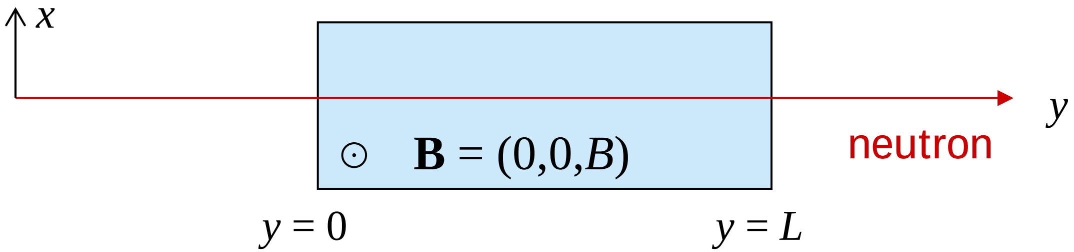
$$p_y = 1 \quad \begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i \\ -1 \end{pmatrix} \qquad p_y = -1 \quad \begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i \\ 1 \end{pmatrix}$$

$$p_z = 1 \quad \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad p_z = -1 \quad \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

NB If $p_x = \pm 1$, or $p_y = \pm 1$, the probability of measuring a plus or minus spin will be 50%

Interaction neutron with magnetic field

quantum-mechanical description



Neutron is plane wave polarized in the $x-$direction, travelling in $y-$direction, in free space ($y < 0$):

$$\Psi_0 = \begin{bmatrix} \psi^+ \\ \psi^- \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \exp\left(k_0 y - \omega_0 t\right) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \varphi_0 = \left(\frac{1}{\sqrt{2}} \left|+\right\rangle + \frac{1}{\sqrt{2}} \left|-\right\rangle \right) \varphi_0$$

where the kinetic energy is given by

$$E_0 = \hbar\omega_0 = \frac{\hbar^2 k_0^2}{2m}$$

Interaction neutron with magnetic field
quantum-mechanical description

The Schrödinger equation:

$$i\hbar \frac{\partial}{\partial t}\Psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r},t)\Psi(\mathbf{r},t)$$

- include spinor description
- potential energy due to spin: Zeeman energy
- for the moment we omit interaction with material $V(\mathbf{r}) = -\boldsymbol{\mu}_n \cdot \mathbf{B}$

Since the neutron is a spin ½ particle, only 2 states of potential energy are allowed in a static magnetic field, also referred to as Zeeman splitting.
In the following superscript + and – refer to plus spin state (spin parallel to $\mathbf{B}$) and minus spin state (anti-parallel), respectively.

The lower energy state is for $\boldsymbol{\mu}$ parallel to $\mathbf{B}$, i.e. the minus spin state.
This means that upon entering a field region 'the neutron in the minus state will be accelerated'. (both wave-function components will have different k)

Interaction neutron with magnetic field

quantum-mechanical description

The potential energy of the neutron in a magnetic field and resulting kinetic energy are

$$E_{\text{pot}} = \hbar\omega_{\text{z}} = \pm\mu_n B; \quad E_{\text{kin}}^{\pm} = E_0 \mp \mu_n B; \quad k^{\pm} = k_0 \mp \Delta k,$$

with

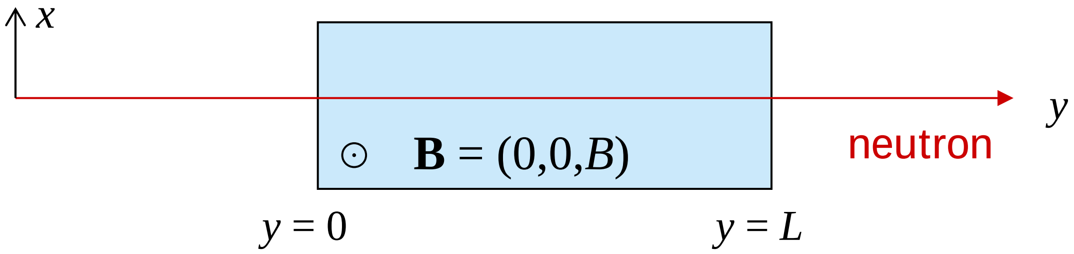
$$\Delta k = \frac{m}{\hbar^2 k_0}\mu_n B = \frac{m\omega_{\text{z}}}{\hbar k_0} = \frac{\omega_{\text{z}}}{v_0} = \frac{-m\gamma_n}{2h}\lambda_0 B.$$

resulting in the Schrödinger equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial y^2} + \begin{bmatrix} \mu_n B & 0 \\ 0 & -\mu_n B \end{bmatrix}\Psi$$

Interaction neutron with magnetic field

quantum-mechanical description



The neutron is polarized in the x – direction at $y = 0$, then the solution of the Schrödinger equation reads

$$\Psi = \begin{bmatrix} \psi^+ \\ \psi^- \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} \exp\left(i\left(\left(k_0 - \Delta k\right)L - \omega_0 t\right)\right) \\ \exp\left(i\left(\left(k_0 + \Delta k\right)L - \omega_0 t\right)\right) \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} \exp\left(-i\Delta kL\right) \\ \exp\left(i\Delta kL\right) \end{bmatrix} \varphi_0 = \begin{bmatrix} a \\ b \end{bmatrix} \varphi_0$$

at $y = L$ the neutron polarization is

$$p_x\left(L\right) = \left\langle \hat{\sigma}_x \right\rangle = ab^* + a^*b = \frac{1}{2}\left[\exp\left(-2i\Delta kL\right) + \exp\left(2i\Delta kL\right)\right] = \cos 2\Delta kL$$

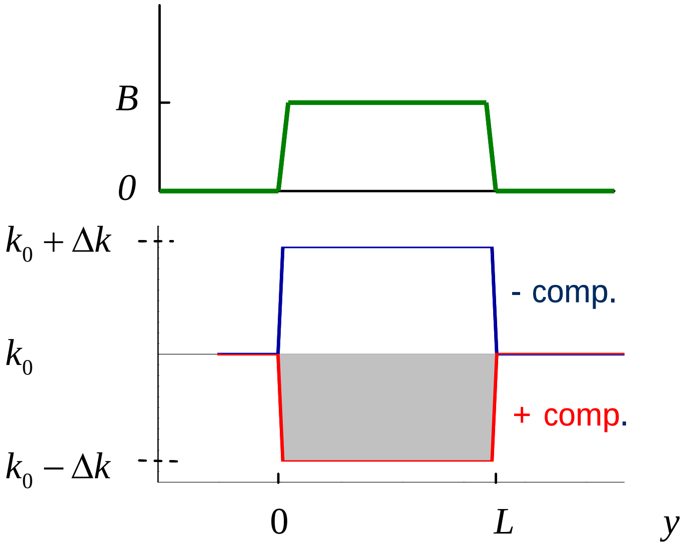
$$p_y\left(L\right) = \left\langle \hat{\sigma}_y \right\rangle = i\left(ab^* - ab^*\right) = \frac{i}{2}\left[\exp\left(-2i\Delta kL\right) - \exp\left(2i\Delta kL\right)\right] = \sin 2\Delta kL$$

Interaction neutron with magnetic field

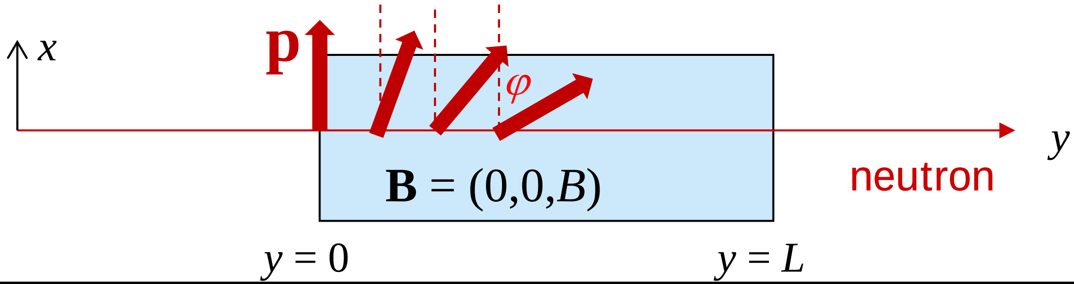
quantum-mechanical description

$$p_x(L) = \langle \hat{\sigma}_x \rangle = ab^* + a^*b = \exp(-2i\Delta kL) + \exp(2i\Delta kL) = \cos 2\Delta kL$$

$$p_y(L) = \langle \hat{\sigma}_y \rangle = i(ab^* - a^*b) = i(\exp(-2i\Delta kL) - \exp(2i\Delta kL)) = \sin 2\Delta kL$$



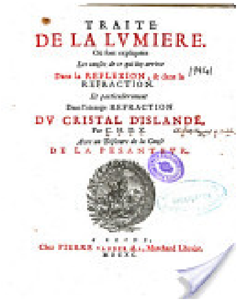
Interpretation: the neutron spin (expect.value) precesses by an angle $\varphi = 2\Delta kL = -m\gamma_n \lambda_0 BL / h$ around the field direction, being the integration of the wave-number difference between the plus and minus wave function integrated over L



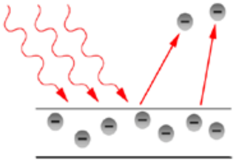
Observation of the Goos-Hänchen shift with neutrons

A.A. van Well, V.O. de Haan, J. Plomp, M.T. Rekveldt,
W.H. Kraan, R.M. Dalgliesh, S. Langridge

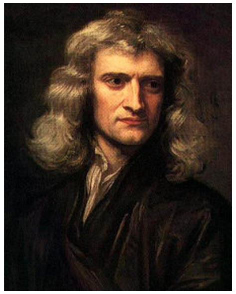
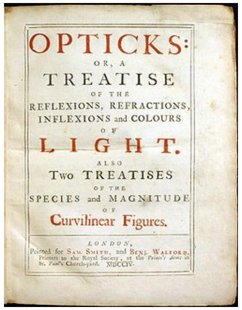
Particle-wave duality

Huygens
1690

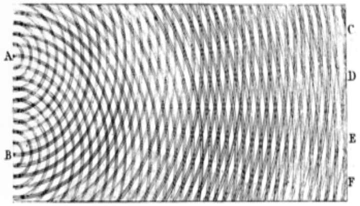
Einstein
1905



OPTICKS:
OR, A
TREATISE
OF THE
REFLEXIONS, REFRACTIONS,
INFLEXIONS and COLOURS
OF
LIGHT.
ALSO
Two TREATISES
OF THE
SPECIES and MAGNITUDE
OF
Curvilinear Figures.

Newton
1704

$$\lambda = \frac{h}{mv}$$

De Broglie
1924

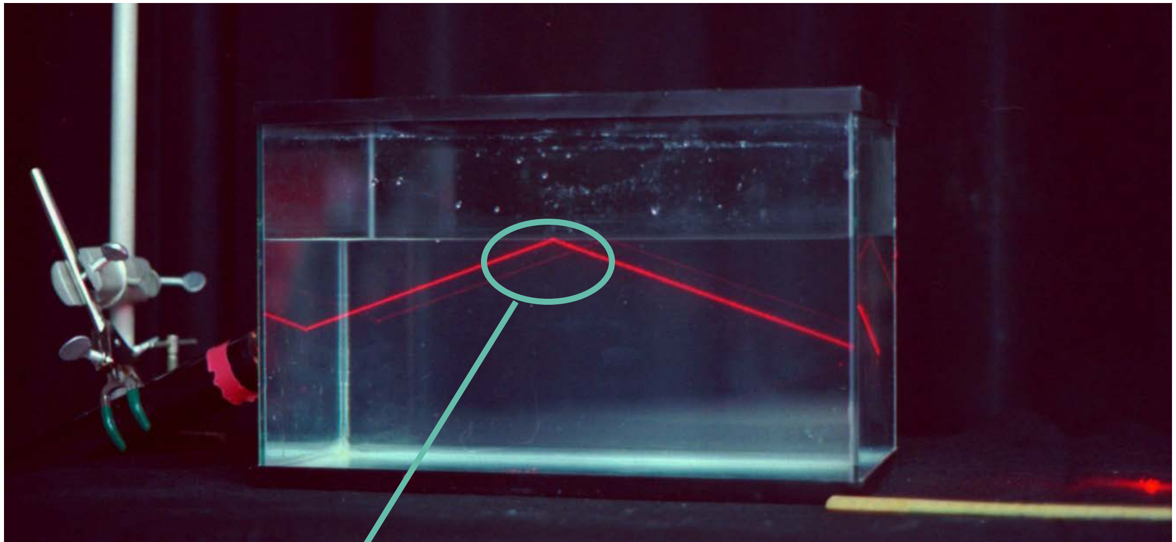
Fresnel
1818

Schrödinger
1925

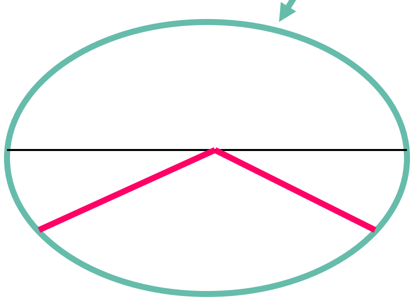
$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r},t) = \hat{H} \Psi(\mathbf{r},t)$$

Goos-Hänchen shift

total reflection for light

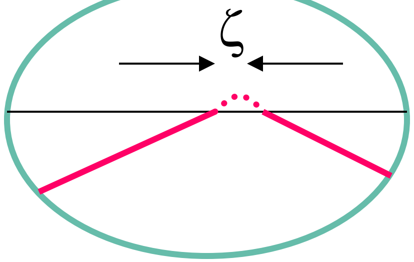


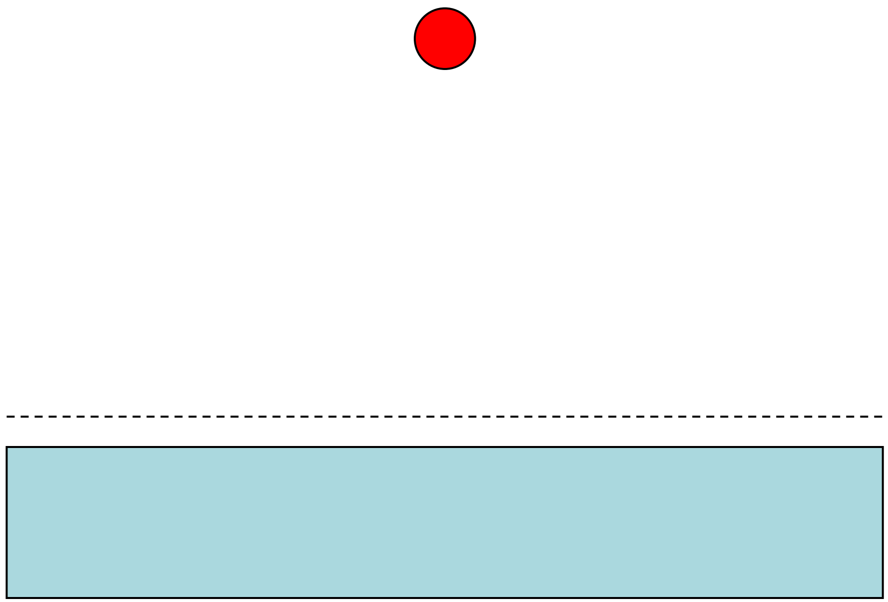
Goos-Hänchen shift ζ
up to 2 μm

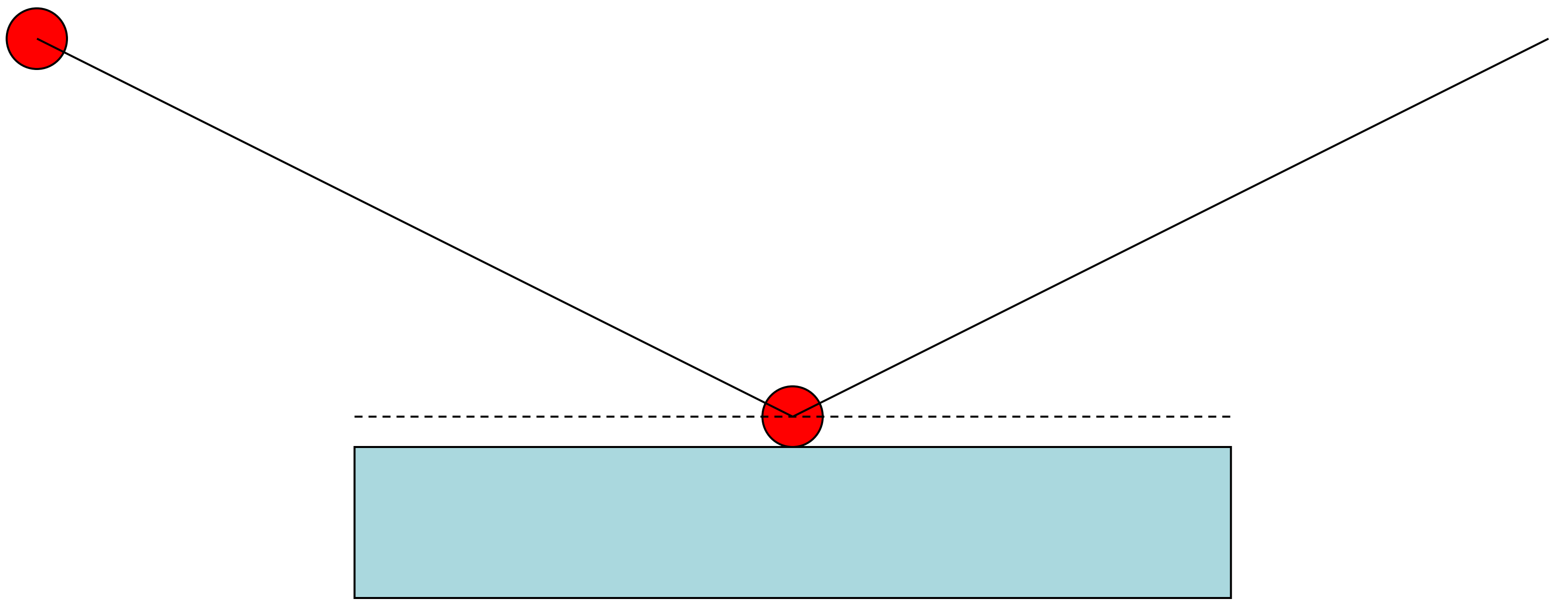


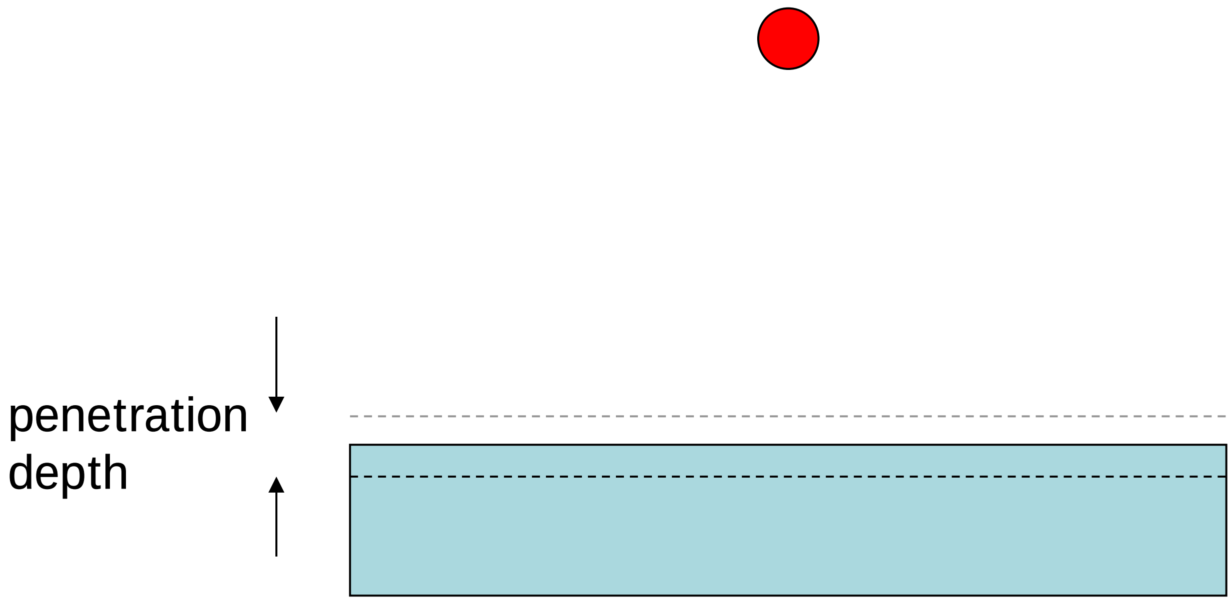
prediction:
I. Newton (~1700)

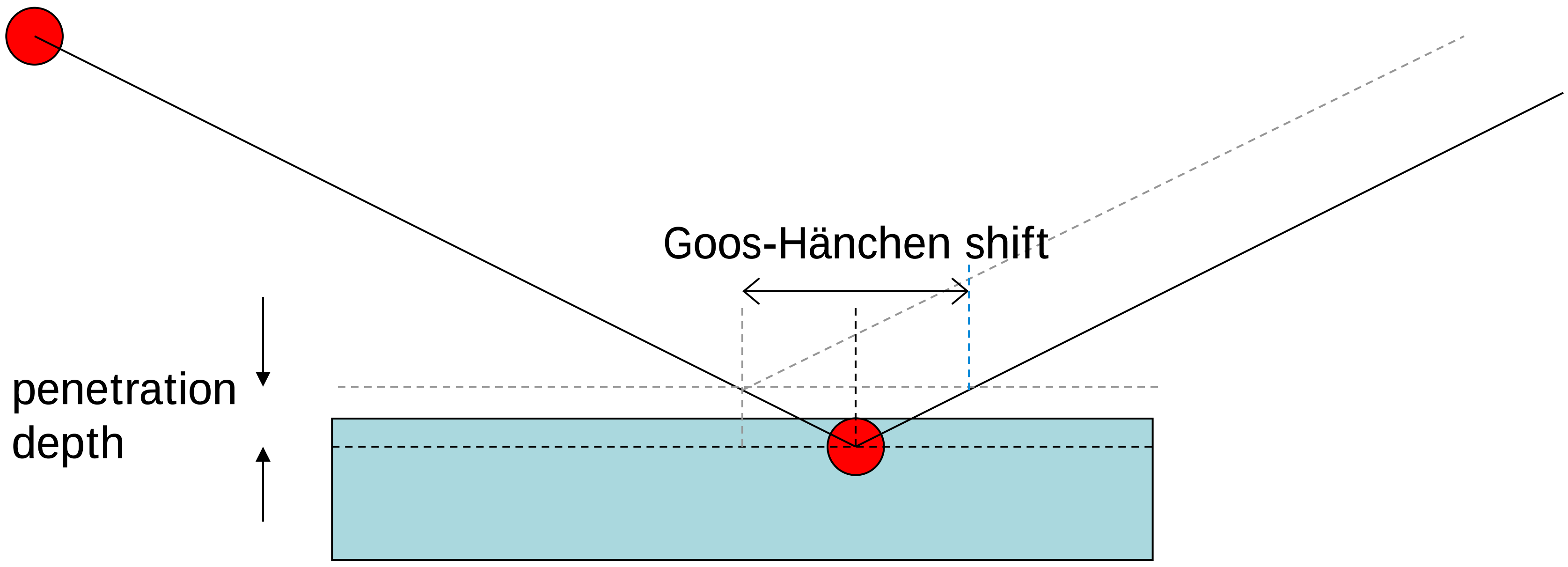
experiment:
F. Goos and H. Hänchen (1949)





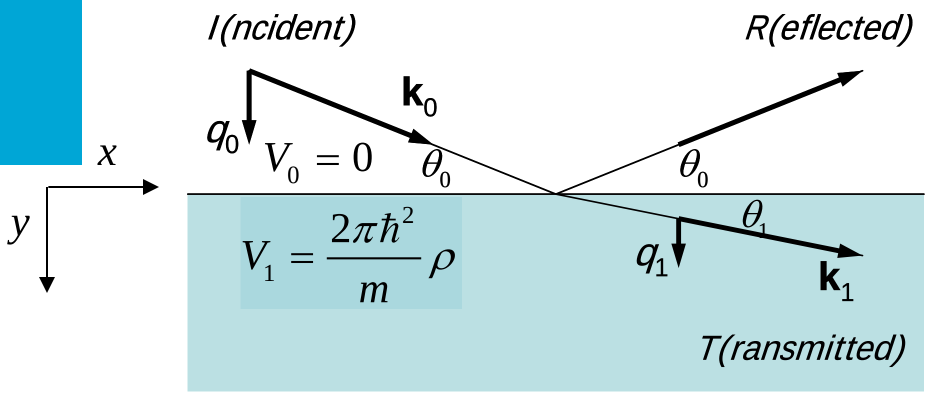


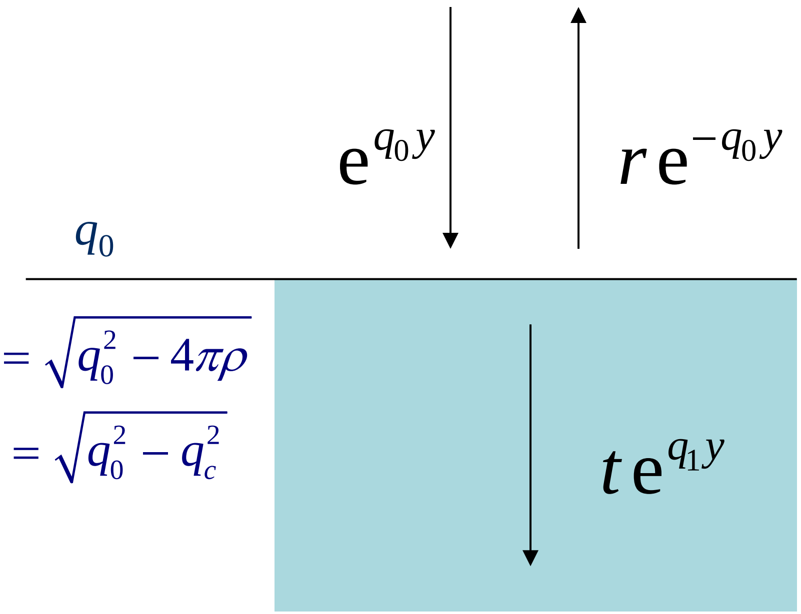

penetration
depth


Goos-Hänchen shift
penetration
depth

Neutron reflection at sharp interface (Fresnel)

no magnetic field





$$q_1 = \sqrt{q_0^2 - 4\pi\rho} = \sqrt{q_0^2 - q_c^2}$$

scattering-length density

$$\rho = \sum_j N_j b_j$$

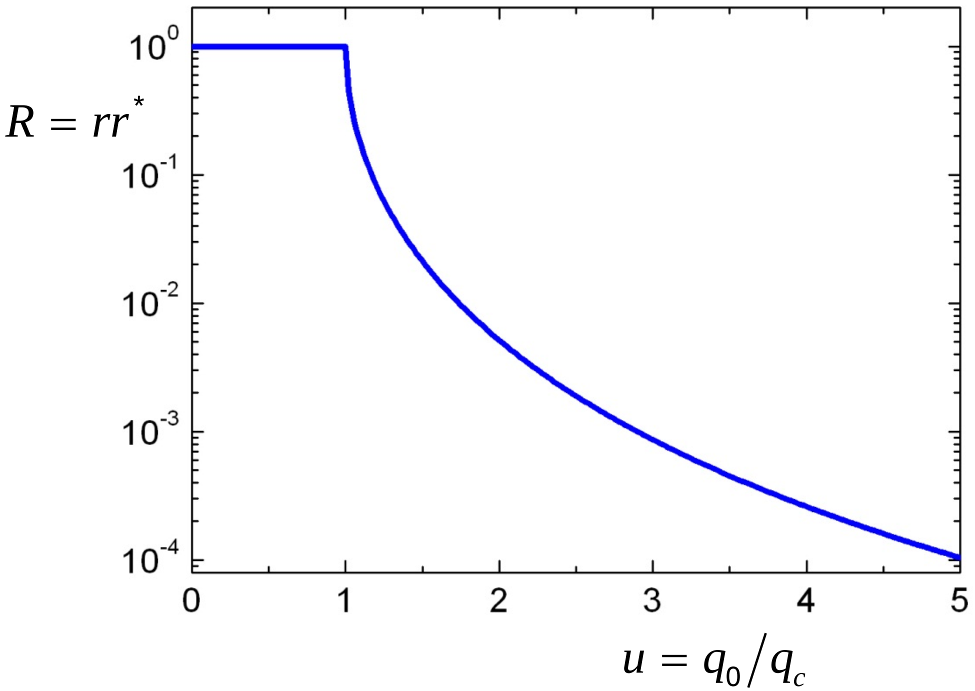
- isotropic in $x - z$
- x-component remains unchanged
- 1-dim Schrödinger Eq.

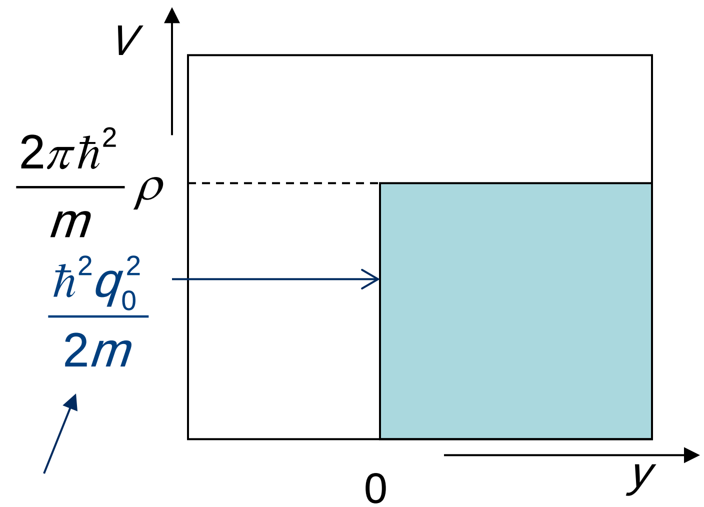
reflection amplitude $r = \frac{q_0 - q_1}{q_0 + q_1}$

transmission amplitude $t = \frac{2q_0}{q_0 + q_1}$

Sharp interface (Fresnel)

$$q_1 = \sqrt{q_0^2 - 4\pi\rho}$$
$$= \sqrt{q_0^2 - q_c^2}$$

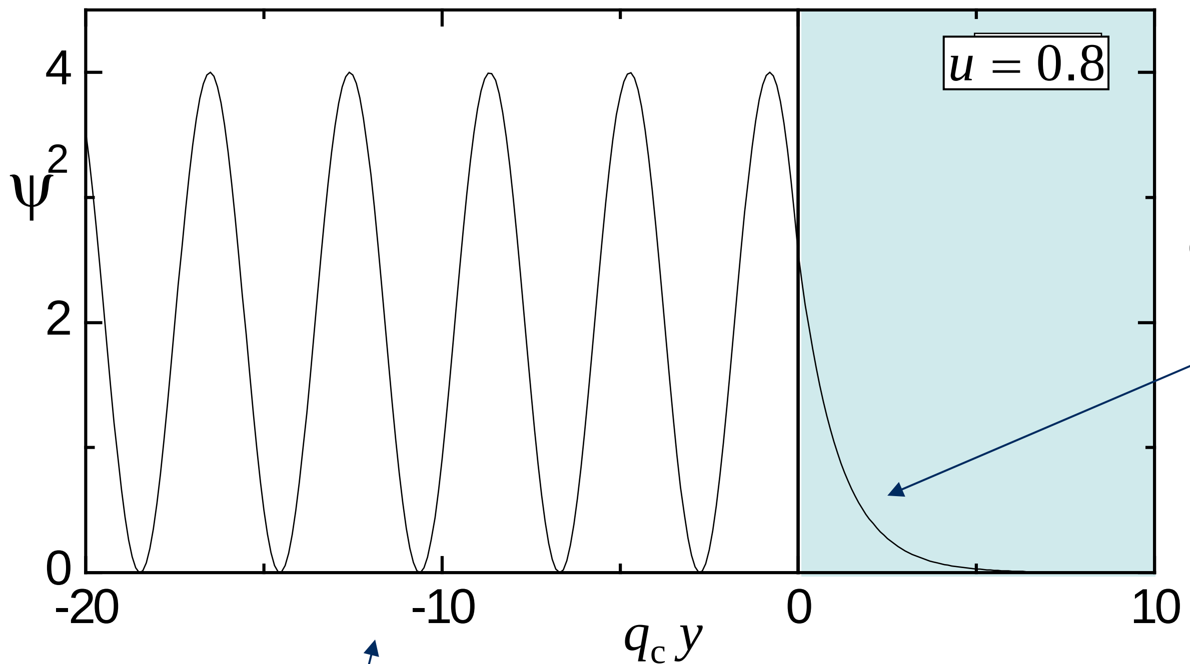




Perpendicular component kinetic energy

Total reflection

$u = q_0 / q_c$



$$q_1 = \sqrt{q_0^2 - q_c^2}$$

evanescent wave

$$\psi(y) = \psi(y=0)\, e^{-y/d}$$

penetration depth:

$$q_o d = \left(1 - u^2\right)^{-1/2}$$

$$\psi(y) = e^{iq_0 y} + re^{-iq_0 y}$$

Total reflection

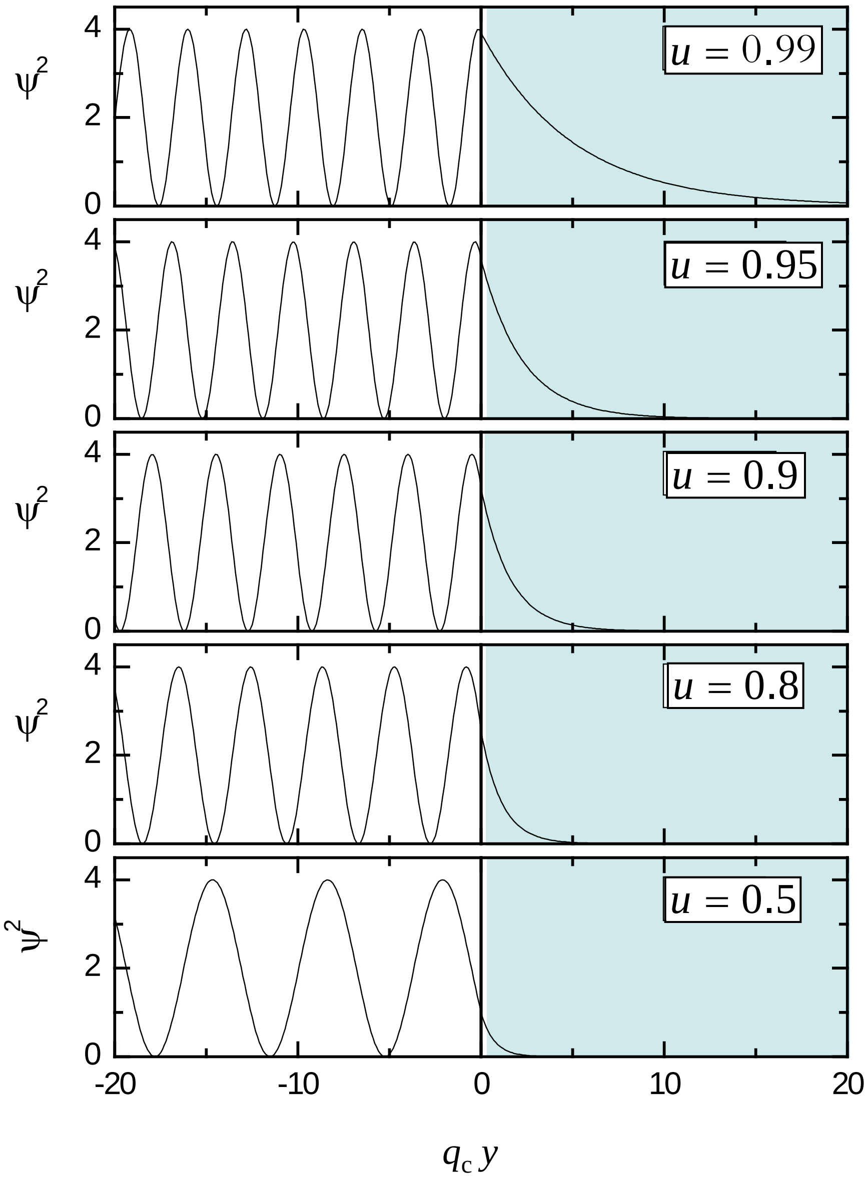
$$\psi(y) = \begin{cases} e^{iq_0 y} + re^{-iq_0 y} & y < 0 \\ \psi(y=0)e^{-y/d} & y > 0 \end{cases}$$

$$\psi(y=0) = 1 + r = 1 + e^{i\phi}$$

with phase $\phi = -2\arccos(u)$

$u = q_0 / q_c$

Unique relation between
- phase
- penetration depth

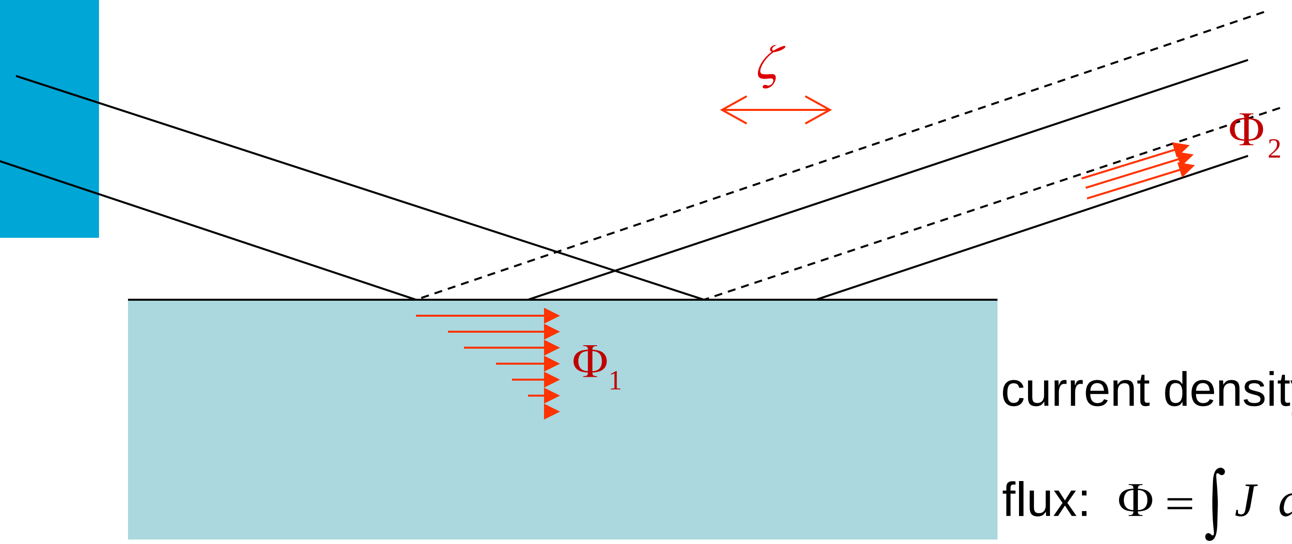


$\phi \to 0$

$\phi \to -\pi$

Goos-Hänchen shift

ref: R.H. Renard, J. Opt. Soc. Am. **54** (1964)1190



current density $\mathbf{J} = \frac{\hbar}{2im}\left(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*\right)$

flux: $\Phi = \int J \; ds$

conservation of particles: $\Phi_1 = \Phi_2$

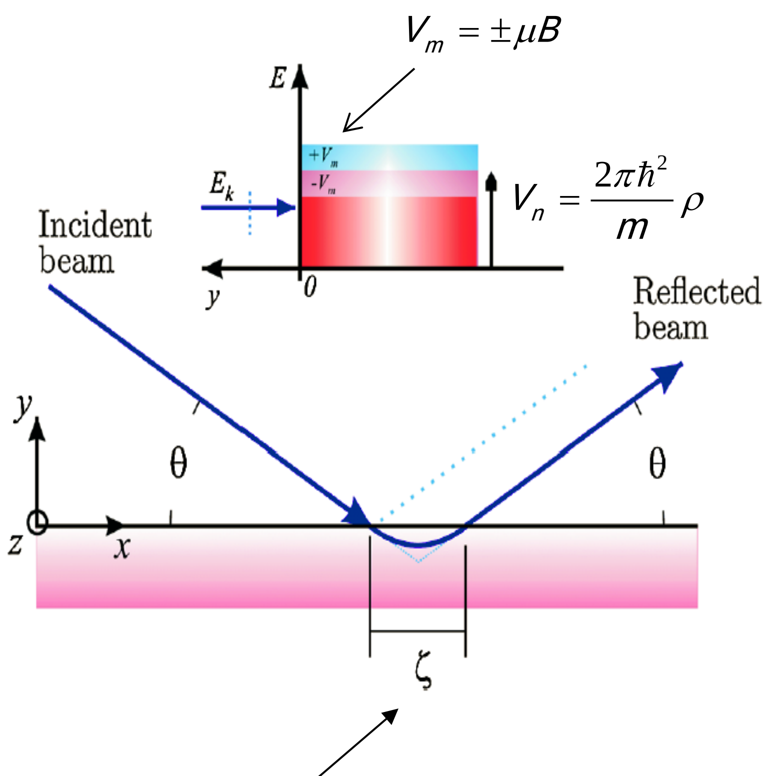
leads to shift $$\zeta = \frac{k}{q_c^2}\frac{2q_0}{\sqrt{q_c^2 - q_0^2}} = \frac{2\pi}{q_c^2\lambda}\frac{2u}{\sqrt{1-u^2}}$$

Unique relation between
- phase
- penetration depth
- GH shift

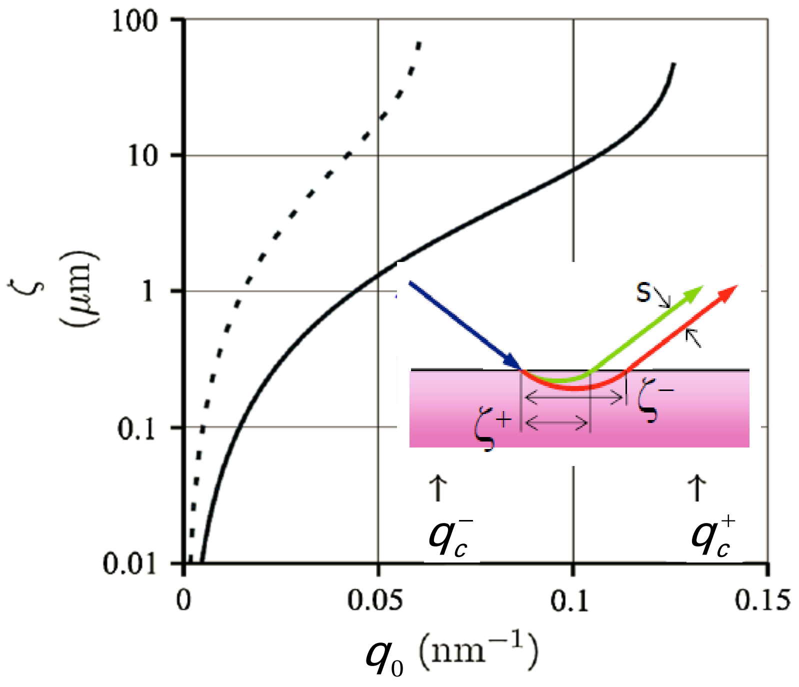
Alternative derivations

V. Ignatovich, Phys. Lett. A **322** (2004) 36

Polarized neutrons and magnetic material



different for + and - state



example: magnetized iron

How to measure it ⇨ Larmor precession

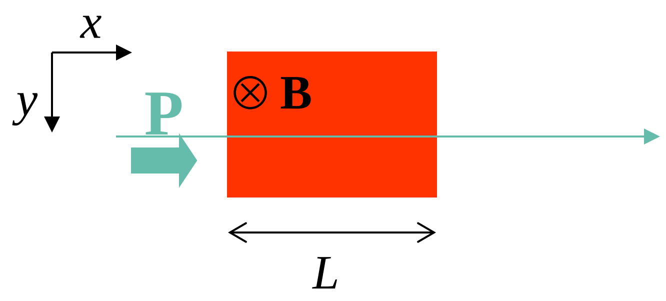
Schrödinger equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + \begin{bmatrix} \mu B & o \\ 0 & -\mu B \end{bmatrix}\Psi$$

Quantization axis in z - direction
Beam polarized in x - direction

No magnetic field:

$$\Psi = \begin{bmatrix} \psi^+ \\ \psi^- \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{i(k_0 x - \omega_0 t)} \\ e^{i(k_0 x - \omega_0 t)} \end{bmatrix}$$



How to measure it ⇨ Larmor precession

Schrödinger equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + \begin{bmatrix}\mu B & o \\ 0 & -\mu B\end{bmatrix}\Psi$$

If the magnetic field is entered at $x = 0$ its solution at $x = L$ is

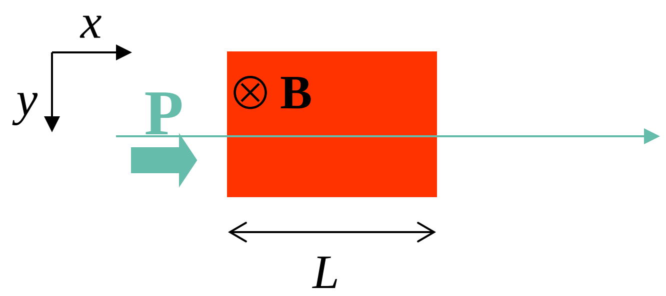
$$\Psi = \begin{bmatrix}\psi^{+} \\ \psi^{-}\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}\mathrm{e}^{i\left((k_0-\Delta k)L-\omega_0 t\right)} \\ \mathrm{e}^{i\left((k_0+\Delta k)L-\omega_0 t\right)}\end{bmatrix}$$

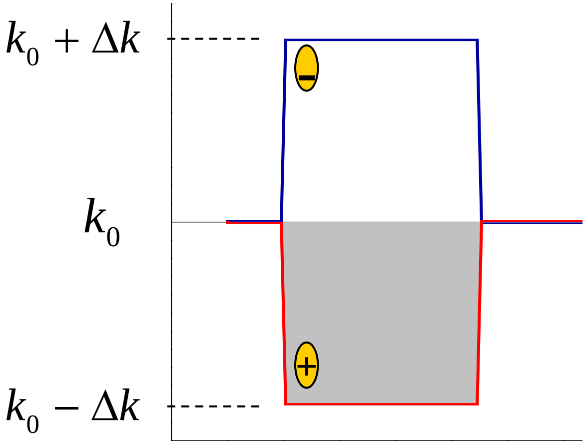
The polarisation is

$$\langle\hat{\sigma}_x\rangle = \frac{\psi^{+}\psi^{-*} + \psi^{-}\psi^{+*}}{\psi^{+}\psi^{+*} + \psi^{-}\psi^{-*}}$$

$$= \tfrac{1}{2}\left(\mathrm{e}^{-2i\Delta kL} + \mathrm{e}^{+2i\Delta kL}\right) = \cos 2\Delta kL$$

$$= \cos\frac{2\mu BL}{\hbar v_0}$$





Zeeman energy μB

How to measure it ⇨ Larmor precession

Schrödinger equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + \begin{bmatrix}\mu B & o \\ 0 & -\mu B\end{bmatrix}\Psi$$

If the magnetic field is entered at $x = 0$ its solution at $x = L$ is

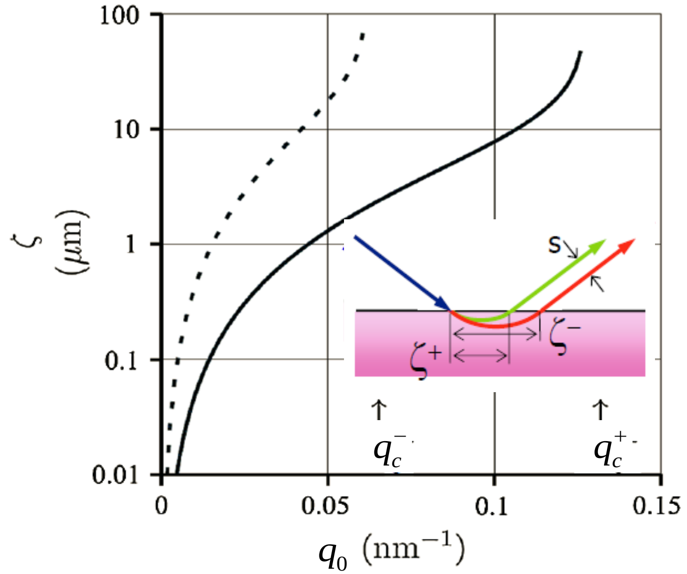
$$\Psi = \begin{bmatrix}\psi^+ \\ \psi^-\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}e^{i((k_0-\Delta k)L-\omega_0 t)} \\ e^{i((k_0+\Delta k)L-\omega_0 t)}\end{bmatrix}$$

The polarisation is

$$\langle\hat{\sigma}_x\rangle = \frac{\psi^+\psi^{-*} + \psi^-\psi^{+*}}{\psi^+\psi^{+*} + \psi^-\psi^{-*}}$$

$$= \tfrac{1}{2}\left(e^{-2i\Delta kL} + e^{+2i\Delta kL}\right) = \cos 2\Delta kL$$

$$= \cos\frac{2\mu BL}{\hbar v_0}$$



in total reflection region both spin states add different phase to wave function:

Extra 'pseudo' Larmor precession

How to measure it

Pseudo Larmor precession

total reflection

$$\psi_r^{\pm}(y=0) = r^{\pm} = e^{i\phi^{\pm}}$$

with phase $\phi^{\pm} = -2\arccos\left(q_0/q_c^{\pm}\right)$

$$\Psi_r(y=0) = \begin{bmatrix} \psi_r^{+}(y=0) \\ \psi_r^{-}(y=0) \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} \exp(i\phi^{+}) \\ \exp(i\phi^{-}) \end{bmatrix}$$

$$= \frac{\exp(i\varepsilon/2)}{\sqrt{2}} \begin{bmatrix} \exp(i\delta/2) \\ \exp(-i\delta/2) \end{bmatrix}$$

with

$$\gamma(q_0) = \phi^{+}(q_0) + \phi^{-}(q_0)$$

$$\delta(q_0) = \phi^{+}(q_0) - \phi^{-}(q_0)$$

analogous to Larmor precession:

$$\langle \hat{\sigma}_x \rangle = \frac{\psi^{+}\psi^{-*} + \psi^{-}\psi^{+*}}{\psi^{+}\psi^{+*} + \psi^{-}\psi^{-*}}$$

$$= \tfrac{1}{2}\left(\exp(+i\delta) + \exp(-i\delta)\right) = \cos\delta$$

$$= \cos\left(\phi^{+}(q_0) - \phi^{-}(q_0)\right)$$

extra precession upon reflection

How to measure it

(i) Spin-echo instrument

beam polarization
after one precession region:

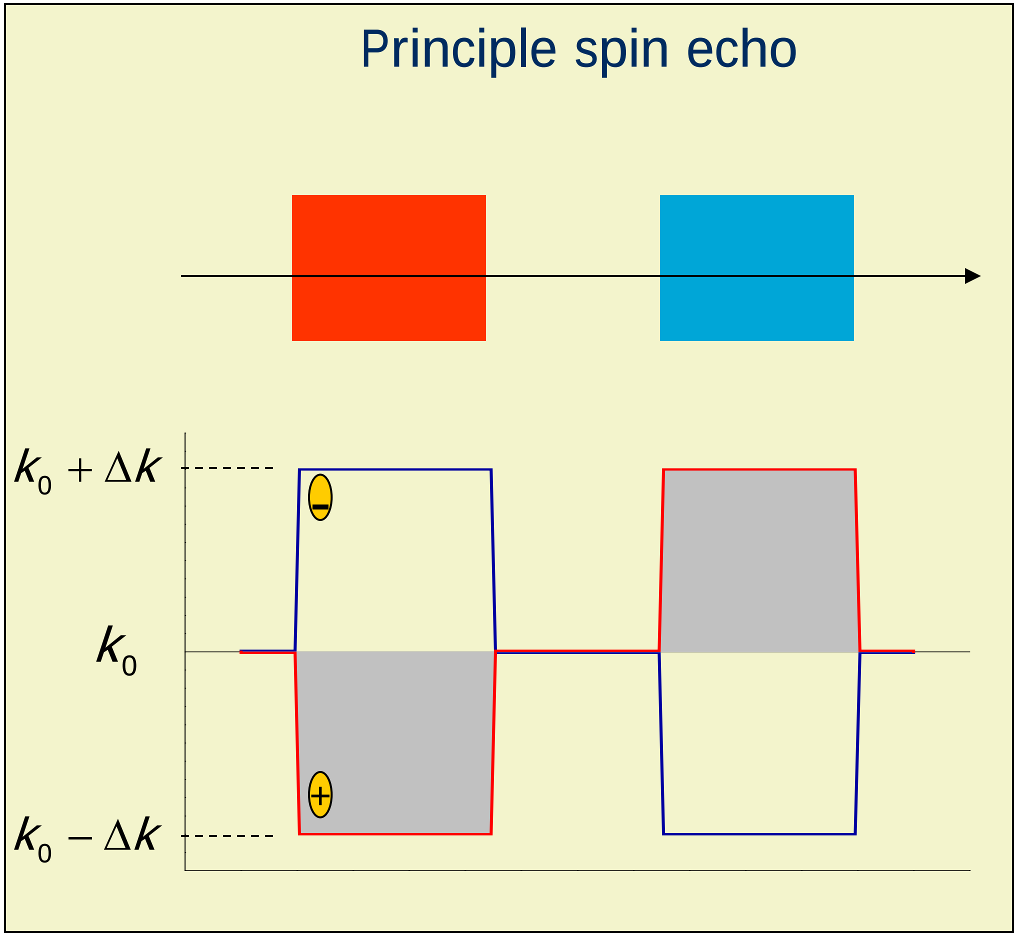
$$\langle \hat{\sigma}_x \rangle = \psi^+ \psi^{-*} + \psi^- \psi^{+*}$$

$$= \tfrac{1}{2}\left(e^{-2i\Delta kL} + e^{+2i\Delta kL} \right) = \cos 2\Delta kL$$

$$= \cos \frac{2\mu BL}{\hbar v_0}$$

is compensated in 2nd region

sensitive to extra precession
upon reflection

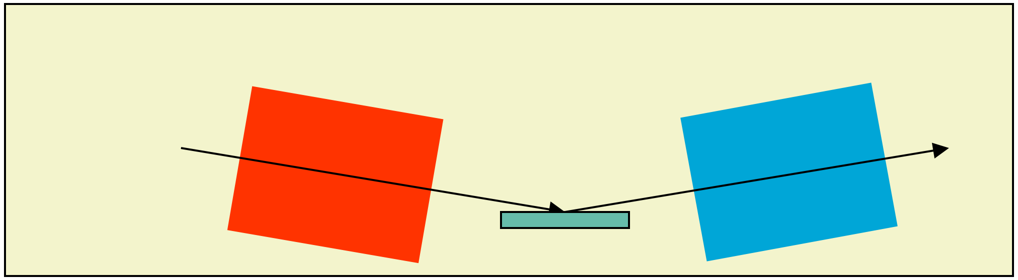


How to measure it

(ii) Neutron reflectometer

OffSpec, ISIS, UK

Experiment



- sample : Si wafer with 3 μm Permalloy ($Fe_{0.2}Ni_{0.8}$) magnetized in plane ($\mathbf{B} \perp$ beam)

- OffSpec 'in echo' with non-magnetic sample in reflection
  $\rightarrow$ determines polarization P_0

- glancing angle ~ 4 mrad, q_0 scanned by time-of-flight

- two measurements: single and double reflection

- measured spin-echo signal $\frac{P}{P_0} = \cos\left(N\delta\left(q_0\right)\right)$

with $\delta\left(q_0\right) = \phi^+\left(q_0\right) - \phi^-\left(q_0\right)$ the Larmor 'pseudo precession' due to different phases at reflection

de Haan *et al*., PRL **104** (2010) 010401

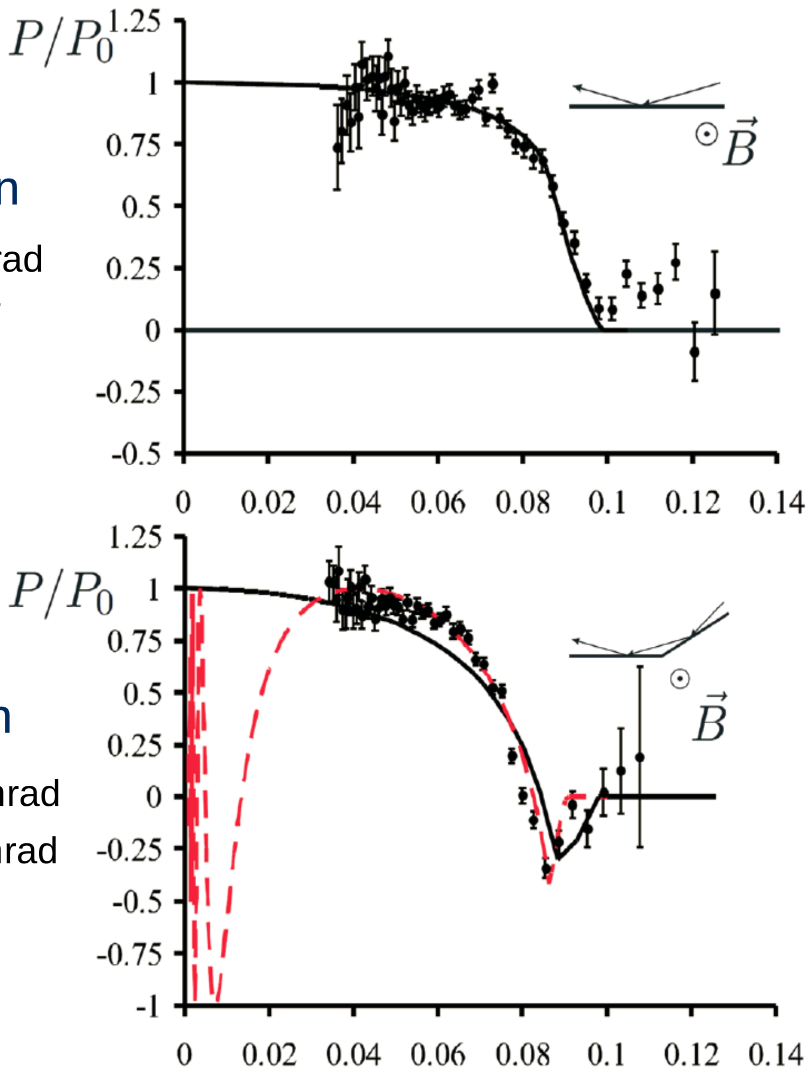
Experiment

black line: theory
red line: theory, with small correction in P_0

single reflection

$\theta_0 = 5.0$ mrad

$B_s = 1.2$ T



double reflection

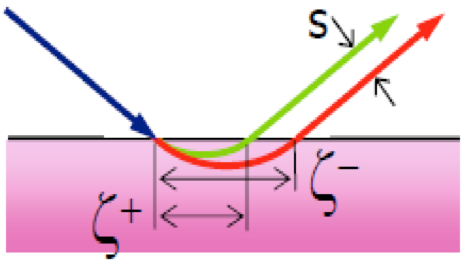
$\theta_0 = 4.05$ mrad

$\theta_1 = 3.75$ mrad

$B_s = 1.2$ T

q_0 $\left(\text{nm}^{-1}\right)$

GH shift



$q_0 = 0.06$ nm^{-1} $\zeta^- = 2.4$ μm

$\zeta^+ = 1.0$ μm

$q_0 = 0.09$ nm^{-1} $\zeta^- = 20$ μm

$\zeta^+ = 2.8$ μm

$s \leq 100$ nm

Gravitation-induced quantum phase shift in a spin-echo neutron interferometer

A.A. van Well, V.O. de Haan, J. Plomp, M.T. Rekveldt, Y.H. Hasegawa, R.M. Dalgliesh, N.J. Steinke

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- Previous experiments
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Introduction

Time-independent Schrödinger equation:

$$-\frac{\hbar^2}{2m_i}\nabla^2\psi(\mathbf{r})+V(\mathbf{r})\psi(\mathbf{r})=E\psi(\mathbf{r})$$

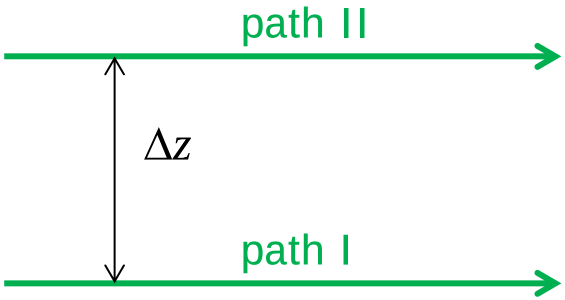
In vacuum, gravitational field:

$$V_g(z)=m_g g z$$

- both h and g in one equation
- m_i inertial mass
- m_g gravitational mass

Plane-wave solution:

$$\psi = A\exp(i\ \mathbf{k}\cdot\mathbf{r})$$ with accumulated phase $$\phi=\int\mathbf{k}\cdot d\mathbf{r}$$



kinetic energy	wave number
$E_{k,II}=E_{k,I}-\Delta E_g$	$k_{II}=k_I-\Delta k_g$
$E_{k,I}=E_0=\frac{\hbar^2 k_0^2}{2m_i}$	$k_I=k_0$

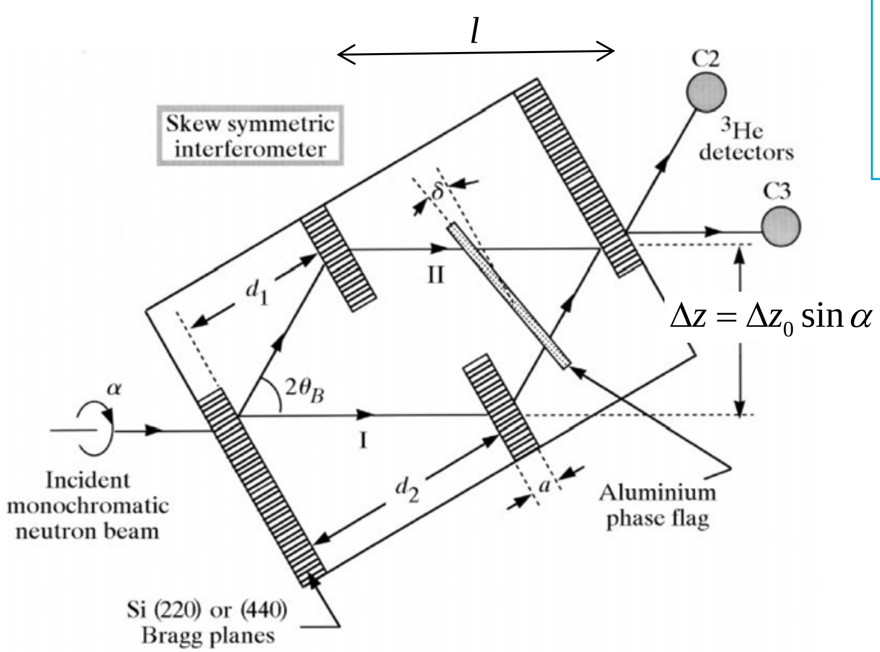
$$\Delta E_g = m_g g\Delta z$$

$$\Delta k_g=\frac{m_i m_g}{\hbar^2 k_0}g\Delta z$$

Previous experiments

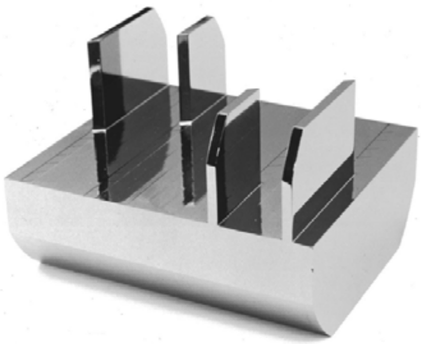
Si single-crystal interferometer (COW experiments)

The wave function is coherently split in two paths at different heights by means of Bragg reflection



phase difference between both paths:

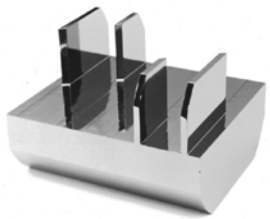
$$\Delta\phi = k_I l - k_{II} l \propto \Delta k_g l \Delta z$$



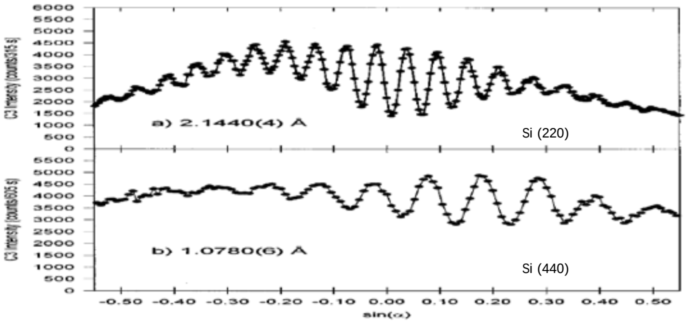
R. Colella *et al.*, Phys. Rev. Lett. **34** (1975) 1472
K.C. Littrell *et al.*, Acta Cryst. **54** (1998) 563

Previous experiments

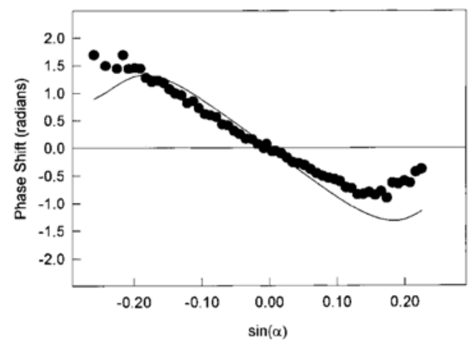
Si single-crystal interferometer (COW experiments)



Results: interference signal as a function of extra phase added to both arms for two wavelengths



phase shift $\Delta\phi$ as a function of rotation angle α



some numbers

$E_{220} = 18$ meV

$E_{440} = 70$ meV

$\Delta z = 18$ mm

$\Delta E_g \approx 2$ neV

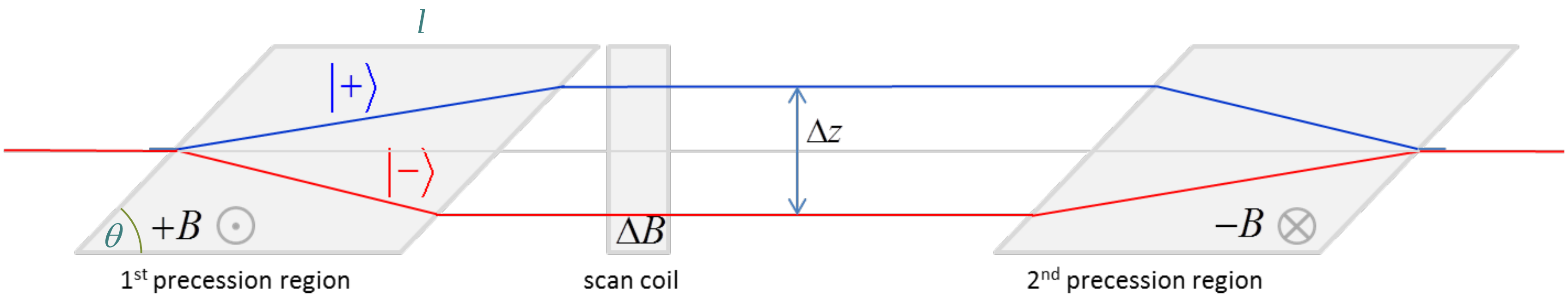
Conclusion:

experimental phase shift is 1.0 ± 0.1% smaller, compared with theory, when taking $m_i = m_g$

Present experiment

Spin-echo neutron interferometer (Offspec, ISIS)

The spin-up and spin-down component of the wave function is coherently split in two paths at different heights by means of a magnetic field



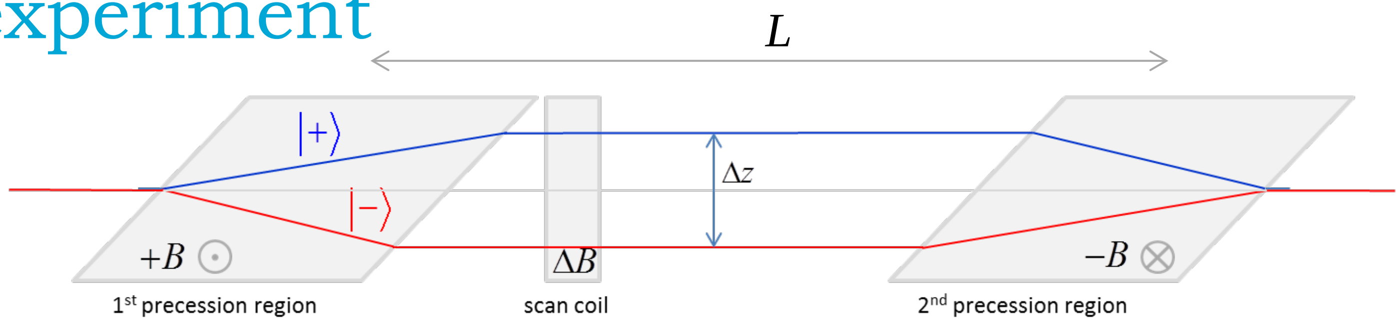
Schrödinger equation:

$$-\frac{\hbar^2}{2m_i}\nabla^2\begin{pmatrix}\psi^+\\ \psi^-\end{pmatrix}+m_g g/2\begin{bmatrix}\Delta z & 0\\ 0 & -\Delta z\end{bmatrix}\begin{pmatrix}\psi^+\\ \psi^-\end{pmatrix}+\begin{bmatrix}\mu_n B & 0\\ 0 & -\mu_n B\end{bmatrix}\begin{pmatrix}\psi^+\\ \psi^-\end{pmatrix}=E\begin{pmatrix}\psi^+\\ \psi^-\end{pmatrix}$$

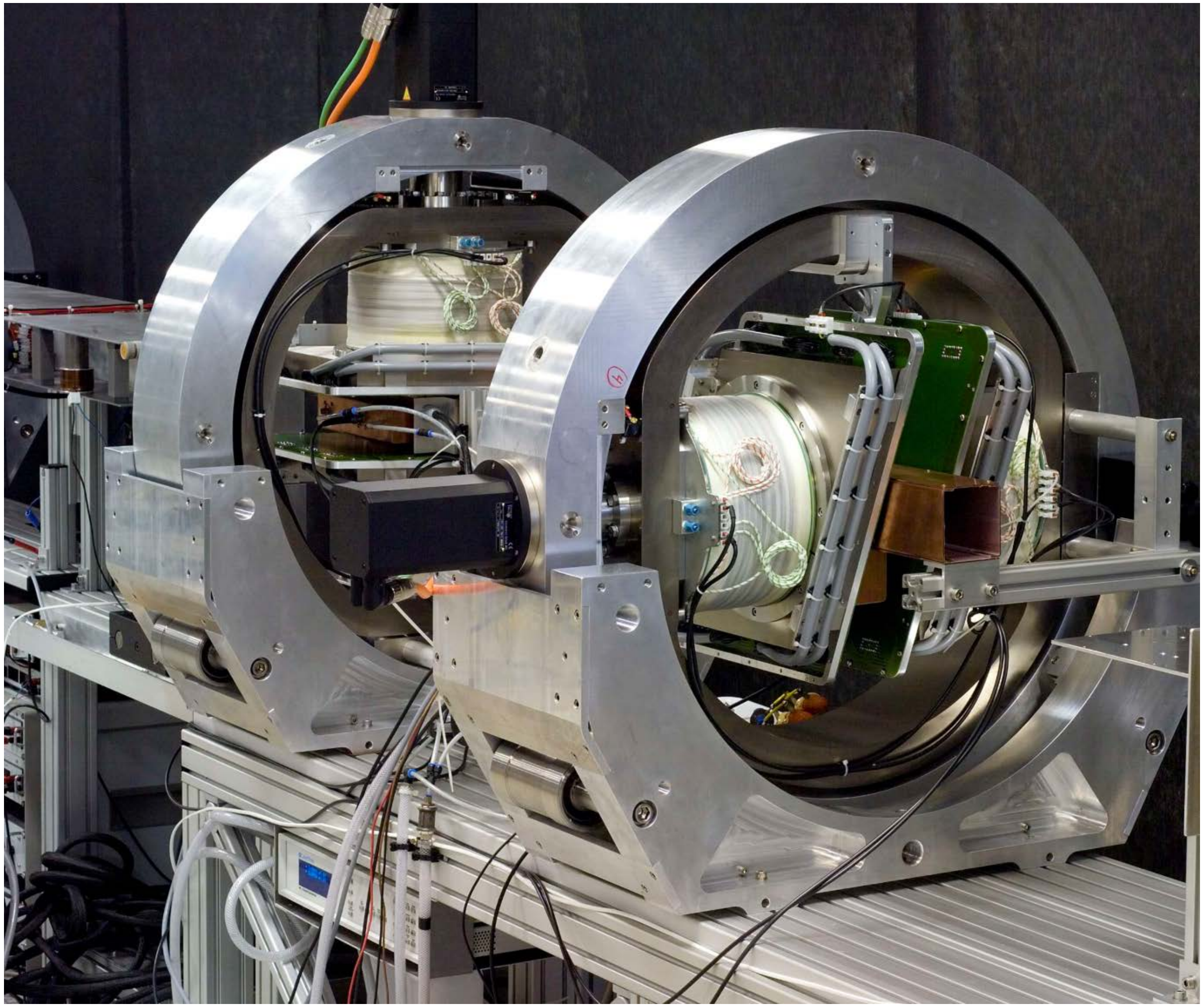
	kinetic energy	wave number	
$\lvert + \rangle$	$E_k^+ = E_0 - \Delta E_g/2 - \Delta E_Z$	$k^+ = k_0 - \Delta k_g/2 - \Delta k_Z$	$\Delta E_Z = \mu_n B$
			$\Delta k_Z = \frac{m_i}{\hbar^2 k_0}\mu_n B$
$\lvert - \rangle$	$E_k^- = E_0 + \Delta E_g/2 + \Delta E_Z$	$k^- = k_0 + \Delta k_g/2 + \Delta k_Z$	$\Delta z = \left(2m\mu_n Bl\cot\theta / h^2\right)\lambda^2$

V.O. de Haan *et al.*, Phys. Rev. A **89** (2014) 063611

Present experiment

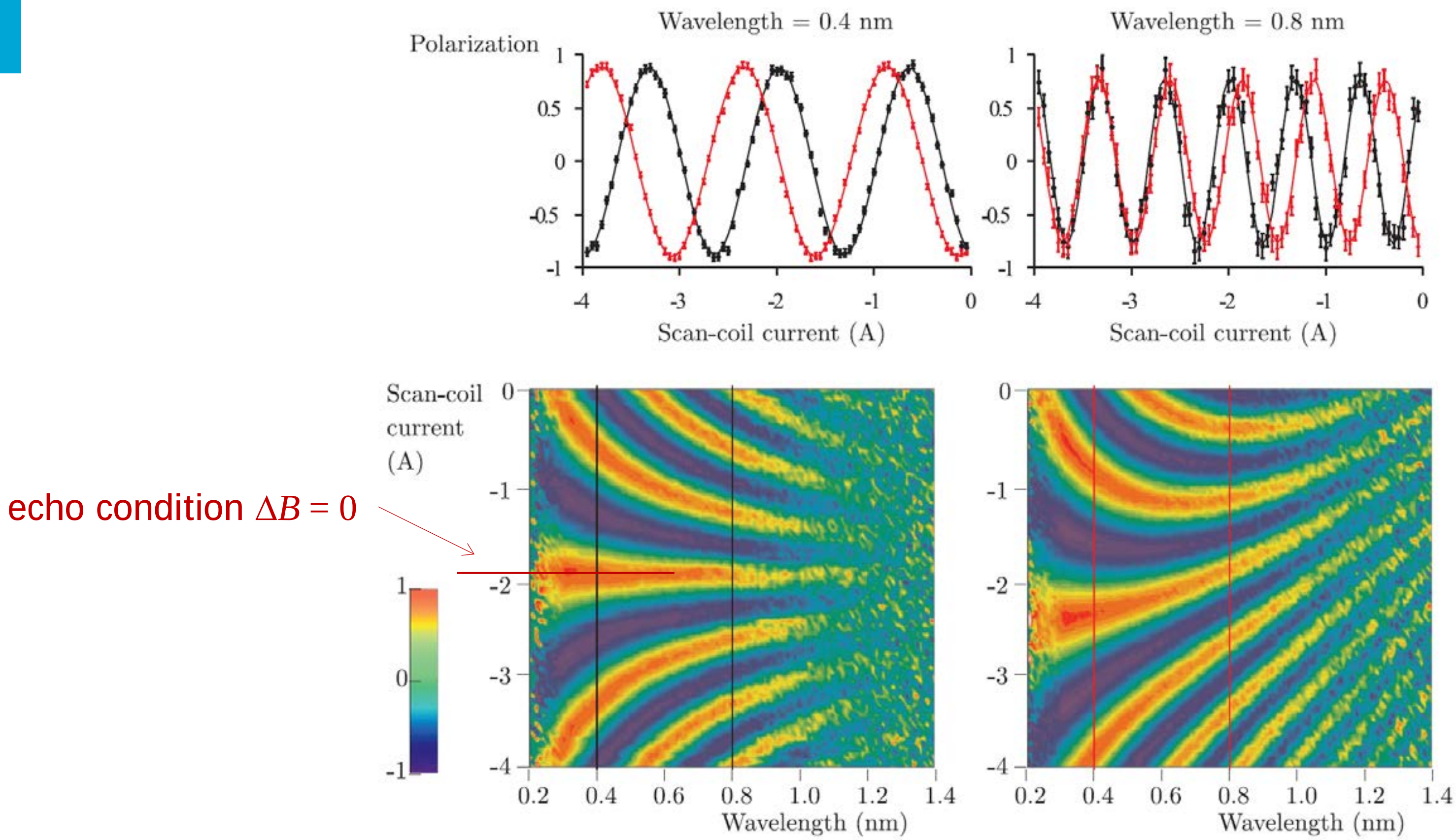


- Offspec is a time-of-flight instrument covering wavelength range $0.2 < \lambda < 1.4$ nm
- The spin-echo polarisation of the recombined neutrons is measured: $P(B,\lambda) = \langle \cos(\phi_L) \rangle$
- with Larmor phase: $\phi_L = \int \left(k^+(x) - k^-(x)\right) dx \propto \Delta k_g L \Delta z$
- extra phase is created by scan coil
- experiments are performed with splitting both in horizontal and vertical plane
- inclination angle of whole setup between -1.0 and +0.5 degrees



Present experiment

Result 1: Contour plot of the spin-echo polarisation as a function of wavelength and extra phase added to both arms



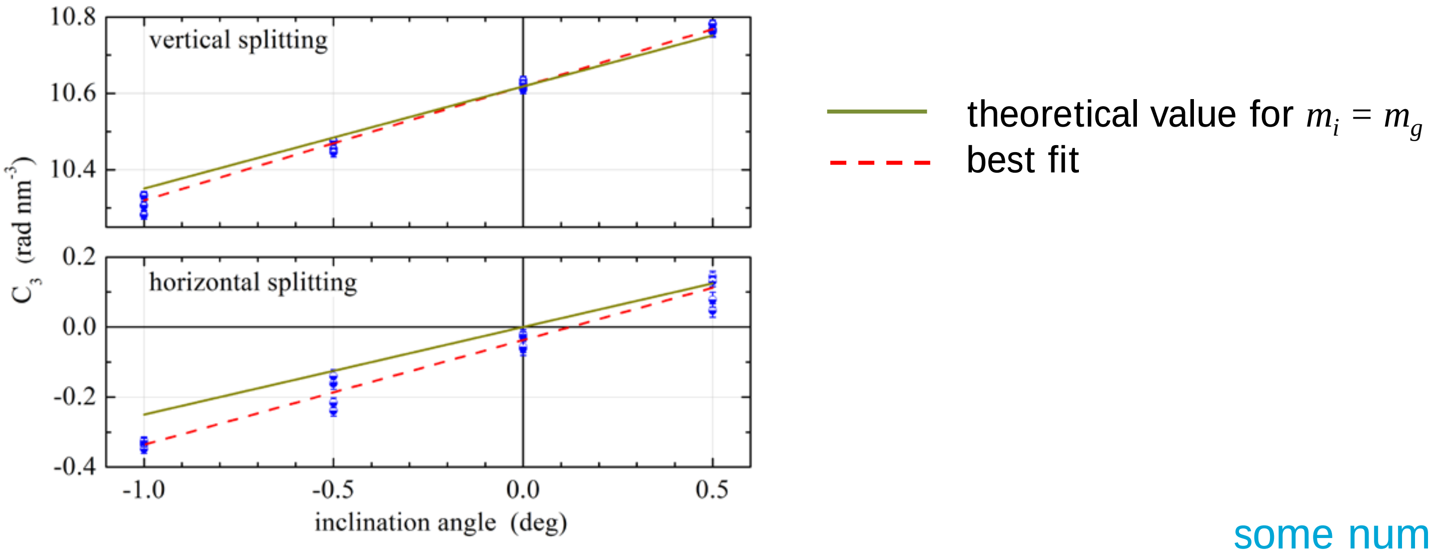
splitting in horizontal / vertical plane

Present experiment

C_1: echo-condition
C_2: Sagnac effect
(small correction)
C_3: result of gravity

Phases are described by $\Delta\phi_L = C_1\lambda + C_2\lambda^2 + C_3\lambda^3$

Result 2: Parameter C_3 as a function of inclination angle



some numbers

$E_0 = 0.4 - 20$ meV

$\Delta E_Z = 20$ neV

$\Delta z = 0.3 - 14$ μm

$\Delta E_g = 0.03 - 1.4$ peV

Conclusion:

experimental phase shift is, within the experimental accuracy of 0.1%, in agreement with theory, when taking $m_i = m_g$

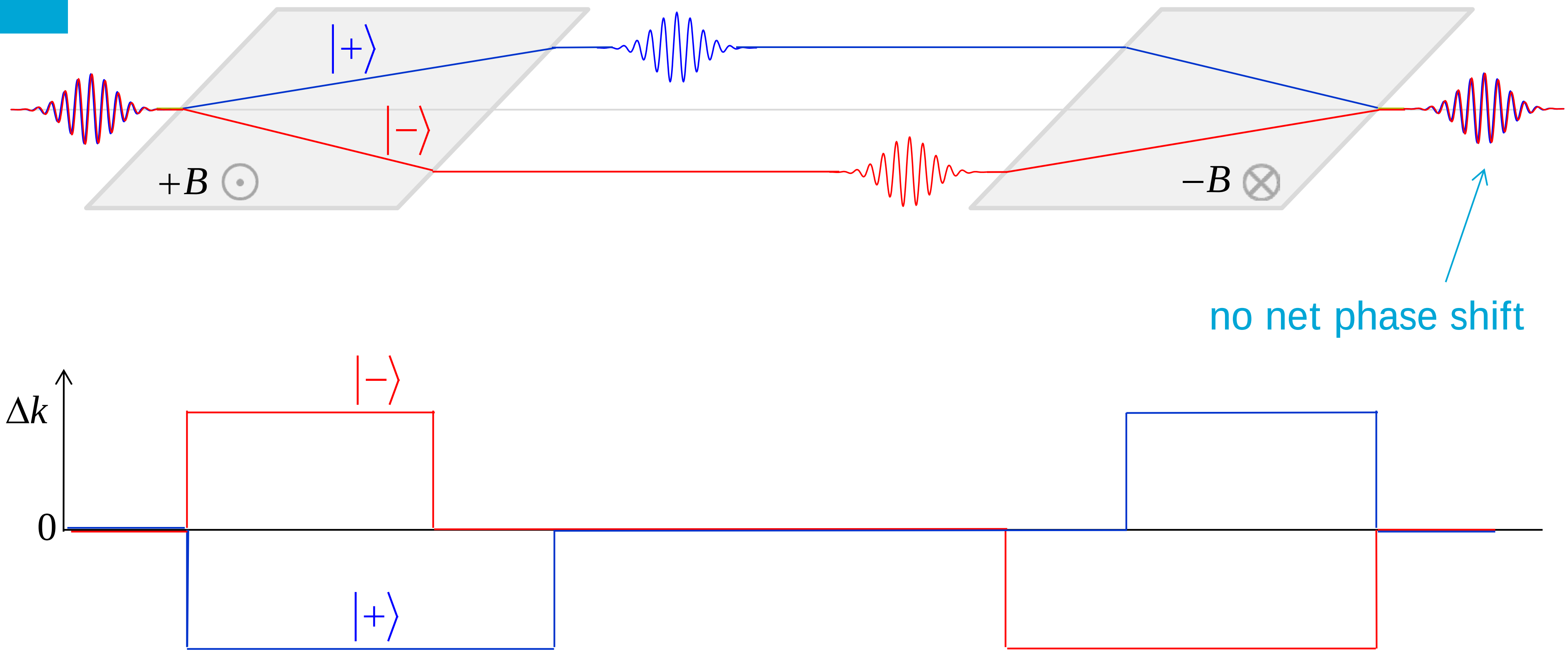
intercept C_3: exp:10.619(9) theory: 10.618(24)

Discussion

$$\phi_L = \int \left(k^+(x) - k^-(x)\right) dx \propto \Delta k_g L \Delta z$$

$$k^+ = k_0 - \Delta k_g/2 - \Delta k_Z$$

$$k^- = k_0 + \Delta k_g/2 + \Delta k_Z$$



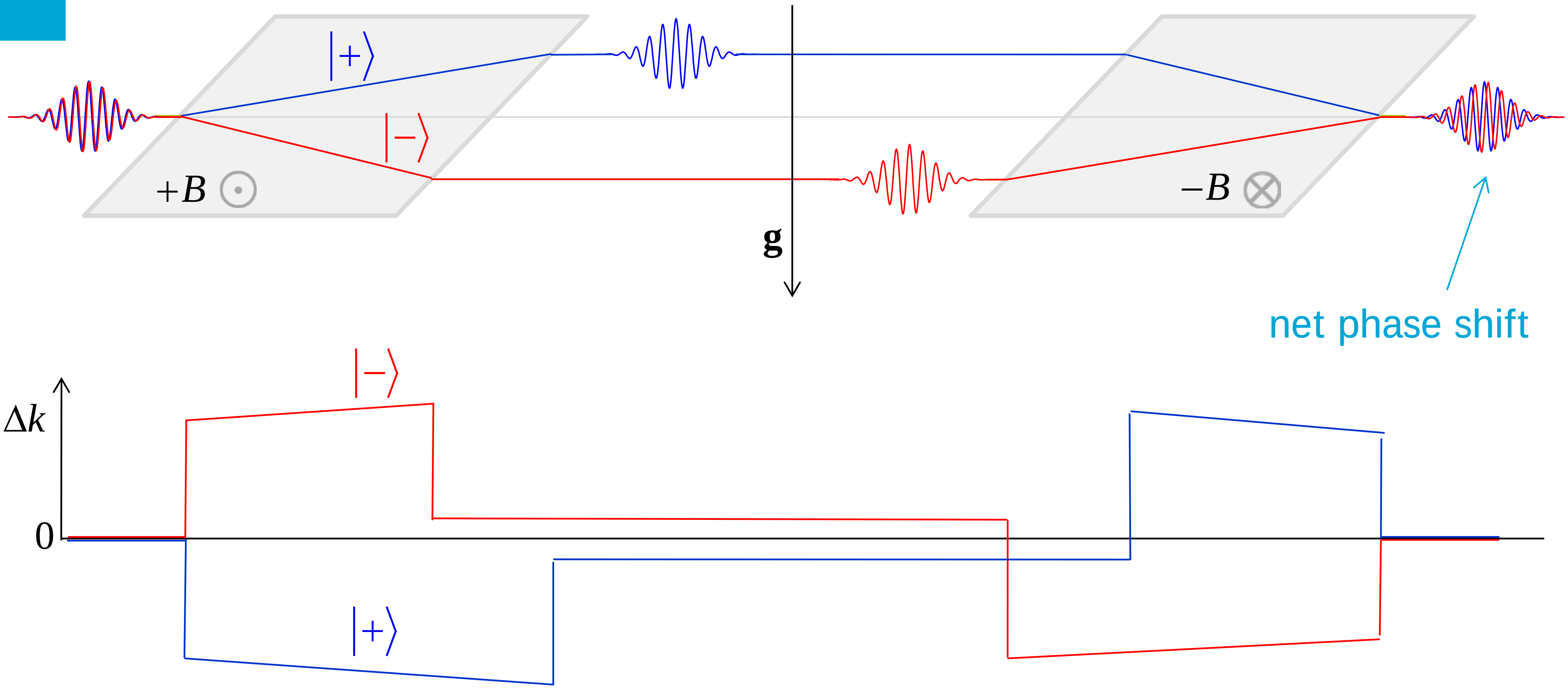
splitting in horizontal plane

Discussion

$$\phi_L = \int \left(k^+(x) - k^-(x)\right) dx \propto \Delta k_g L \Delta z$$

$$k^+ = k_0 - \Delta k_g/2 - \Delta k_Z$$

$$k^- = k_0 + \Delta k_g/2 + \Delta k_Z$$



splitting in vertical plane