

# NSE: Magnetic Field Simulation and Optimization

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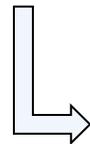
*JCNS-MLZ Forschungszentrum Jülich*

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# Outline

1. Problem: Asymmetries and inhomogeneities

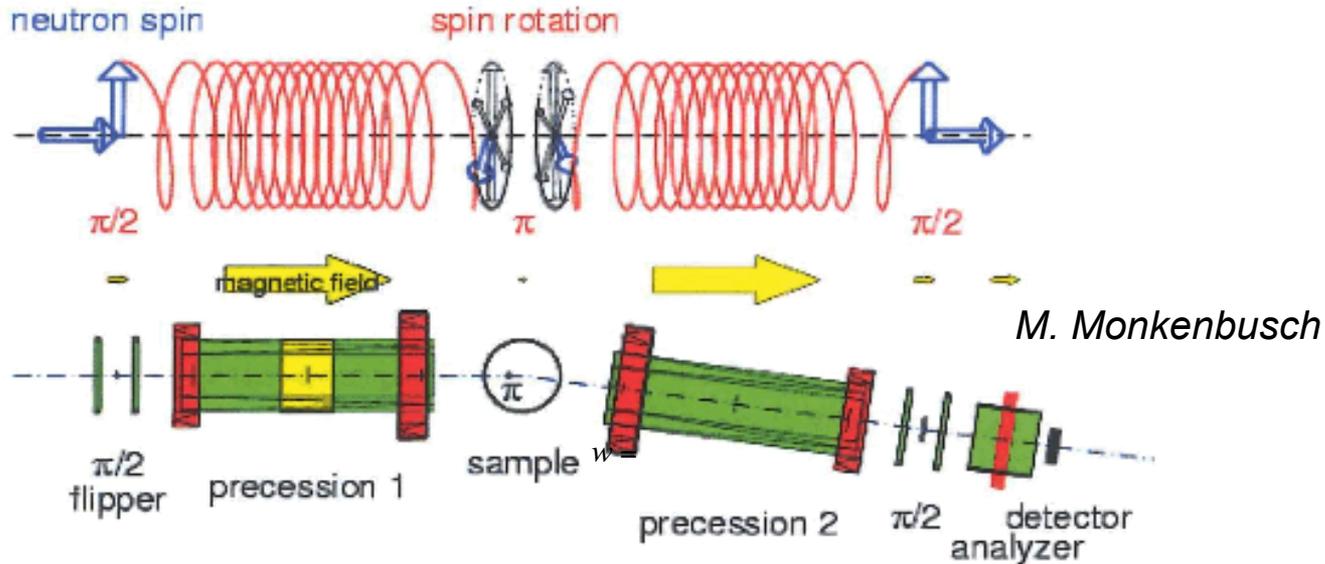
2. Strategy: Optimum magnetic field-shape



B-shape with higher field homogeneity

3. Magnetic coil optimization for standard NSE

# Neutron Spin Echo Spectrometer



M. Monkenbusch

polarized beam

$$\phi_1 = \frac{2\omega_z^{(1)} L_1}{v_1}$$

$$\phi_2 = -\frac{2\omega_z^{(2)} L_2}{v_2}$$

$$\phi = \phi_1 + \phi_2 = 2\omega_z L \left[ \frac{1}{v_1} - \frac{1}{v_2} \right]$$

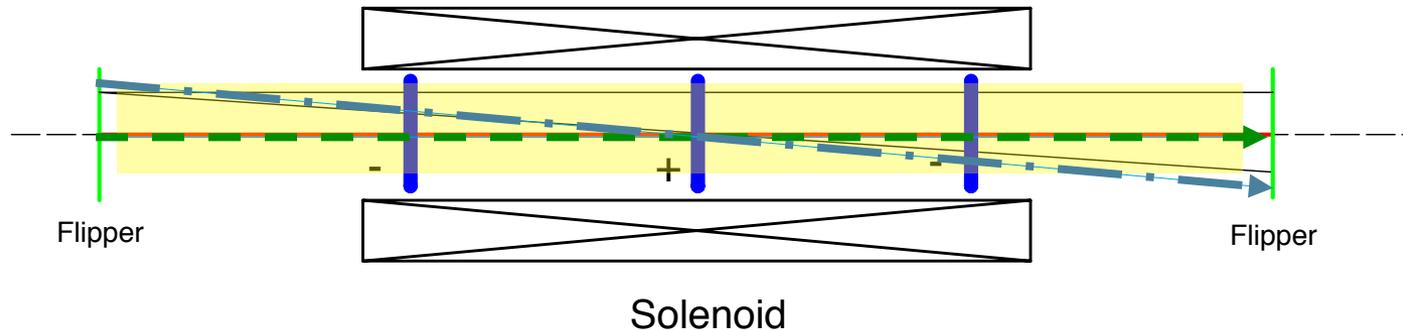
Quasi-elastic scattering  $\rightarrow \phi = \delta v \left[ \frac{d\phi}{dv_2} \right]_{v_2=v_1} = \frac{2\omega_z L}{v_1^2} \delta v \quad (\omega_z = \gamma B)$

If elastic scattering and the two arms are exactly symmetric  $\rightarrow \phi = 0$

# Problem: Asymmetries in NSE

$\Phi \neq 0$   
if the symmetry in the two arms  
is broken!

(e.g. no corrections on)



- ❖ Different trajectories in the two arms → different phase accumulation
  - Different time spent in each arm
  - Different magnetic field experienced, (*magnetic field inhomogeneities*)

$$P_{echo} \sim R * S(q,t)/S(q)$$

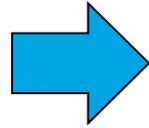
**Resolution  $\rightarrow 1$  for  $\phi_{ideal} \rightarrow 0$**

**The last resolution is achieved by reduction of magnetic field inhomogeneities through correction coils, but...**

**STRATEGY**  **Reduce the inhomogeneities by designing the magnetic field as homogeneous as possible**

# The magnetic field integral functional

$$J = \int^L |\vec{B}| dl.$$

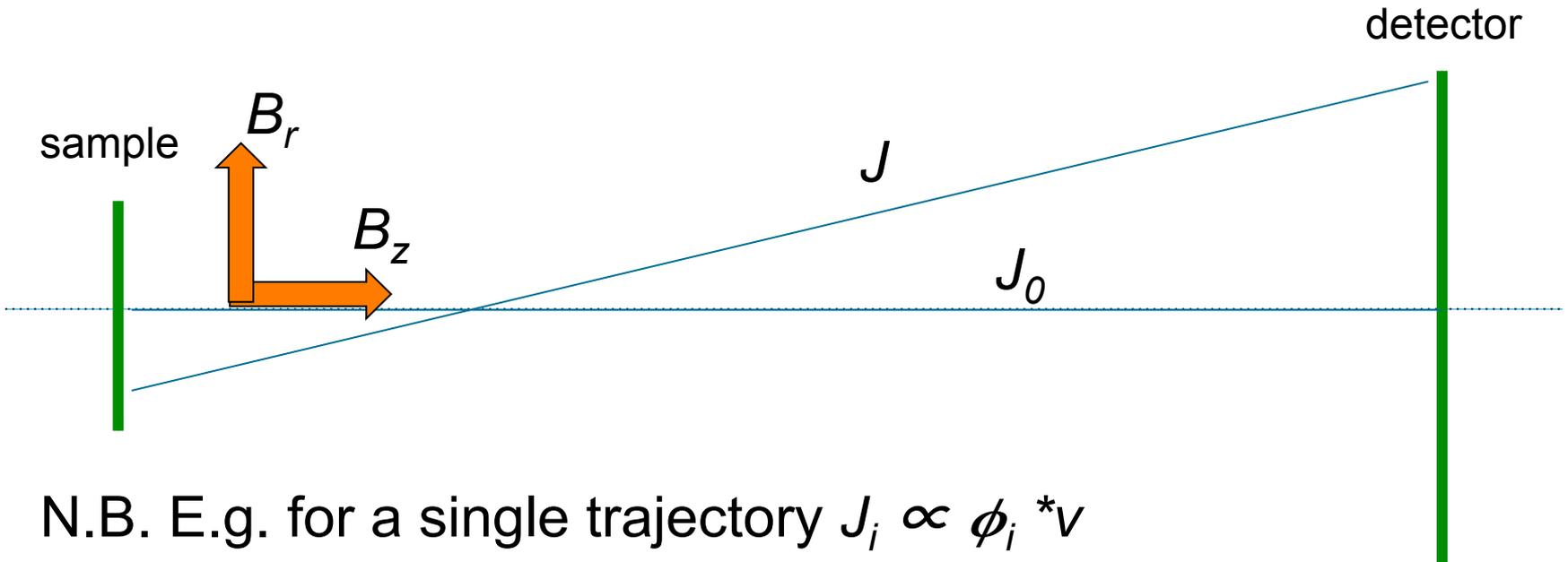


Field integral inhomogeneity

$$\Delta J = J - J_0.$$

↑  
Divergent trajectory

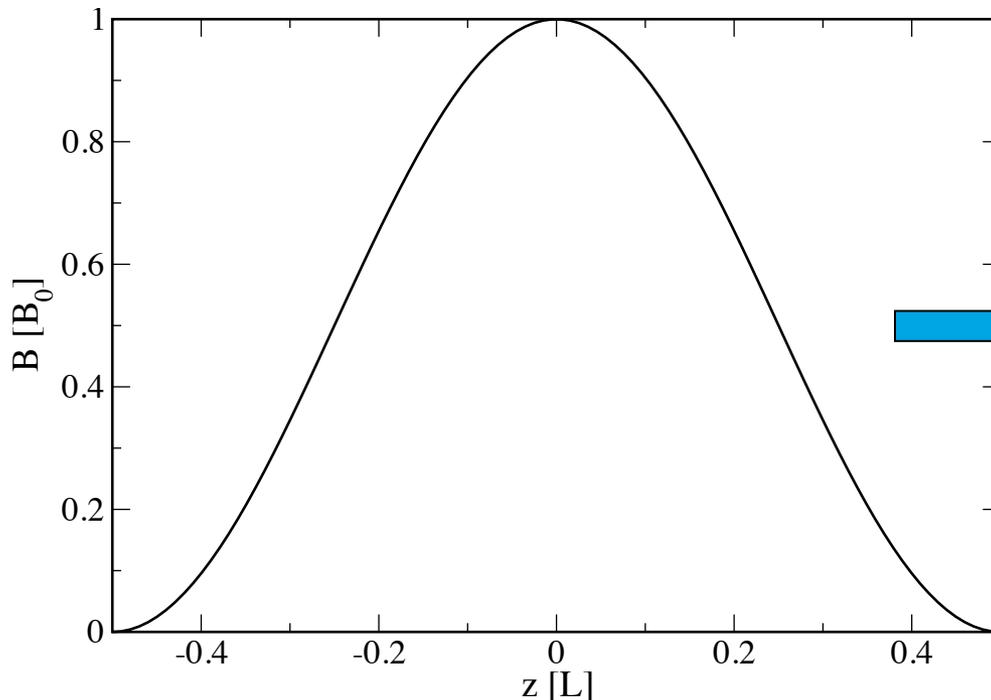
↑  
Trajectory along axis



N.B. E.g. for a single trajectory  $J_i \propto \phi_i * v$

# Zeyen & Rem and the idea of FIELD SHAPING

$B_z(z) = B_0 \cos^2\left(\frac{\pi z}{L}\right)$  axial magnetic field along a distance L  **iNSE @ Tokai**



This shape minimizes the difference of field Integral

$$\Delta J = J - J_0$$

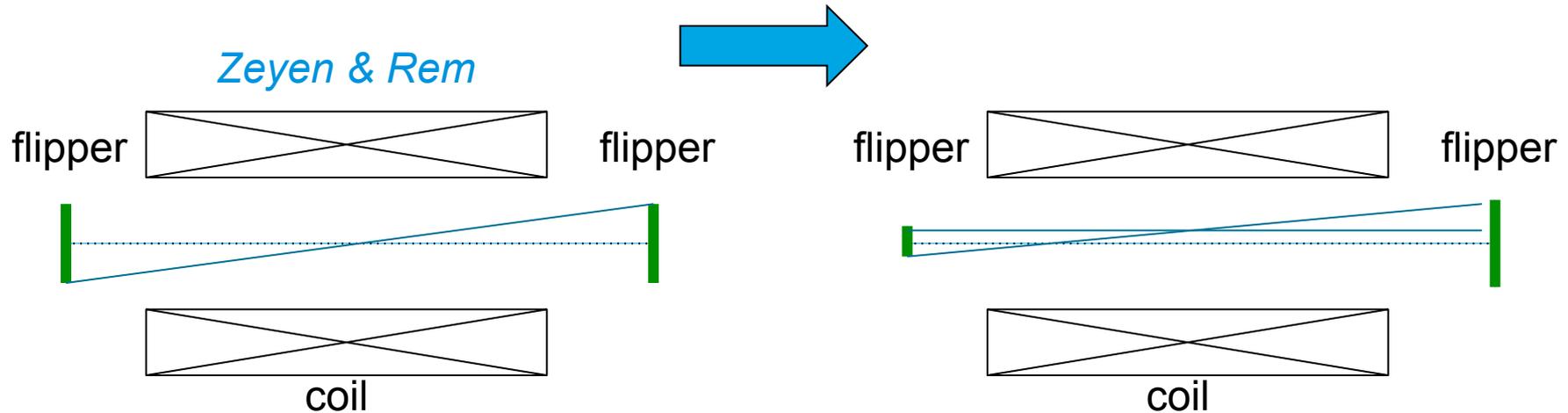
for some configurations

## The design strategy of the new coils for J-NSE

- 1 – A new semi-analytical derivation of the optimal magnetic field shape that minimizes the field integral inhomogeneity and a derivation of the lower bound for the inhomogeneity.
- 2 – A numerical optimization of the coil geometry of standard NSE.

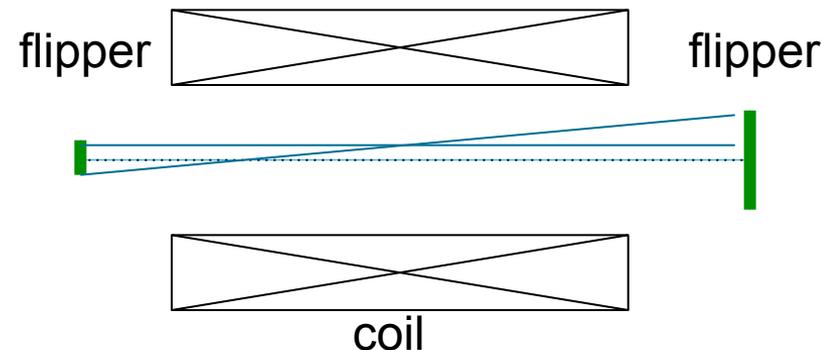
# Our approach

1 – We consider also asymmetric beam configurations



# What is new in our approach?

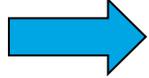
1 – We consider also asymmetric beam configurations



2 – We average over all possible neutron trajectories before minimizing

3 – We minimize all the terms in the leading order of the magnetic field integral inhomogeneity

# The magnetic field integral functional

For  $B_r(z, r) \ll B_z(z, r)$   Taylor expansion of B components

$$B_z(z, r) = B_z(z, 0) - \frac{1}{4}r^2 \partial_z^2 B_z(z, 0) + O(r^4)$$

$$B_r(z, r) = -\frac{1}{2}r \partial_z B_z(z, 0) + O(r^3). \quad (\text{Btygin and Toptygin})$$

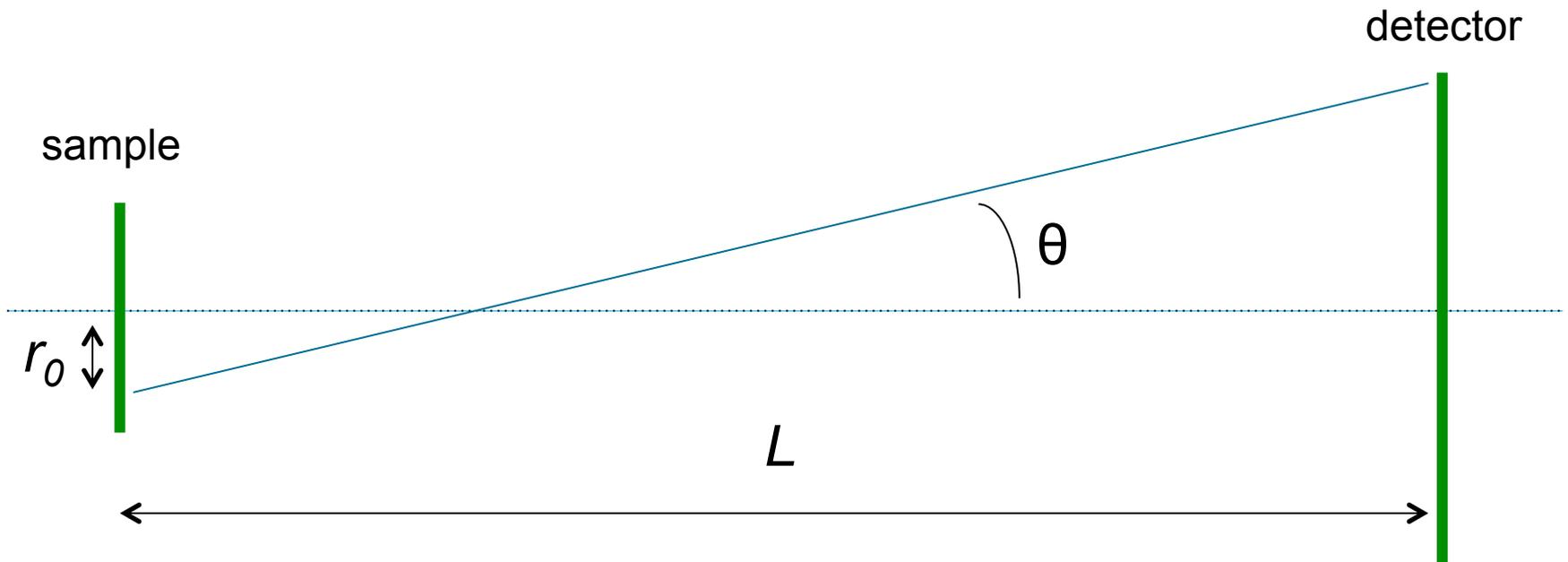
To be minimized

$$\langle \Delta J[B(z)] \rangle \simeq \langle \tan^2 \theta \rangle G + \langle r_0^2 \rangle H + \langle 2r_0 \tan \theta \rangle U$$

# The magnetic field integral functional

$$\langle \Delta J[B(z)] \rangle \simeq \langle \tan^2 \theta \rangle G + \langle r_0^2 \rangle H + \langle 2r_0 \tan \theta \rangle U$$

Divergence and axial-offset of each neutron path



# The magnetic field integral functional

$$\langle \Delta J[B(z)] \rangle \simeq \langle \tan^2 \theta \rangle G + \langle r_0^2 \rangle H + \langle 2r_0 \tan \theta \rangle U$$

- Averages over all paths
- They can be calculated exactly for given dimensions of the sample and of the detector they are numbers

# The magnetic field integral functional

$$\langle \Delta J[B(z)] \rangle \simeq \langle \tan^2 \theta \rangle G + \langle r_0^2 \rangle H + \langle 2r_0 \tan \theta \rangle U$$

Integral functions of  $B_z(z)$

$$U := \int_{-L/2}^{L/2} dz \left[ \frac{1}{8} \frac{(\partial_z B_z(z))^2}{B_z(z)} - \frac{1}{4} \partial_z^2 B_z(z) \right] z$$

$$H := \int_{-L/2}^{L/2} dz \left[ \frac{1}{8} \frac{(\partial_z B_z(z))^2}{B_z(z)} - \frac{1}{4} \partial_z^2 B_z(z) \right],$$

$$G := \frac{1}{2} \int_{-L/2}^{L/2} dz B_z(z) + \int_{-L/2}^{L/2} dz z^2 \left[ \frac{1}{8} \frac{(\partial_z B_z(z))^2}{B_z(z)} - \frac{1}{4} \partial_z^2 B_z(z) \right]$$

## A parametrization for B:

$$B_z(z) = B_0 y(z)^2 \quad \text{with}$$

$$y(z) = \sum_{i=1}^N a_i \sin \left[ i \frac{\pi z}{L} \right]$$

it satisfies boundary cond.  $\rightarrow B_z = \partial_z B_z = 0$  @  $z = 0, L$

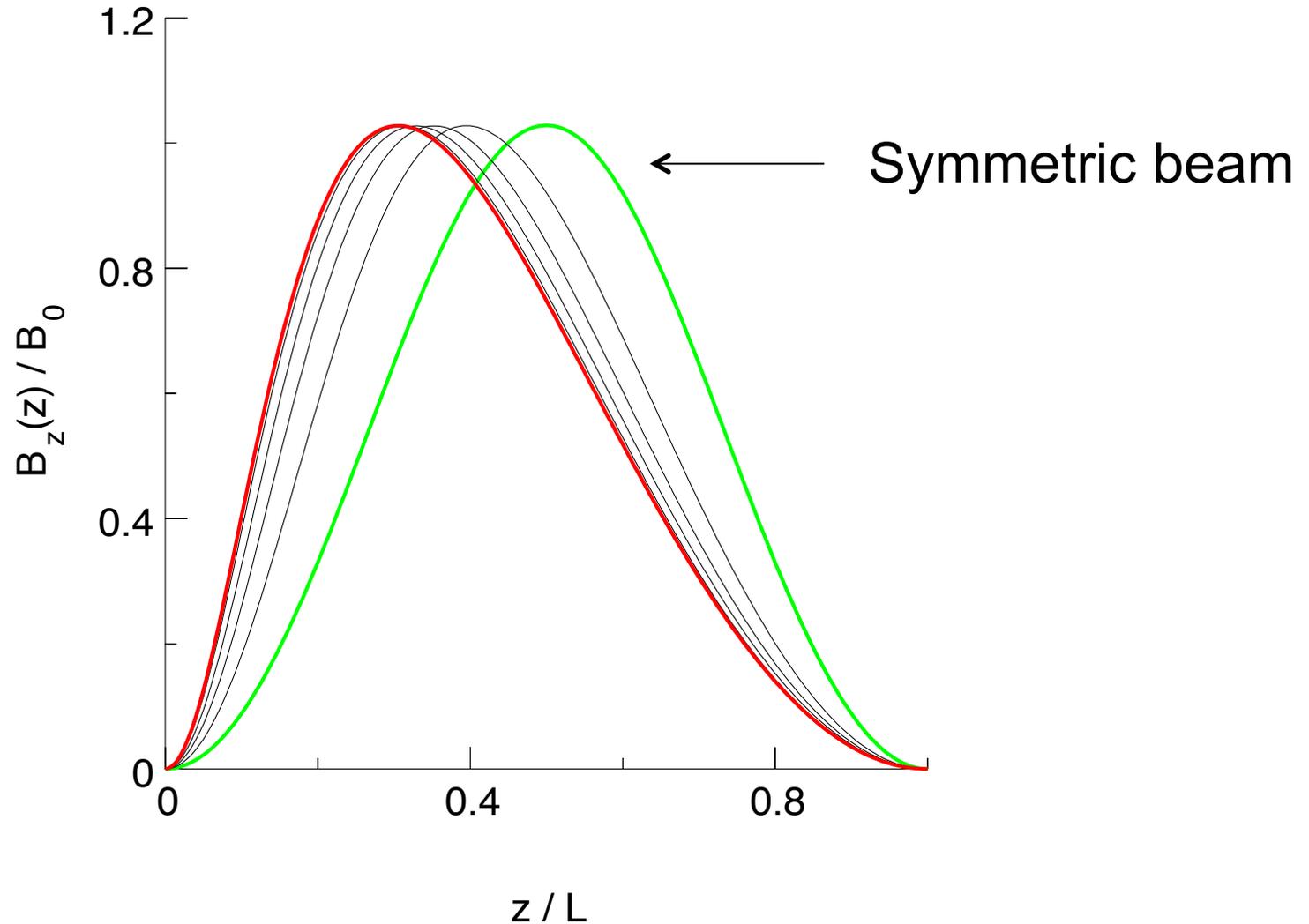
The problem of minimizing  $\Delta J$  at constant  $J_0$ ...

$$\langle \Delta J \rangle - \lambda J_0 = L \sum_{i,j}^N \left[ C_{i,j} - \lambda \frac{1}{2} B_0 \delta_{i,j} \right] a_i a_j$$

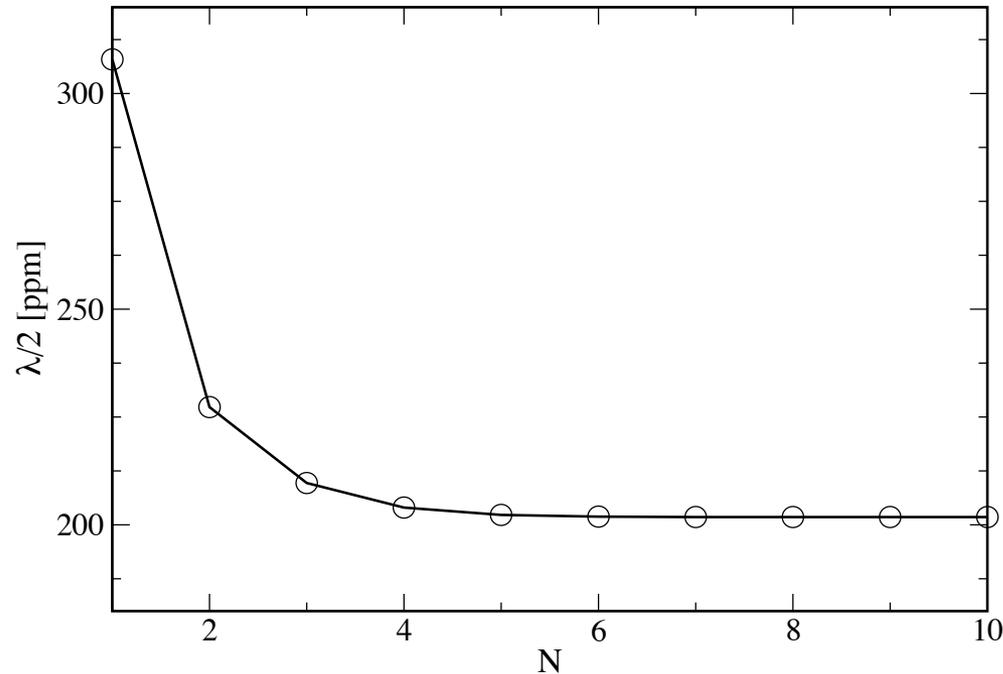
...translates into the diagonalization of a matrix!

The coefficients  $C_{ij} = B_0 C_{ij}(\theta, r_0)$  are known.

# The optimum shape tends to be asymmetric!



# A lower bound for the inhomogeneity



$$\langle \Delta J \rangle_{RMS} = \sqrt{\langle \Delta J^2 \rangle - \langle \Delta J \rangle^2}$$

$$\langle \Delta J \rangle_{RMS} / J_0 \simeq 215 \text{ ppm}$$

For  $L=4\text{m}$ ,  $r_0=2\text{cm}$  and  $r_{\pi/2}=10\text{cm}$

# Modified operational program for the optimization of the coil geometry

from semi-analytical  
solution →

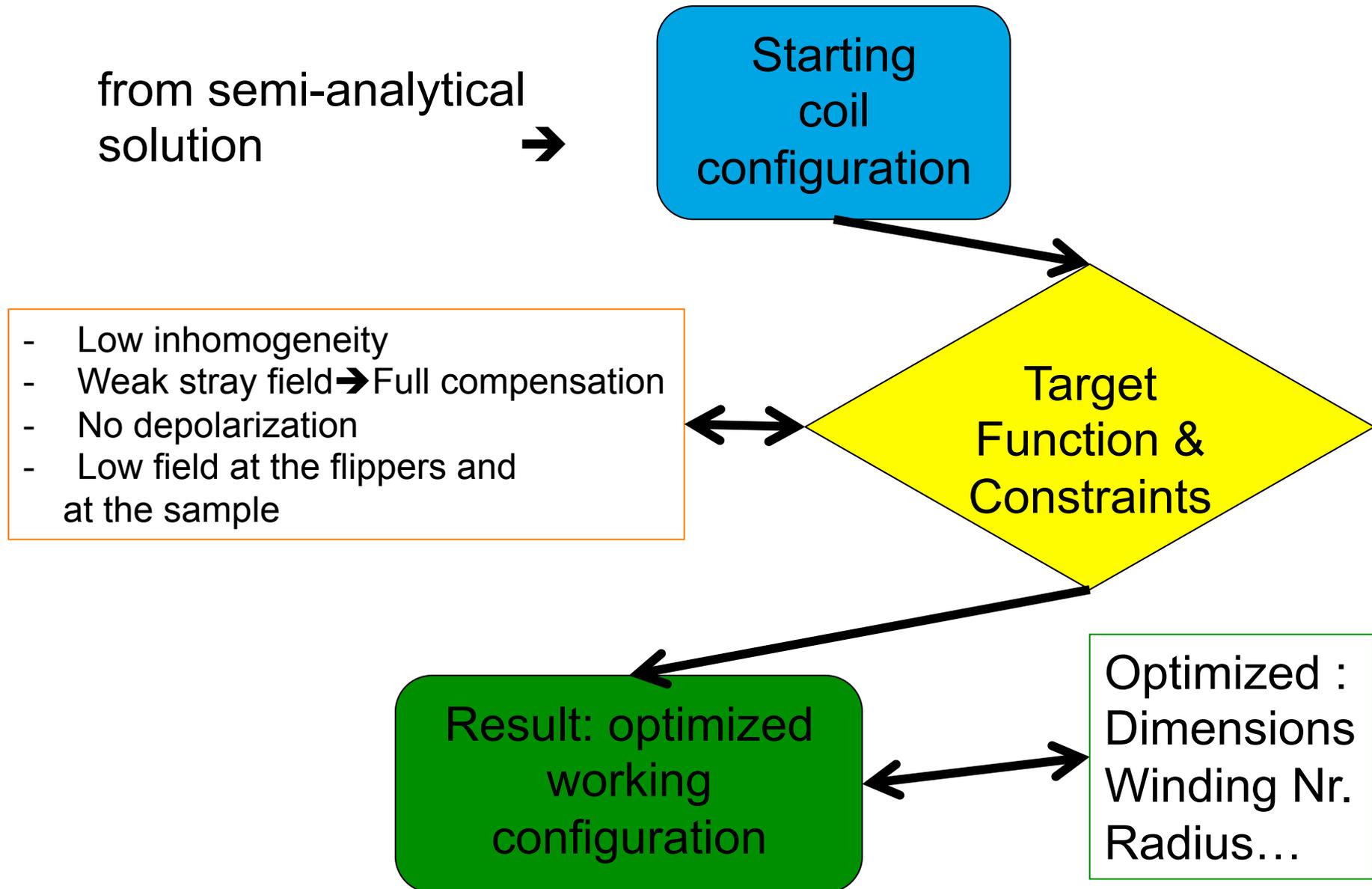
Starting  
coil  
configuration

- Low inhomogeneity
- Weak stray field → Full compensation
- No depolarization
- Low field at the flippers and at the sample

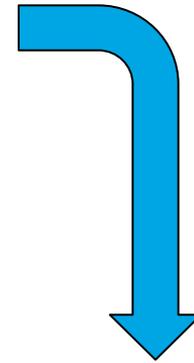
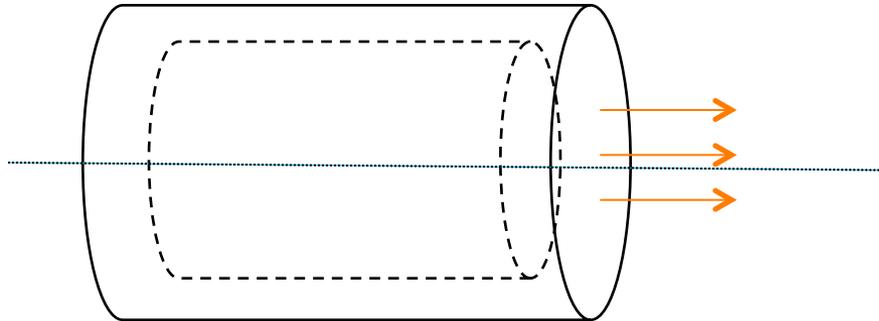
Target  
Function &  
Constraints

Result: optimized  
working  
configuration

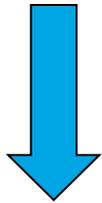
Optimized :  
Dimensions  
Winding Nr.  
Radius...



# Full compensated coils



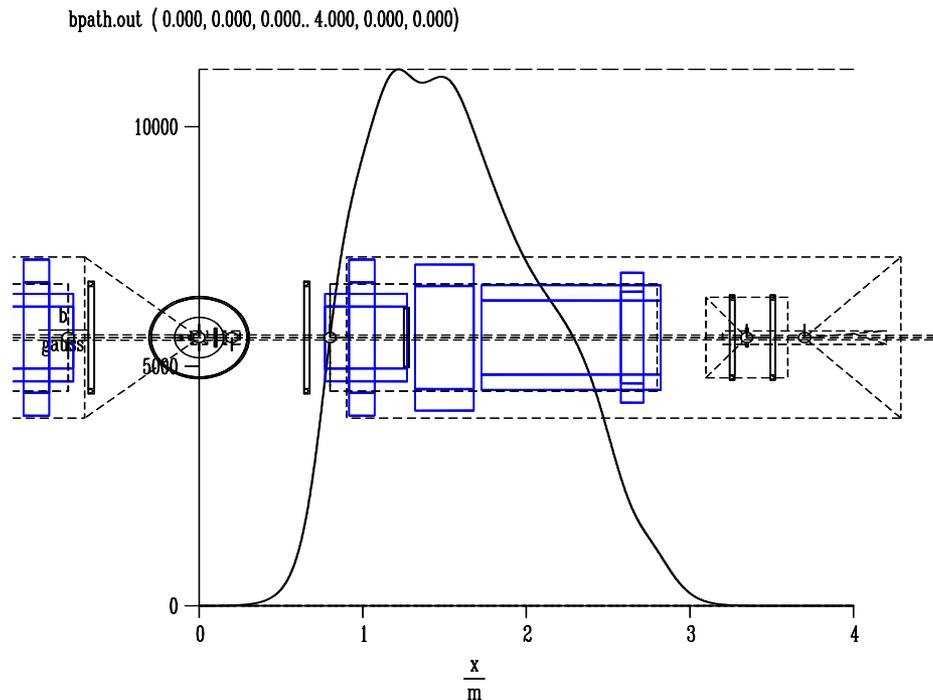
weaker stray fields



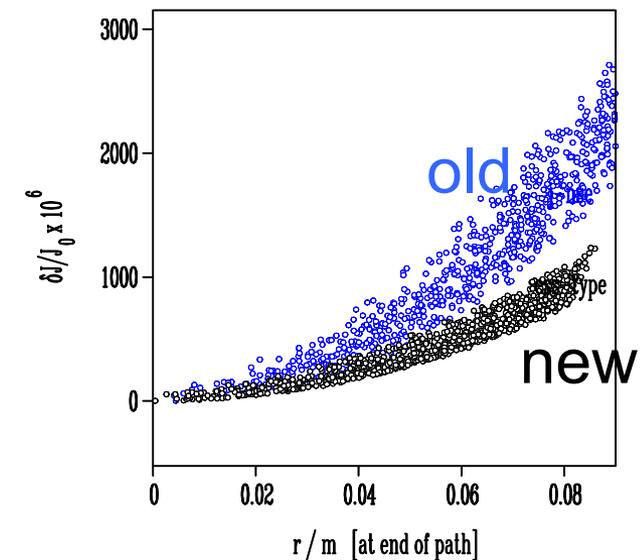
possible with superconducting coils



# New optimized field shape for J-NSE by superconducting compensated coil set (5 x 2)



intrinsic field  
integral  
deviations



# The need for Correction coils

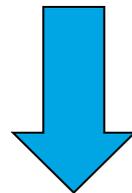
Requirements for a functioning NSE → ~1ppm homogeneity

$$P_{echo} \sim R * S(q,t)/S(q)$$

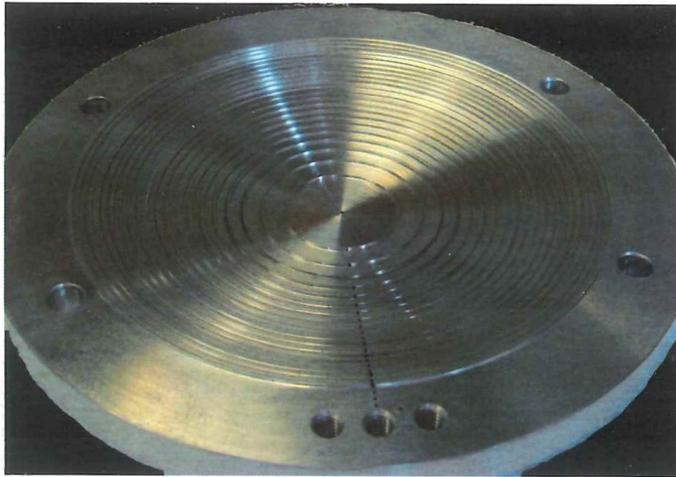
$$Resolution = e^{\frac{-(\langle \Delta J^2 \rangle) \lambda^2 m_n (2\pi\gamma)^2}{2 h^2}} > e^{-1}$$

$$\rightarrow \lambda \sqrt{\langle \Delta J^2 \rangle} \leq 2 \times 10^{-15} \text{Tm}^2$$

$$\rightarrow \sqrt{\langle \Delta J^2 \rangle} \leq 2 \times 10^{-6} \text{Tm for } \lambda \leq 1 \text{nm}$$



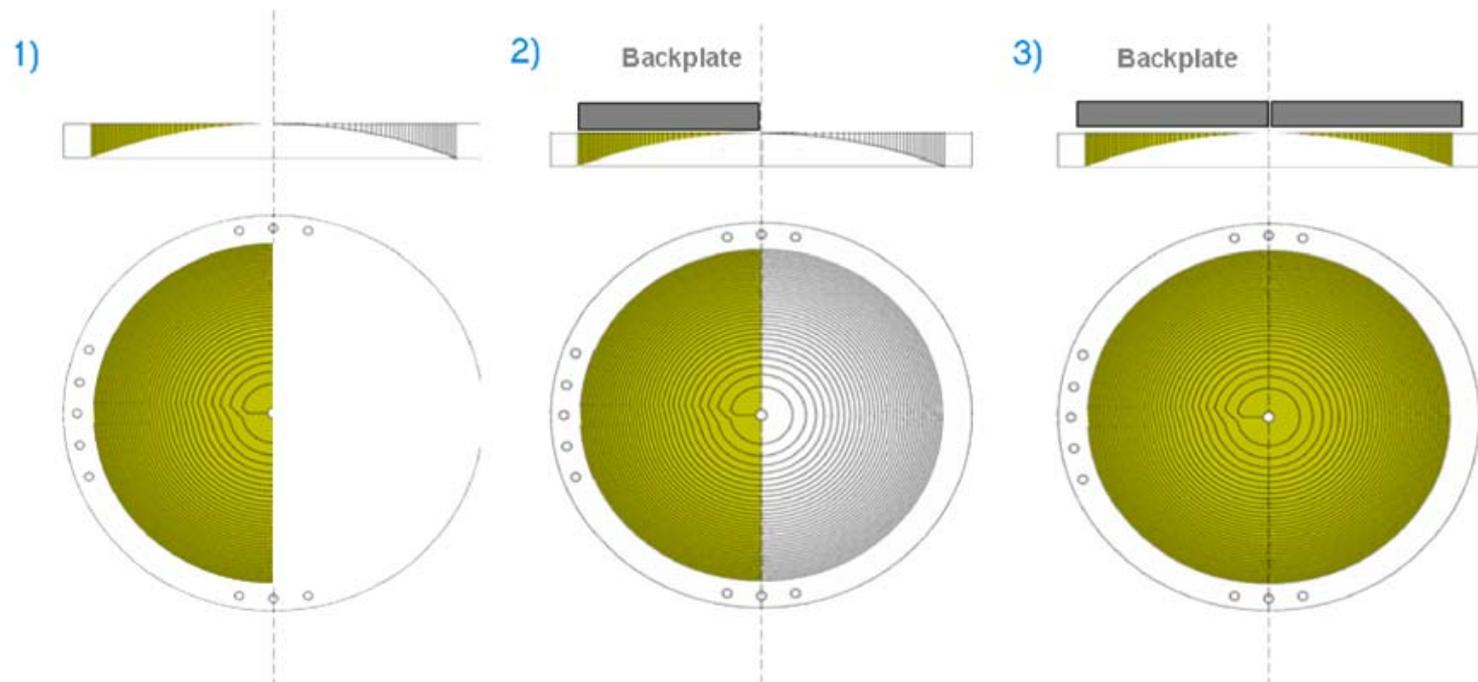
Correction coils still needed



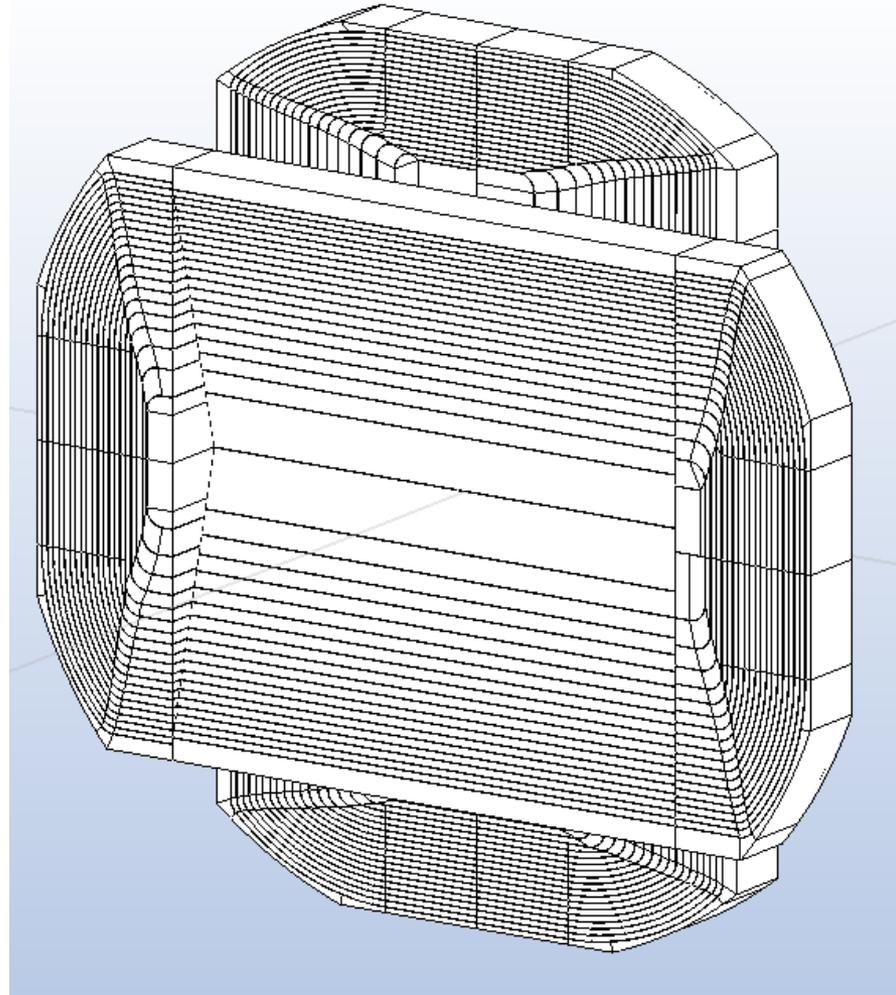
# Fresnel correction coils

Fig. 24. Cutting scheme with oblique cuts, first mechanical test for a small (100mm diameter) correction coil.

*M. Ohl et al. / Physica B 356 (2005) 234–238*



# Pythagoras correction coils



Thank You