Spherical Neutron Polarimetry

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Content

- Beam polarisation vector
- Cross-section & scattered polarisation vector
- What happens to the polarisation upon scattering ?
- Instrumentation — Cryopad and others
- Some simple examples
- A word about polarimetric neutron spin-echo

Spherical Neutron Polarimetry

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Beam polarisation vector

neutron spin & neutron magnetic moment

- The neutron carries a spin $\vec{s}$ which is an internal angular momentum with a quantum number $s = 1/2$. The general spin wave of an itinerant neutron is:

$$|\chi\rangle = a|+\rangle + b|-\rangle \text{ where } |a|^2 + |b|^2 = 1$$

- The 3 components of this angular momentum are given by the Pauli matrices representing $\vec{\sigma} = 2\vec{s}/\hbar$:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma_y = \begin{pmatrix} 0 & -\imath \\ \imath & 0 \end{pmatrix},\ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Beam polarisation vector

neutron spin & neutron magnetic moment

- The neutron carries a magnetic moment:

$$\mu_n = \gamma_n \mu_B \vec{\sigma} \text{ where } \gamma_n = -1.913$$

- The gyromagnetic ratio of the neutron is the ratio between the magnetic moment and the spin moment:

$$\vec{\mu_n} = \gamma_L \vec{s}$$

$$\text{where } \gamma_L = \frac{2\gamma_n \mu_B}{\hbar} = -1.832 \cdot 10^8 \text{ rad.s}^{-1}.\text{T}^{-1}$$

NB: the magnetic moment and spin are opposed.

Beam polarisation vector

polarisation of a neutron beam

- The neutron beam is a statistical ensemble of several quantum states and the beam polarisation $\vec{P} = \langle \vec{\sigma} \rangle$.
- We use the density matrix formalism to describe this statistical quantum system (analog to phase-space probability measure):

$$\hat{\rho} = \frac{1}{2}\left(\mathbb{1} + \vec{\sigma} \cdot \vec{P}\right) = \frac{1}{2}\begin{bmatrix} 1 + P_z & P_x - \imath P_y \\ P_x + \imath P_y & 1 - P_z \end{bmatrix}$$

Only 3 real numbers are required to describe this 2x2 matrix, i.e. the statistical quantum situation.

Beam polarisation vector

polarisation of a neutron beam

- The beam polarisation can therefore be seen as a vector in space:

$$\vec{P} = \langle \vec{\sigma} \rangle = trace\,(\hat{\rho}\ \vec{\sigma})$$

- We can measure each of the 3 orthogonal components in any arbitrary direction $\vec{u}$ in space:

$$P_u = trace\,[\hat{\rho}\ \ (u_x \sigma_x + u_y \sigma_y + u_z \sigma_z)]$$

Beam polarisation vector

polarisation of a neutron beam

- For historical reasons, some people prefer to measure the *flipping ratio*:

$$R = \frac{r_{p,+} - r_{b,+}}{r_{p,-} - r_{b,-}} \quad \left(\text{and } P = \frac{R-1}{R+1}\right)$$

$$\sigma_R^2 = \frac{(r_{p,+} - r_{b,+})^2 \left(\sigma^2_{r_{p,-}} + \sigma^2_{r_{b,-}}\right) + (r_{p,-} - r_{b,-})^2 \left(\sigma^2_{r_{p,+}} + \sigma^2_{r_{b,+}}\right)}{(r_{p,-} - r_{b,-})^4}$$

but it has no physical meaning and *P* is recommended.

Beam polarisation vector

polarisation of a neutron beam

- Experimentally, we always measure the component parallel to the field direction (quantisation axis):

$$P = \frac{(r_{p,+} - r_{b,+}) - (r_{p,-} - r_{b,-})}{(r_{p,+} - r_{b,+}) + (r_{p,-} - r_{b,-})}$$

$$\sigma_P^2 = 4\frac{(r_{p,+} - r_{b,+})^2 \left(\sigma^2_{r_{p,-}} + \sigma^2_{r_{b,-}}\right) + (r_{p,-} - r_{b,-})^2 \left(\sigma^2_{r_{p,+}} + \sigma^2_{r_{b,+}}\right)}{(r_{p,+} - r_{b,+} + r_{p,-} - r_{b,-})^4}$$

where *r* is a neutron count rate, *p/b* stand for peak/background, +/- for the spin states.

Beam polarisation vector

polarisation of a neutron beam

- Generally, the count rates are measured separately and normalised to the time: …or to the monitor:

$$r = N/T \qquad r = N/M$$

$$\sigma_r = \sqrt{N}/T \qquad \sigma_r = \sqrt{\frac{N\,(M+N)}{M^3}}$$

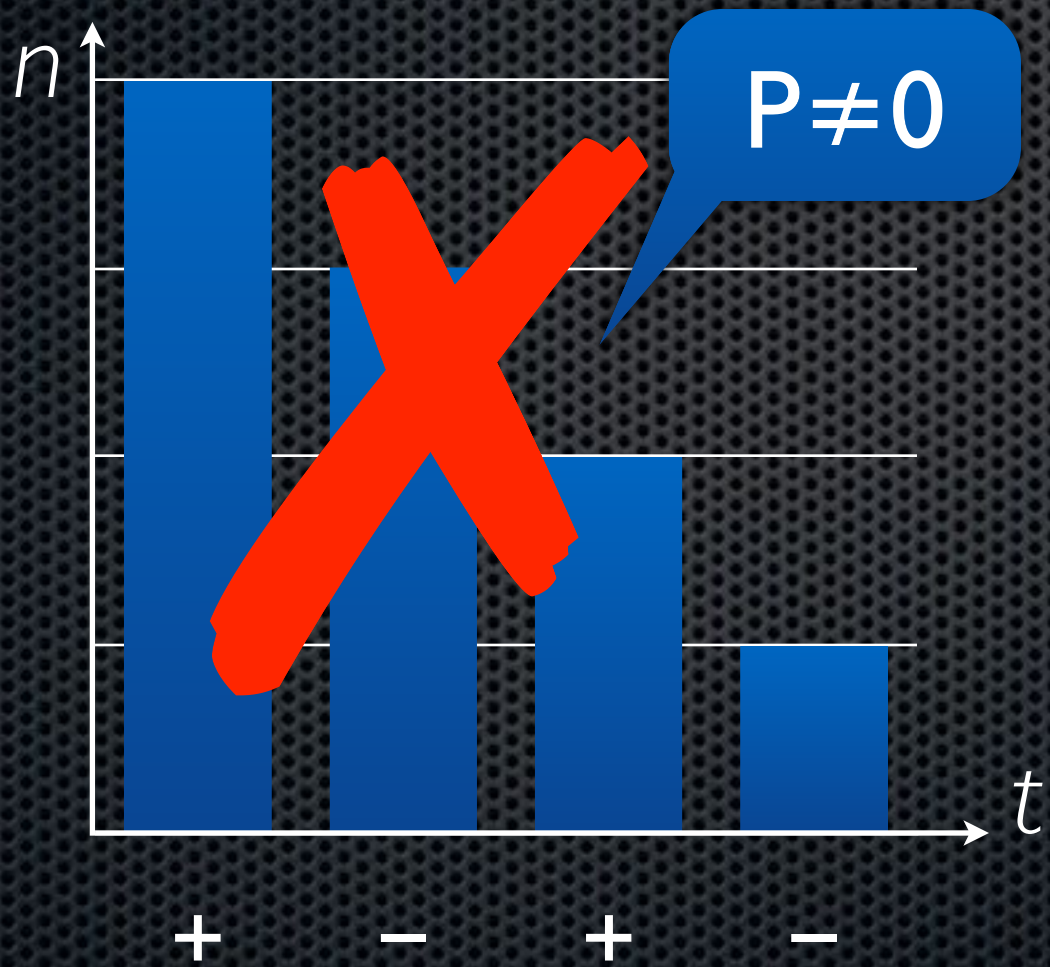
- But don't forget… to optimise the distribution of the times spent on the peak, the background and the [+] and [-] spin states.
- That way, you will reduce the error bar of P.

http://www.ill.eu/sane/software/xop-plugins-for-igor-pro/neutron-scattering-xop/

Beam polarisation vector

polarisation of a neutron beam

- NB: compensate for the variations of the incident flux! Choose the right sequence and a stable detector.



Beam polarisation vector

polarisation of a neutron beam

— *Physica B* **397** (2007) 138 —

- Using a *flipping control unit*, you can further minimise the error bar by taking advantage of the high-precision clock of your electronics:

$$P_{opt} = \frac{M_b\, t_p\, t_{b,+}\, t_{b,-}\, (N_{p,+} t_{p,-} - N_{p,-} t_{p,+}) - M_p\, t_b\, t_{p,+}\, t_{p,-}\, (N_{b,+}\, t_{b,-} - N_{b,-}\, t_{b,+})}{M_b\, t_p\, t_{b,+}\, t_{b,-}\, (N_{p,+} t_{p,-} + N_{p,-} t_{p,+}) - M_p\, t_b\, t_{p,+}\, t_{p,-}\, (N_{b,+}\, t_{b,-} + N_{b,-}\, t_{b,+})}$$

$$\begin{aligned} \sigma^2_{P_{opt}} = & \frac{M_b\, t_p^2\, t_{b,+}^2\, t_{b,-}^2}{P_d^2} \left[\begin{array}{c} ((1+P_{opt})\; t_{p,+}\, N_{p,-} - (1-P_{opt})\; t_{p,-}\, N_{p,+})^2 + \\ M_b \left((1+P_{opt})^2\; t_{p,+}^2\, N_{p,-}^2 + (1-P_{opt})^2\; t_{p,-}^2\, N_{p,+}^2 \right) \end{array} \right] \\ + & \frac{M_p\, t_b^2\, t_{p,+}^2\, t_{p,-}^2}{P_d^2} \left[\begin{array}{c} ((1+P_{opt})\; t_{b,+}\, N_{b,-} - (1-P_{opt})\; t_{b,-}\, N_{b,+})^2 + \\ M_p \left((1+P_{opt})^2\; t_{b,+}^2\, N_{b,-}^2 + (1-P_{opt})^2\; t_{b,-}^2\, N_{b,+}^2 \right) \end{array} \right] \end{aligned}$$

with P_d = denominator of P_{opt}

Beam polarisation vector

polarisation of a neutron beam

— none of these polarisers is perfect—

- Heusler Cu_2MnAl crystals: monochromatised beam, large λ/2 contamination, ≈15 cm height max. (mag. saturation), 95% polarisation with some variations in the beam section.
- Polarising supermirrors: efficient above ≈2Å, 85-95% polarisation but angular dependent unless in crossed geometry (reduced transmission).
- ^3He spin filters: polarisation decoupled from optical functions, compromise polarisation/transmission.

Beam polarisation vector

manipulate it with care

- In a magnetic field, the polarisation rotates around the field in a Larmor precession with the frequency:

$$\omega_L(\mathrm{rad/s}) = 18\,325\ B(G)$$

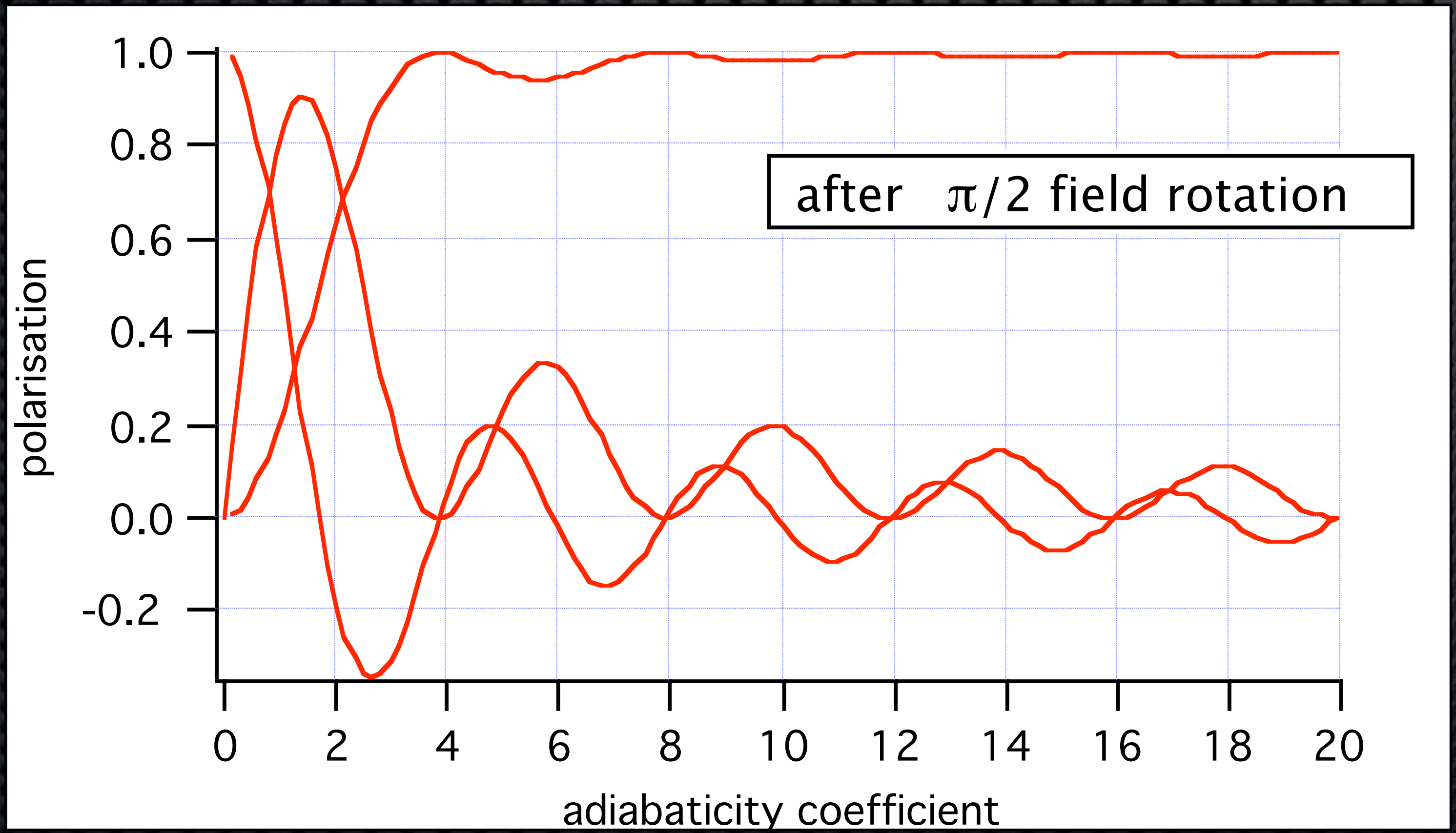
- With *B* aligned along the *z*-axis, we find:

$$\begin{cases} P_x(t) = \cos(\omega_L.t)P_x(0) - \sin(\omega_L.t)P_y(0) \\ P_y(t) = \sin(\omega_L.t)P_x(0) + \cos(\omega_L.t)P_y(0) \\ P_z(t) = P_z(0) \end{cases}$$

Beam polarisation vector

manipulate it with care

- If the guiding field rotates <u>slowly</u> compared to the Larmor precession frequency, the polarisation is transported adiabatically.



$$E = \frac{\omega_L}{\omega_B} \gg 30$$

Zeeman energy conserved

Beam polarisation vector

manipulate it with care

- If the guiding field rotates slowly compared to the Larmor precession frequency, the polarisation is transported adiabatically.

Typically, for a 90° rotation over 10 cm

λ [Å]	0.4	1	4	10
B [G]	255	102	25	10

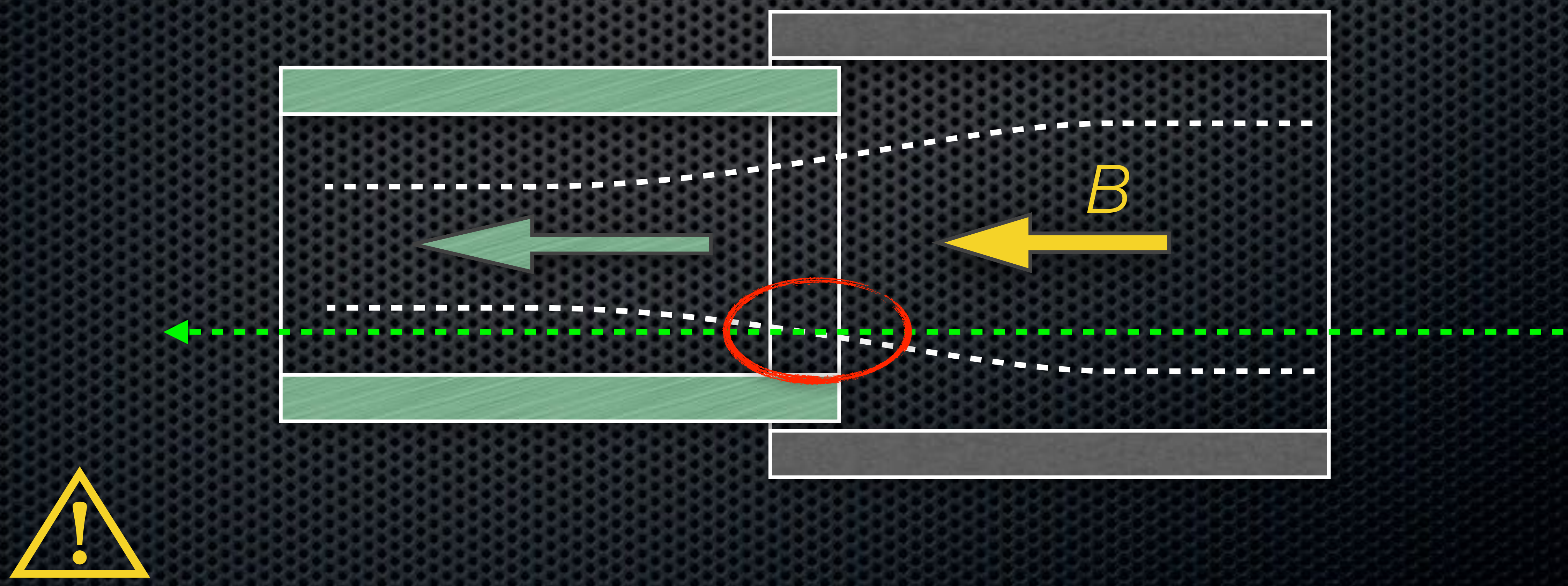
$$E = \frac{\omega_L}{\omega_B} \gg 30$$

Zeeman energy conserved

Beam polarisation vector

manipulate it with care

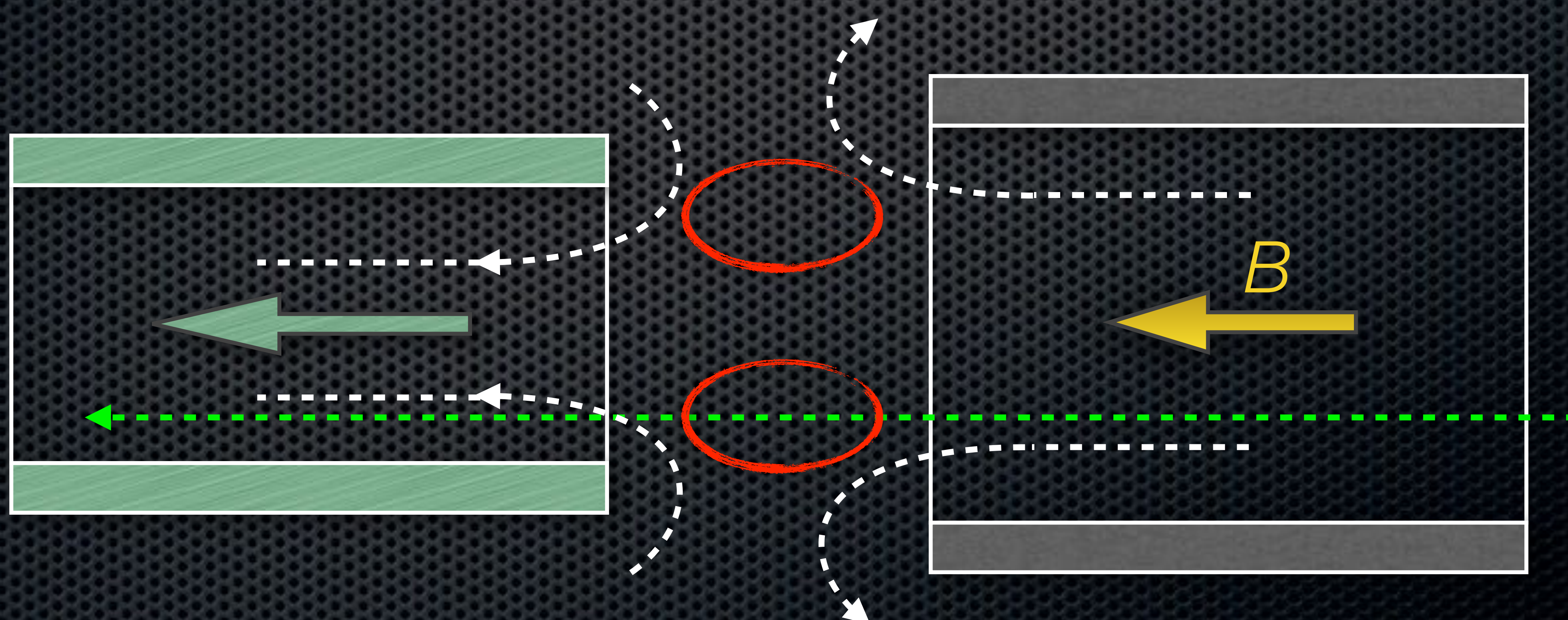
- When setting up guiding fields, always be careful with the reduction of the field amplitude at the location where neutrons see a field rotation.



Beam polarisation vector

manipulate it with care

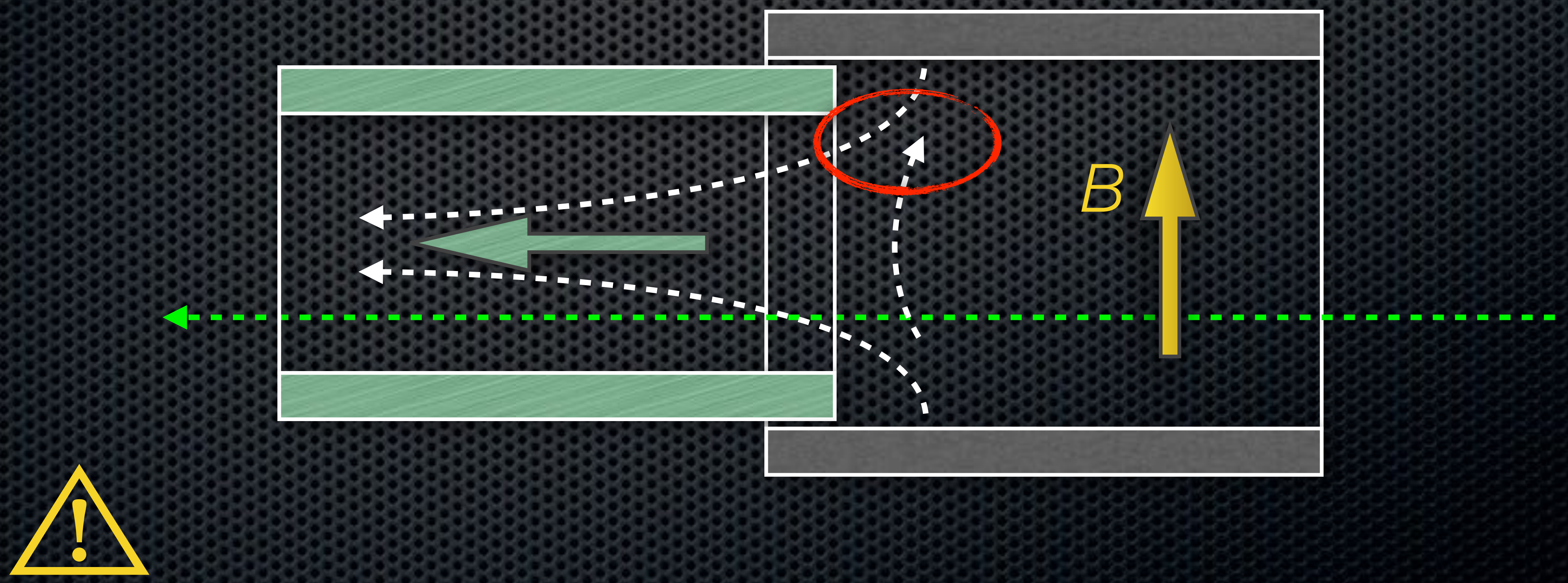
- The gaps between guiding field coils can lead to depolarisation, even when the fields are parallel. Also true for permanent magnets.



Beam polarisation vector

manipulate it with care

- In spin rotators, the loss of polarisation generally comes from the region where the fields cancel, which is also where the field (polarisation) rotates.



Beam polarisation vector

manipulate it with care

- The Magnaprobe is a very useful tool. It illustrates very well the true shape of the magnetic field…

  but NOT its magnitude !

λ [Å]	0.4	1	4	10
B [G]	255	102	25	10

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Cross-section & scattered polarisation vector

theory: Maleyev, Blume,… $(\vec{Q} = \vec{k}_i - \vec{k}_f)$

Contribution	Elastic scattering	Inelastic scattering
(n) Nuclear	$\sigma_n = N N^*$ $\{\vec{P}_f \sigma\}_n = \vec{P}_i \, \sigma_n$	$\sigma_n = \frac{k_f}{k_i} \mathcal{H}\left(N_{-\vec{Q}}, N_{\vec{Q}}\right)$ $\{\vec{P}_f \sigma\}_n = \vec{P}_i \, \sigma_n$
(m) Magnetic (I)	$\sigma_m = \vec{M}_\perp \cdot \vec{M}_\perp^*$ $\{\vec{P}_f \sigma\}_m = - \vec{P}_i \, \sigma_m \; + \ldots$ $.. \; 2\Re\left(\vec{M}_\perp \left(\vec{P}_i \cdot \vec{M}_\perp^*\right)\right)$	$\sigma_m = \frac{k_f}{k_i} S_{\alpha\beta} \, \delta_{\alpha\beta}$ $\{\vec{P_{f,\alpha}} \sigma\}_m = \frac{k_f}{k_i} P_{i\beta} \cdot \ldots$ $.. \; [(S_{\alpha\beta} + S_{\beta\alpha}) - \delta_{\alpha\beta}\delta_{\beta\alpha}]$ $S_{\alpha\beta} = \mathcal{H}\left(M^\alpha_{\perp,-\vec{Q}}, M^\beta_{\perp,\vec{Q}}\right)$
(c) Magnetic (II)	$\sigma_c = \imath \vec{P}_i \cdot \left(\vec{M}_\perp^* \wedge \vec{M}_\perp\right)$ $\{\vec{P}_f \sigma\}_c = - \, \imath \left(\vec{M}_\perp^* \wedge \vec{M}_\perp\right)$	$\sigma_c = \frac{k_f}{k_i} \, \imath \, S_{\alpha\beta} \, \epsilon_{\alpha\beta\gamma} \, P_{i\gamma}$ $\{\vec{P}_{f,\alpha} \sigma\}_c = - \frac{k_f}{k_i} \imath \epsilon_{\alpha\beta\gamma} S_{\beta\gamma}$ $S_{\alpha\beta} = \mathcal{H}\left(M^\alpha_{\perp,-\vec{Q}}, M^\beta_{\perp,\vec{Q}}\right)$
(i) Nuclear–magnetic	$\sigma_i = 2\vec{P}_i \cdot \Re\left(N^* \vec{M}_\perp\right)$ $\{\vec{P}_f \sigma\}_i = 2\Re\left(N^* \vec{M}_\perp\right) \; +$ $2\vec{P}_i \wedge \Im\left(N^* \vec{M}_\perp\right)$	$\sigma_i = \frac{k_f}{k_i} \, \imath \, \vec{S}_+ \cdot \vec{P}_i$ $\{\vec{P}_f \sigma\}_i = \frac{k_f}{k_i} \left(\vec{S}_+ + \imath \vec{S}_- \wedge \vec{P}_i\right)$ $\vec{S}_\pm = \mathcal{H}_\pm\left(N_{-\vec{Q}}, \vec{M}_{\perp,\vec{Q}}\right)$

Cross-section & scattered polarisation vector

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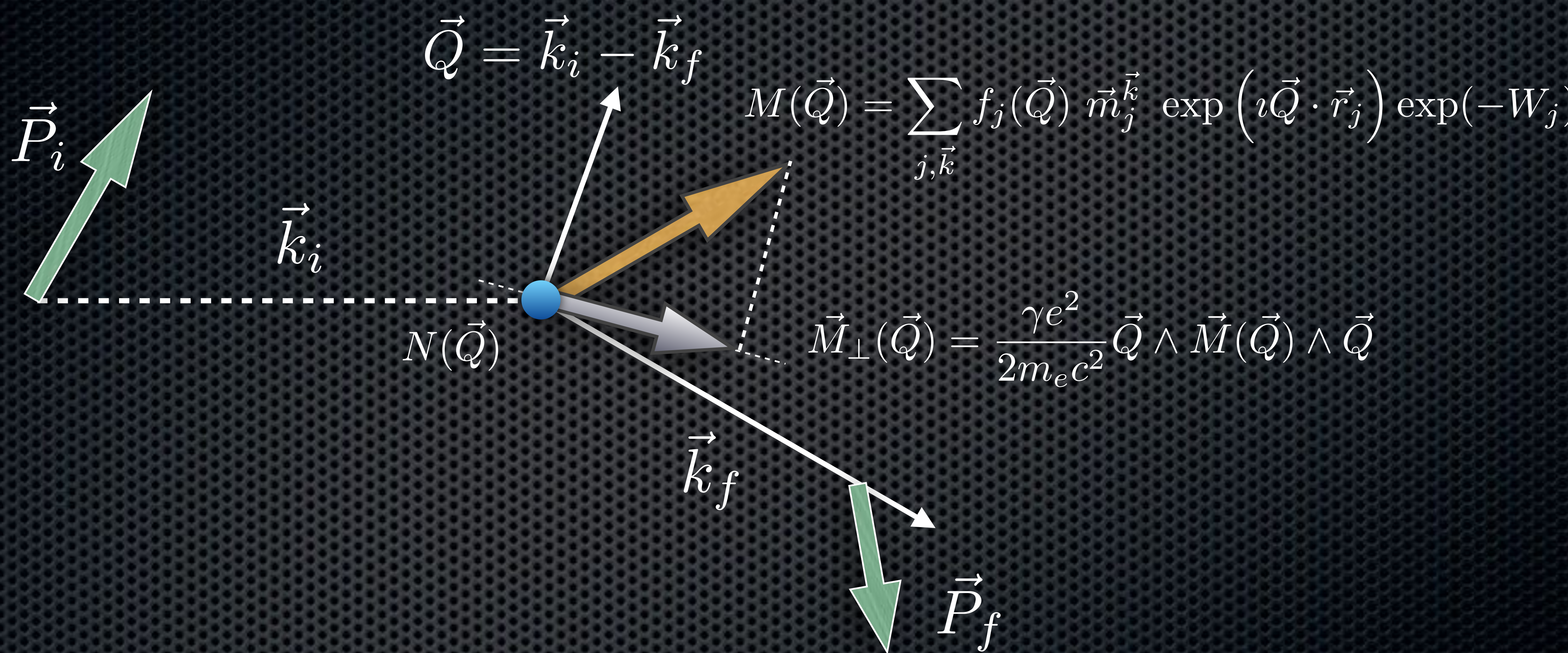
Cross-section & scattered polarisation vector

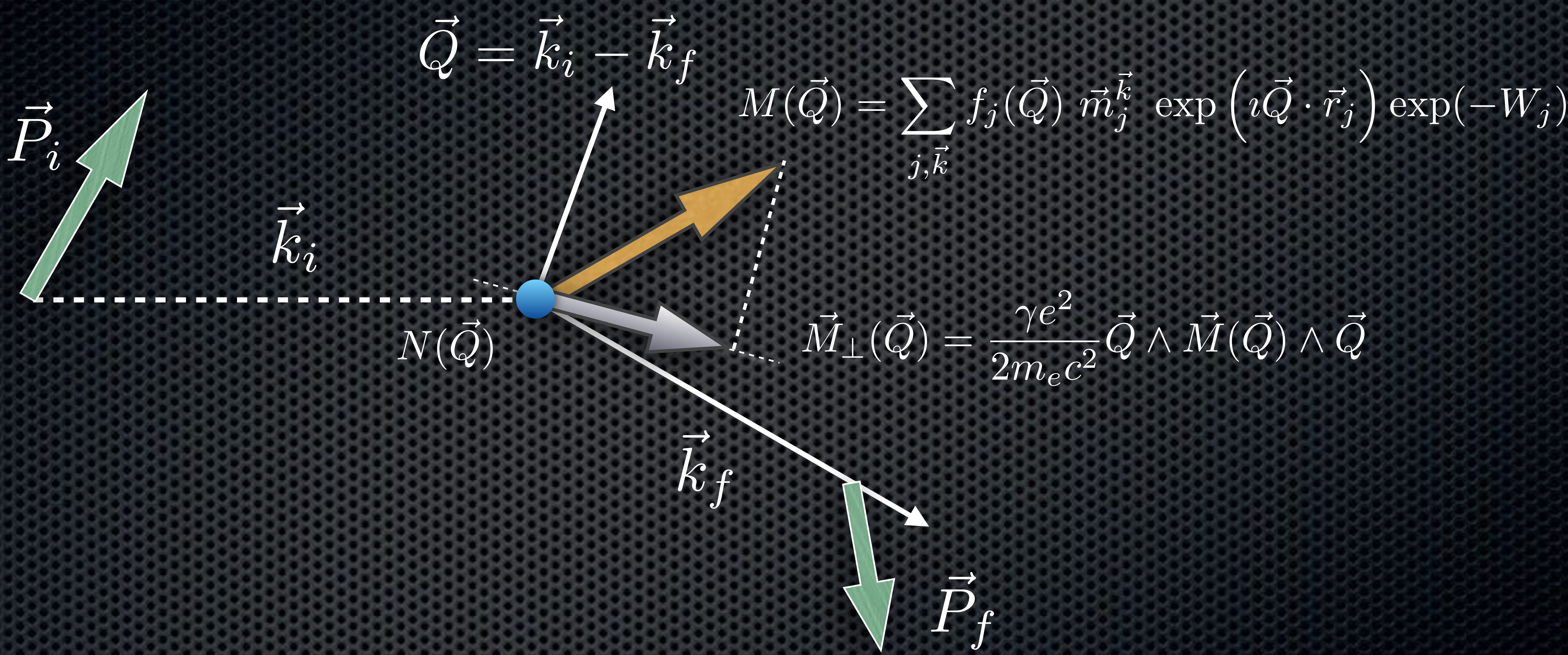
theory: Maleyev, Blume,…



In general, the polarisation of a neutron beam will change both in magnitude and direction upon scattering from a magnetic material.

Cross-section & scattered polarisation vector

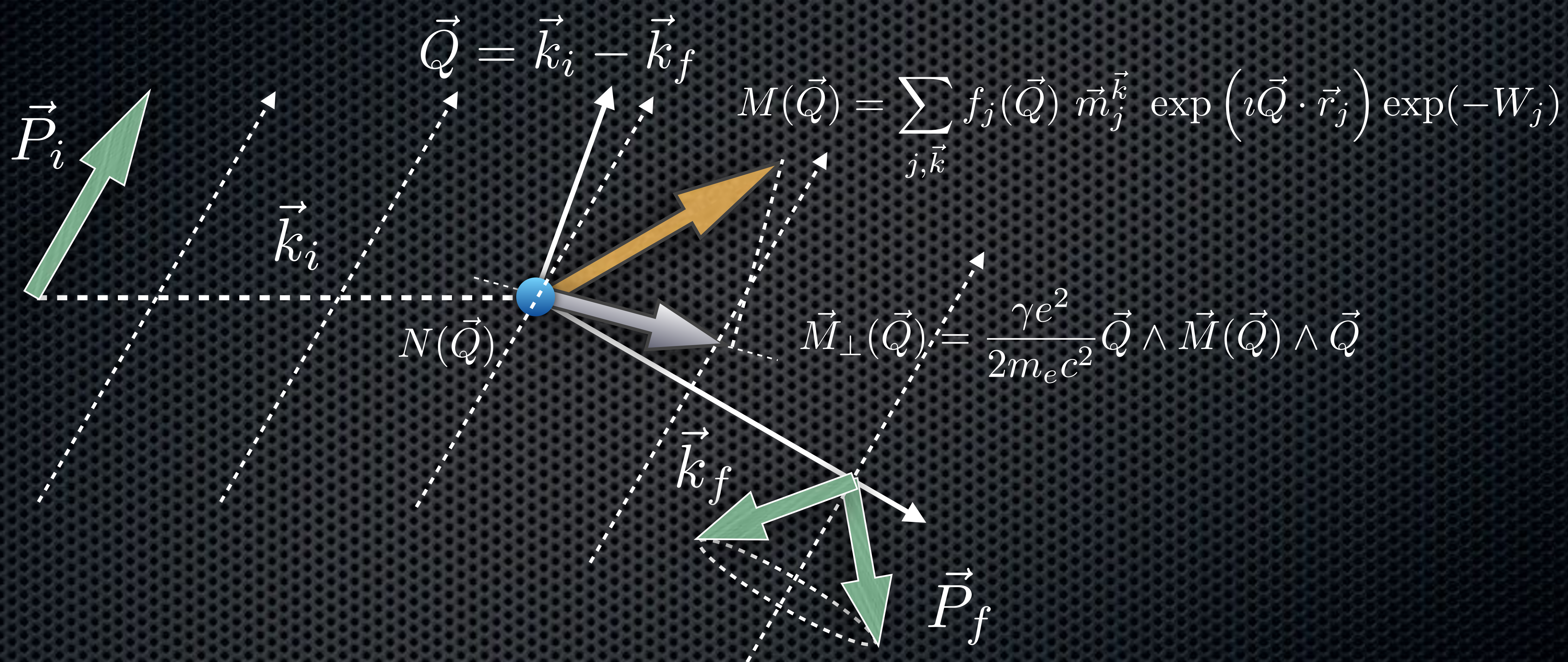
theory: Maleyev, Blume,…



The changes in direction that take place on scattering by a magnetic interaction vector are highly dependent on their relative orientations.

Cross-section & scattered polarisation vector

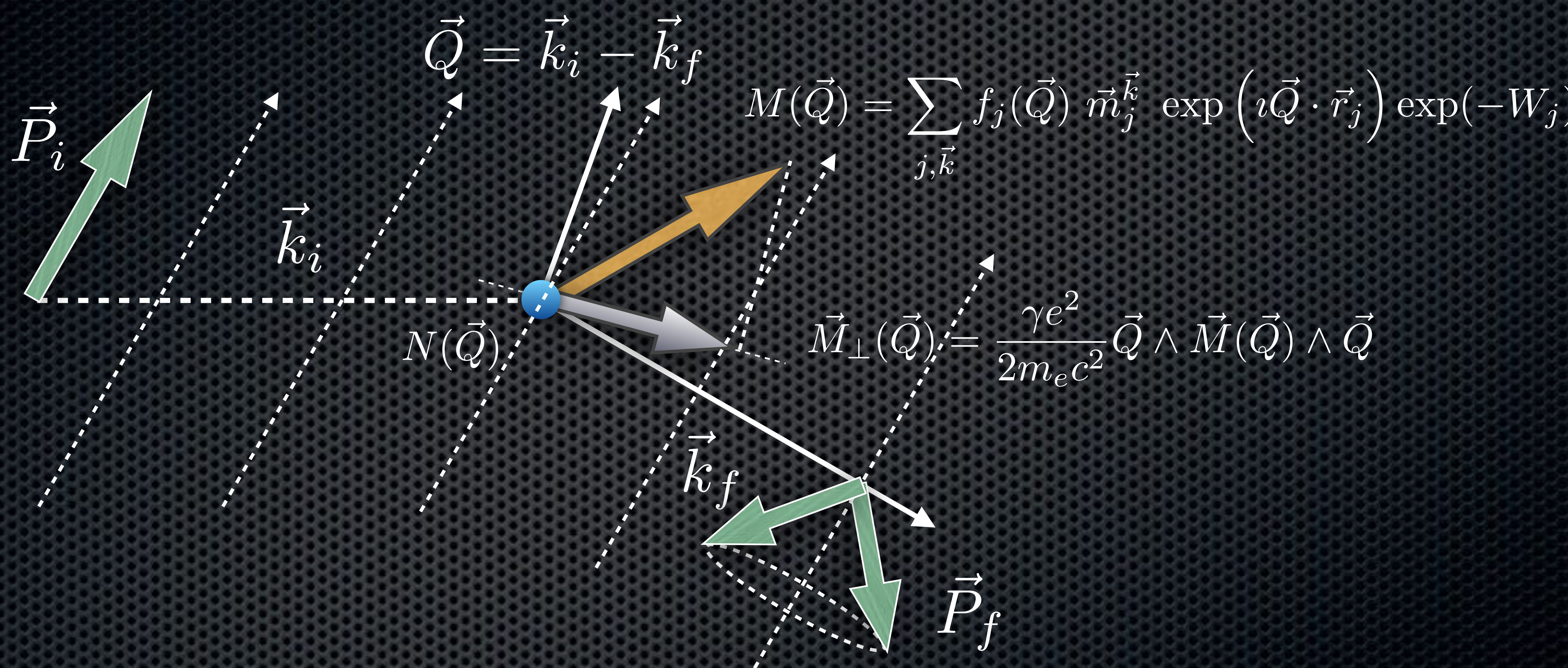
theory: Maleyev, Blume,…



When a magnetic field is applied at the sample, the Larmor precessions lead to the loss of the components perpendicular to the field.

Cross-section & scattered polarisation vector

theory: Maleyev, Blume,…



So obviously, we need a zero-field region around the sample and devices to handle the incident and scattered polarisation vectors.

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Spherical neutron polarimetry

$$\mathrm{P}_{i,j} = \frac{P_i \; \mathbb{P}_{i,j} \; + P_j^{\dagger}}{||\vec{P}_f||} \text{ with } (i,j) \in \{x, y, z\}$$

A useful strategy is to measure the scattered polarisation with incident polarisation parallel to each of the polarisation axes in turn.

$$\mathbb{P} = \begin{bmatrix} N.N^* - \vec{M}_\perp.\vec{M}_\perp^* & 2\Im(NM_{\perp,z}^*) & -2\Im(NM_{\perp,y}^*) \\ -2\Im(NM_{\perp,z}^*) & N.N^* - \vec{M}_\perp.\vec{M}_\perp^* + 2\Re(M_{\perp,y}M_{\perp,y}^*) & 2\Re(M_{\perp,y}M_{\perp,z}^*) \\ 2\Im(NM_{\perp,y}^*) & 2\Re(M_{\perp,y}M_{\perp,z}^*) & N.N^* - \vec{M}_\perp.\vec{M}_\perp^* + 2\Re(M_{\perp,z}M_{\perp,z}^*) \end{bmatrix}$$

$$\vec{P}^{\dagger}\,\sigma = \begin{bmatrix} 2\Im(M_{\perp,y}M_{\perp,z}^*) \\ 2\Re(NM_{\perp,y}^*) \\ 2\Re(NM_{\perp,z}^*) \end{bmatrix} \text{ with } \sigma = N.N^* + \vec{M}_\perp.\vec{M}_\perp^* + \vec{P}_i.\begin{bmatrix} 2\Im(M_{\perp,y}M_{\perp,z}^*) \\ 2\Re(NM_{\perp,y}^*) \\ 2\Re(NM_{\perp,z}^*) \end{bmatrix}$$

Spherical neutron polarimetry

$$\mathrm{P}_{i,j} = \frac{P_i \; \mathbb{P}_{i,j} \; + P_j^{\dagger}}{||\vec{P}_f||} \text{ with } (i,j) \in \{x, y, z\}$$

A useful right-handed cartesian set is defined with $\vec{x}$ parallel to $\vec{Q}$ because

$$\vec{M}_{\perp}(\vec{Q}) \;\; \perp \;\; \vec{Q}$$

$\vec{z}$ is conventionally chosen vertical (often perpendicular to the scattering plane) and $\vec{y}$ completes the cartesian set.

We then measure: $P_{x,x}, P_{x,y}, P_{x,z}, P_{y,x}, P_{y,y}, ..$

Spherical neutron polarimetry

A lot of directional information is lost when only intensities are measured.

$$\frac{\partial\sigma}{\partial\Omega} = NN^* + \vec{M}_\perp \cdot \vec{M}_\perp^* + \imath\vec{P}_i \cdot (\vec{M}_\perp^* \wedge \vec{M}_\perp) + 2\vec{P}_i \cdot \Re(N^*\vec{M}_\perp)$$

The vector properties of the neutron polarisation provide a way of recovering some of this information.

$$\vec{P}_f \frac{\partial\sigma}{\partial\Omega} = \vec{P}_i NN^* - \vec{P}_i(\vec{M}_\perp \cdot \vec{M}_\perp^*) + 2\Re(\vec{M}_\perp(\vec{P}_i \cdot \vec{M}_\perp^*)) + \imath(\vec{M}_\perp \wedge \vec{M}_\perp^*) + 2\Re(N^*\vec{M}_\perp) + 2\vec{P}_i \wedge \Im(N^*\vec{M}_\perp)$$

Spherical neutron polarimetry

Antiferromagnetic single crystals
with non-zero propagation vector

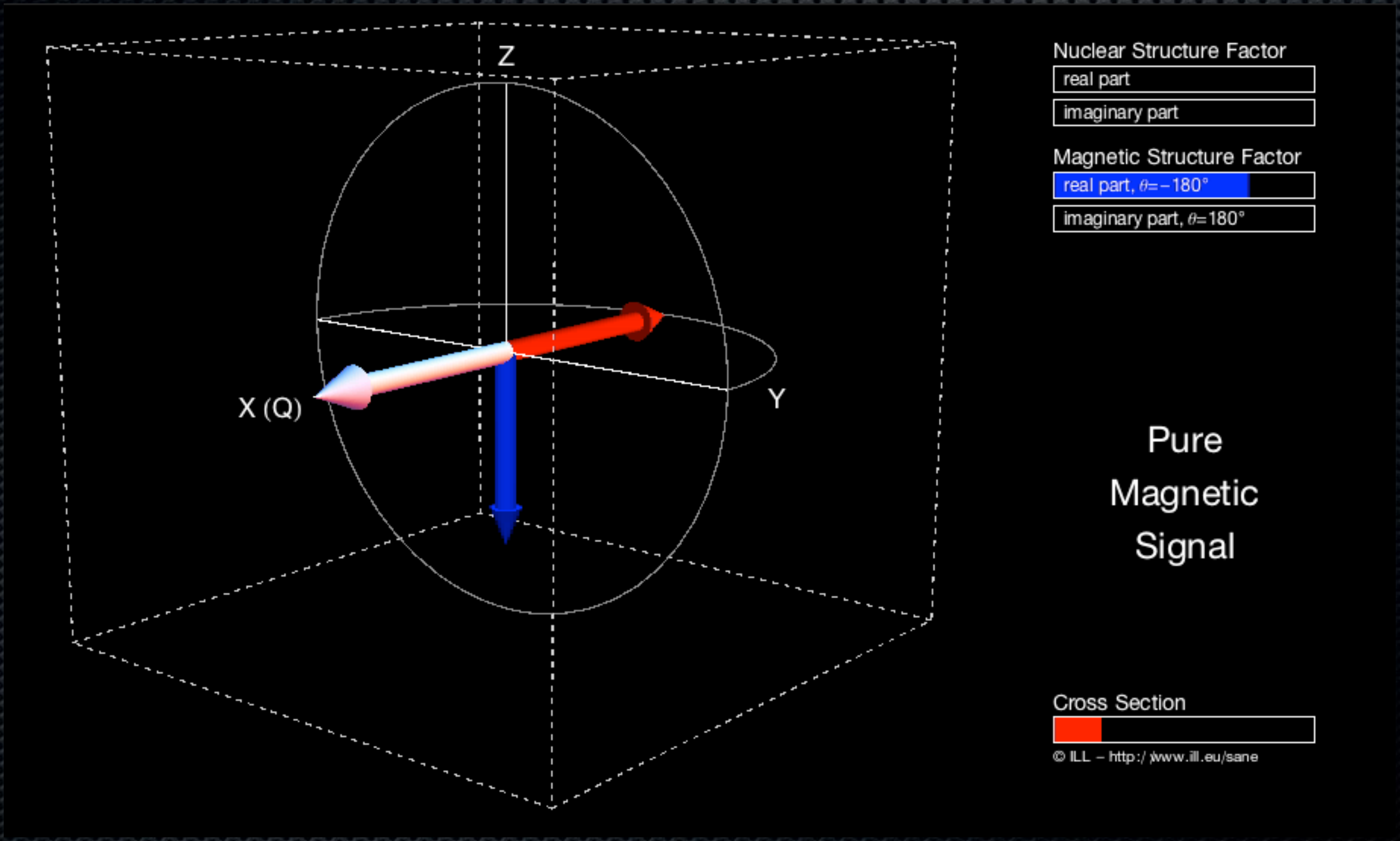
$$\frac{\partial\sigma}{\partial\Omega} = NN^* + \vec{M}_\perp \cdot \vec{M}_\perp^* + \imath\vec{P}_i \cdot (\vec{M}_\perp^* \wedge \vec{M}_\perp) + 2\vec{P}_i \cdot \Re(N^*\vec{M}_\perp)$$

When $\vec{M}_\perp$ is purely real or imaginary, the polarisation rotates around $\vec{M}_\perp$ by 180° - not a spin flip !

$$\vec{P}_f\frac{\partial\sigma}{\partial\Omega} = \vec{P}_i NN^* - \vec{P}_i(\vec{M}_\perp \cdot \vec{M}_\perp^*) + 2\Re(\vec{M}_\perp(\vec{P}_i \cdot \vec{M}_\perp^*)) + \imath(\vec{M}_\perp \wedge \vec{M}_\perp^*) + 2\Re(N^*\vec{M}_\perp) + 2\vec{P}_i \wedge \Im(N^*\vec{M}_\perp)$$

Spherical neutron polarimetry

Antiferromagnetic single crystals: $\tau \neq 0$



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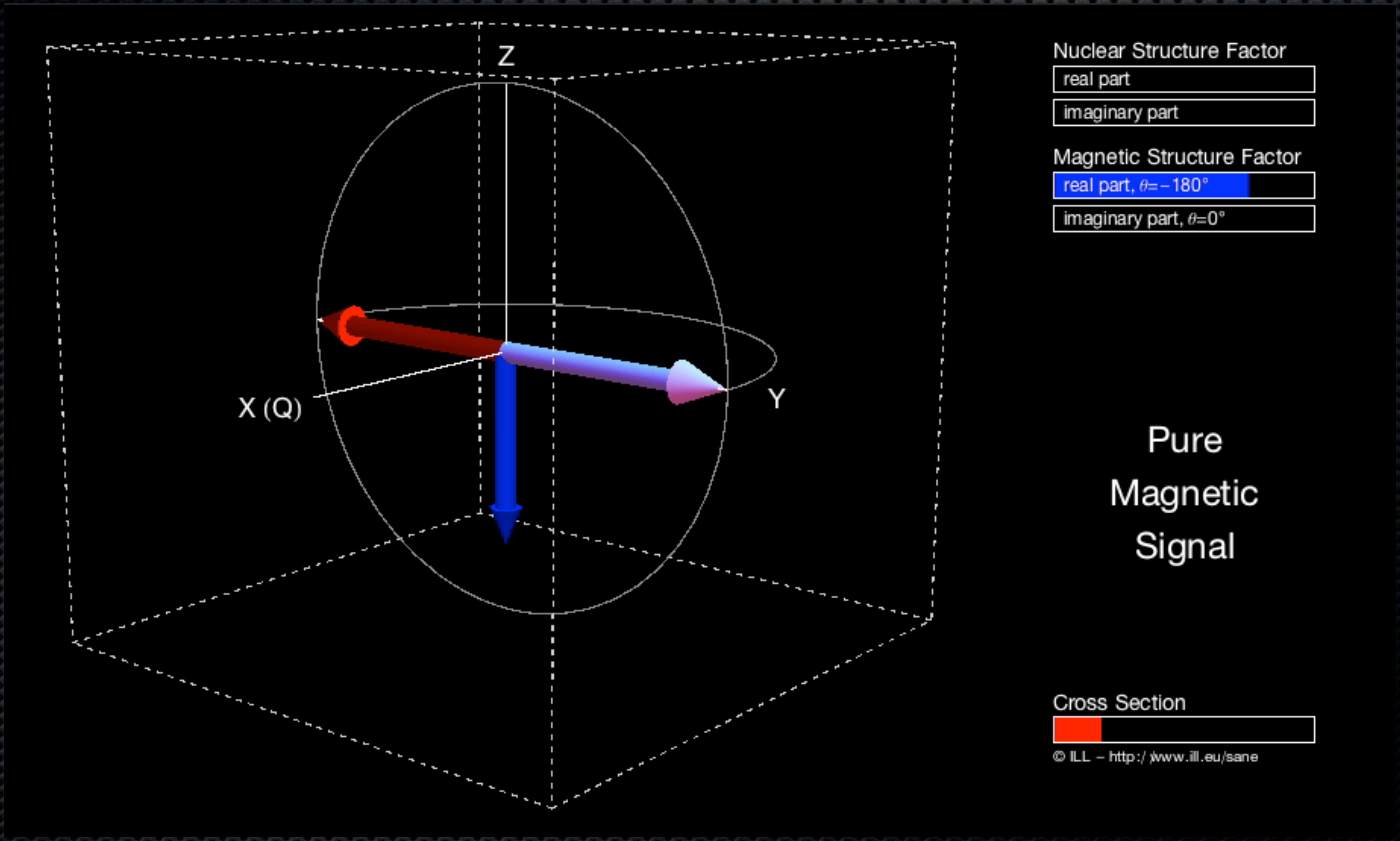

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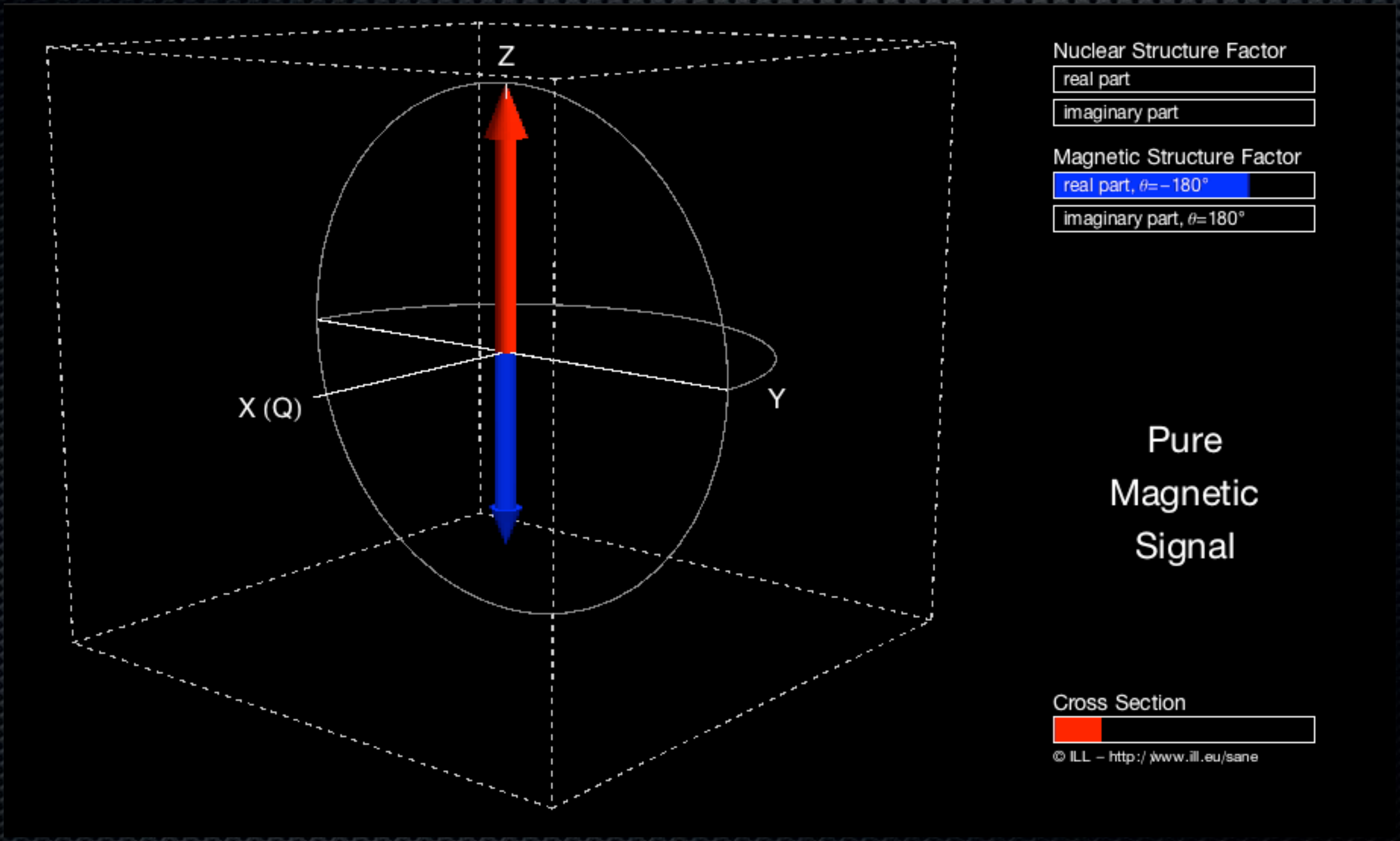

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Antiferromagnetic single crystals
with non-zero propagation vector

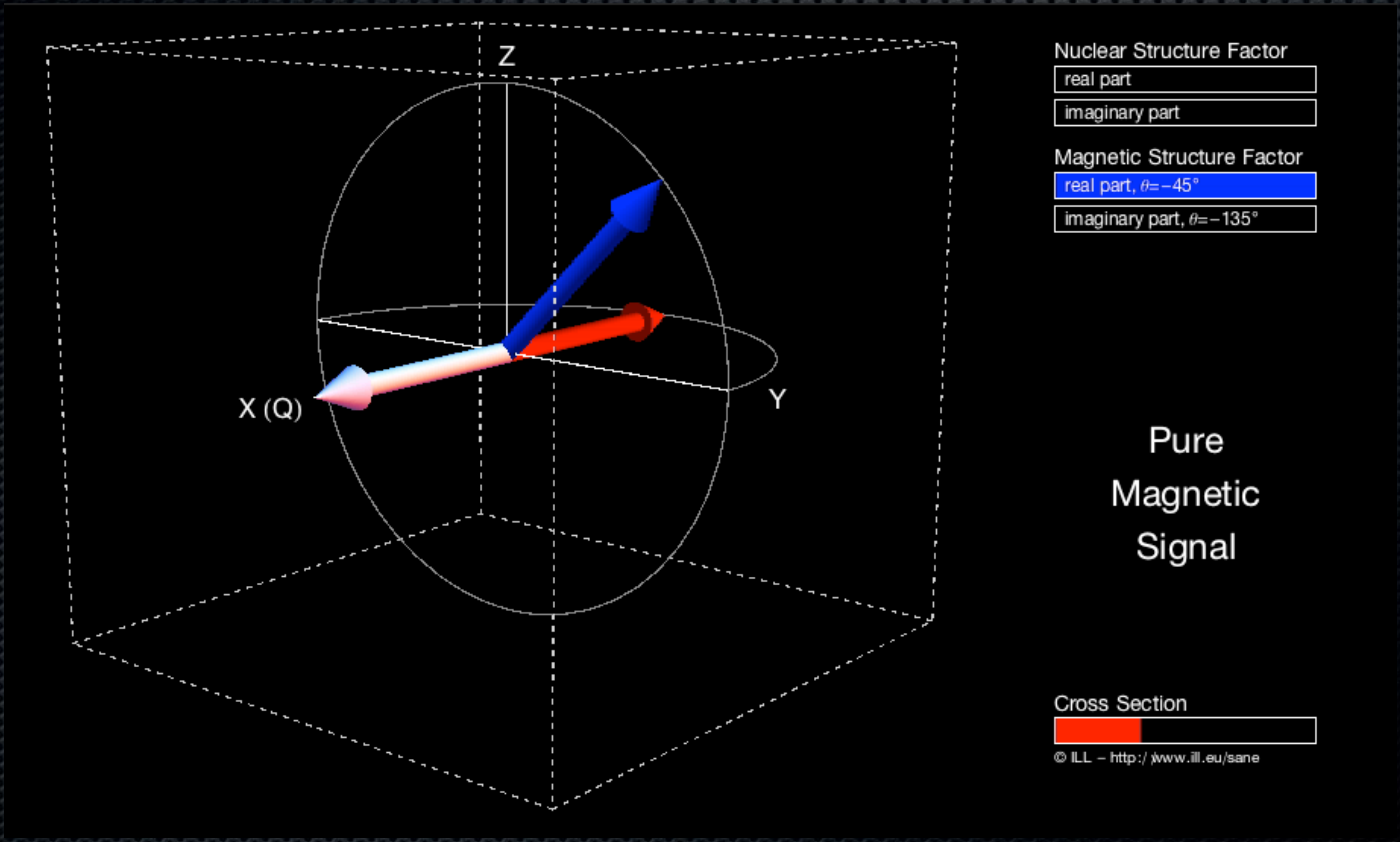
$$\frac{\partial\sigma}{\partial\Omega} = \cancel{NN^*} + \vec{M}_\perp \cdot \vec{M}_\perp^* + \imath\vec{P}_i \cdot (\vec{M}_\perp^* \wedge \vec{M}_\perp) + \cancel{2\vec{P}_i \cdot \Re(N^*\vec{M}_\perp)}$$

When $\vec{M}_\perp$ is complex, the polarisation rotates by 90°
and its final orientation depends on $\|\vec{M}_\perp\|/N$.

$$\vec{P}_f \frac{\partial\sigma}{\partial\Omega} = \cancel{\vec{P}_i NN^*} - \vec{P}_i(\vec{M}_\perp \cdot \vec{M}_\perp^*) + 2\Re(\vec{M}_\perp(\vec{P}_i \cdot \vec{M}_\perp^*)) + \imath(\vec{M}_\perp \wedge \vec{M}_\perp^*) + \cancel{2\Re(N^*\vec{M}_\perp)} + \cancel{2\vec{P}_i \wedge \Im(N^*\vec{M}_\perp)}$$

Spherical neutron polarimetry

Antiferromagnetic single crystals: $\tau \neq 0$



Spherical neutron polarimetry

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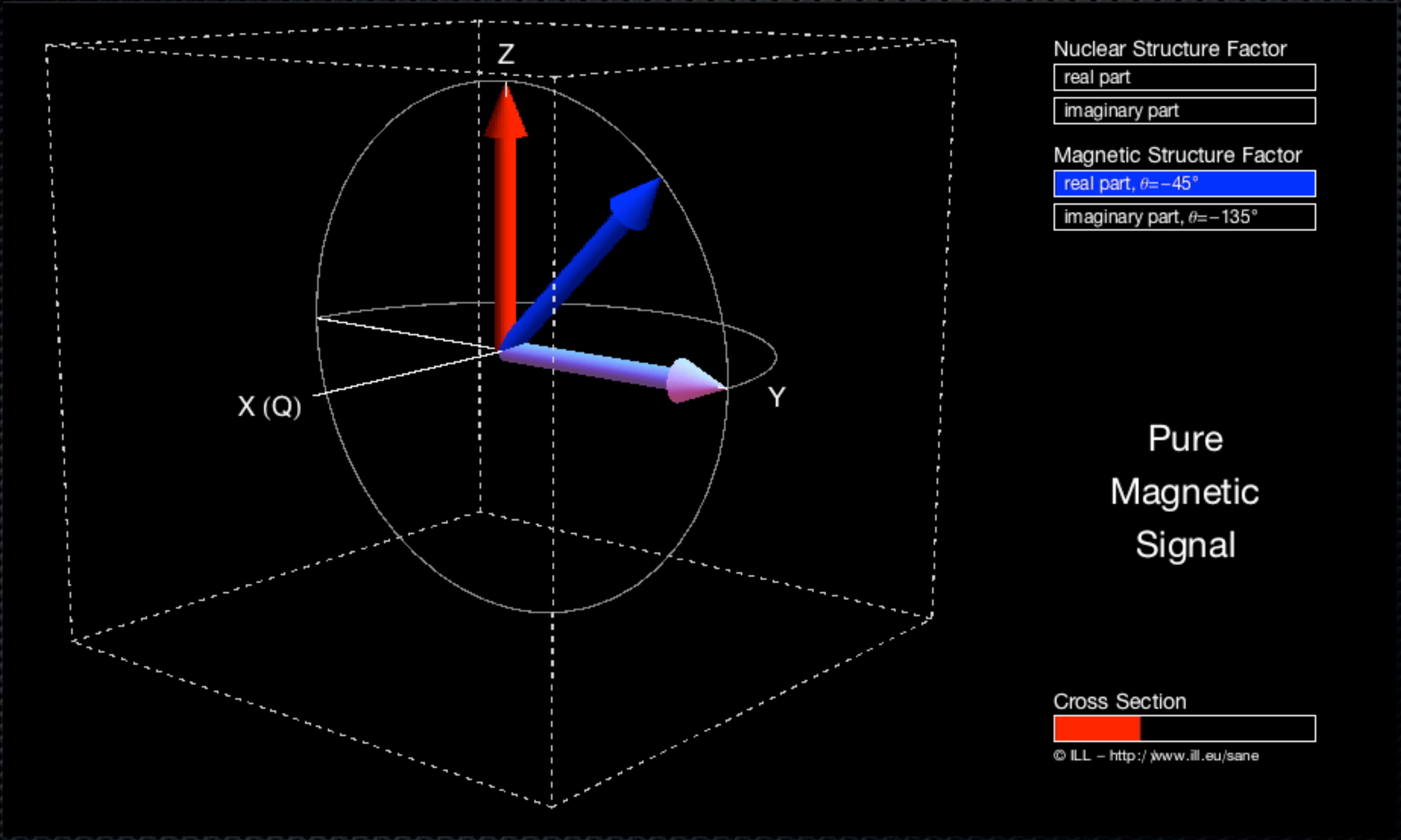

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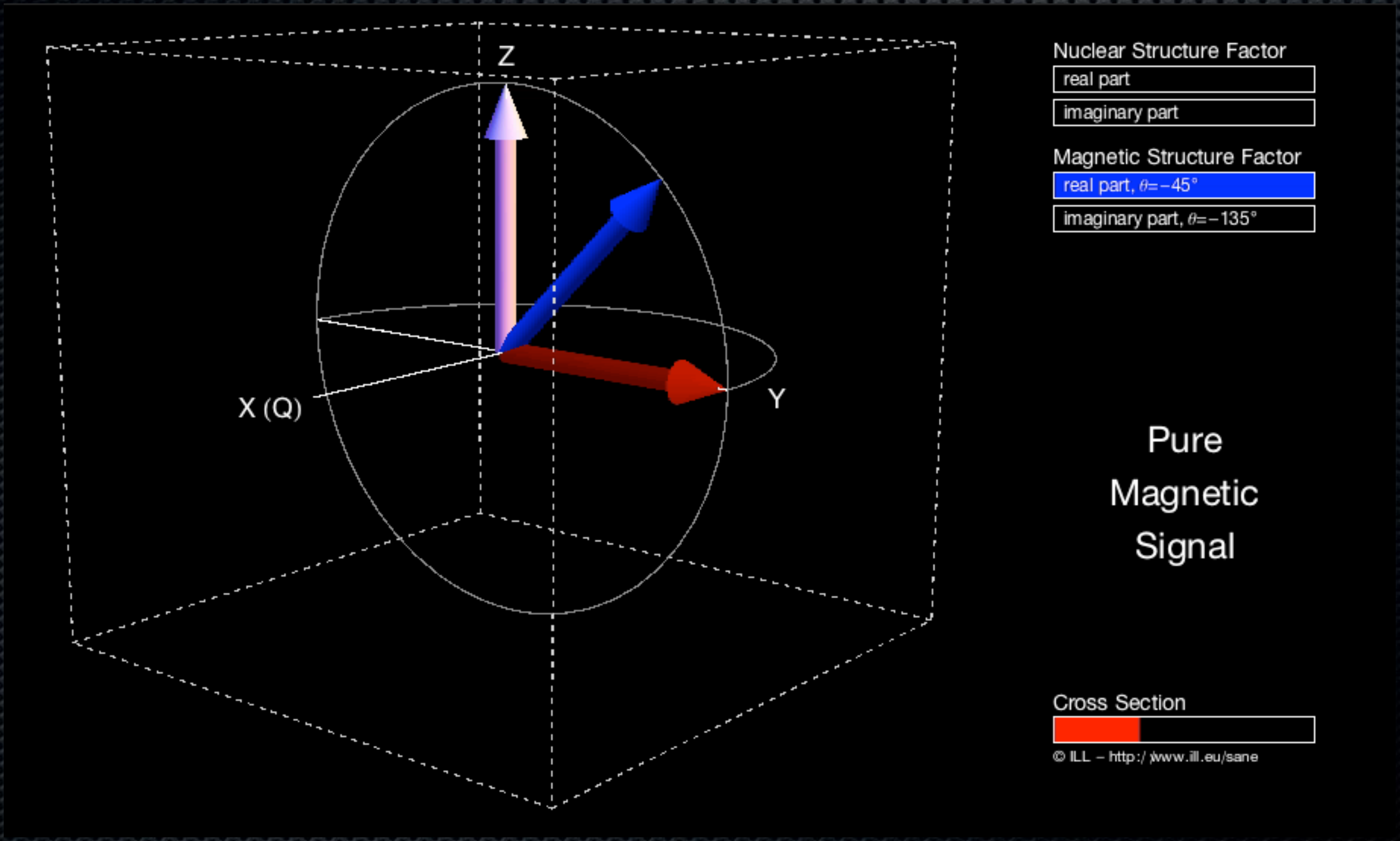


Spherical neutron polarimetry

Antiferromagnetic single crystals
with zero propagation vector

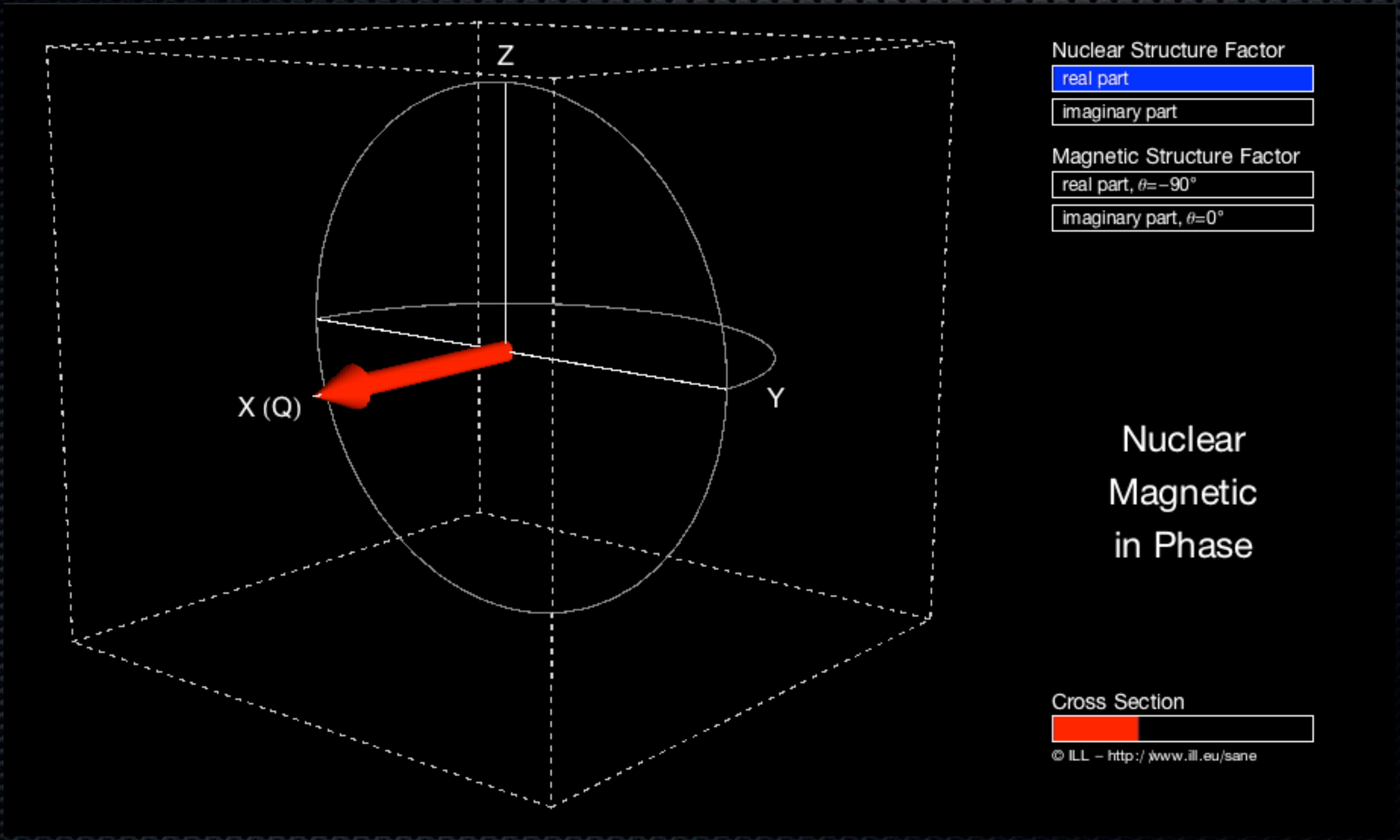
$$\frac{\partial\sigma}{\partial\Omega} = NN^* + \vec{M}_\perp \cdot \vec{M}_\perp^* + \imath\vec{P}_i \cdot (\vec{M}_\perp^* \wedge \vec{M}_\perp) + 2\vec{P}_i \cdot \Re(N^*\vec{M}_\perp)$$

When $\vec{M}_\perp$ is real, the polarisation rotates toward $\vec{M}_\perp$ by an angle depending on $\|\vec{M}_\perp\|/N$.

$$\vec{P}_f \frac{\partial\sigma}{\partial\Omega} = \vec{P}_i NN^* - \vec{P}_i(\vec{M}_\perp \cdot \vec{M}_\perp^*) + 2\Re(\vec{M}_\perp(\vec{P}_i \cdot \vec{M}_\perp^*)) + \imath(\vec{M}_\perp \wedge \vec{M}_\perp^*) + 2\Re(N^*\vec{M}_\perp) + 2\vec{P}_i \wedge \Im(N^*\vec{M}_\perp)$$

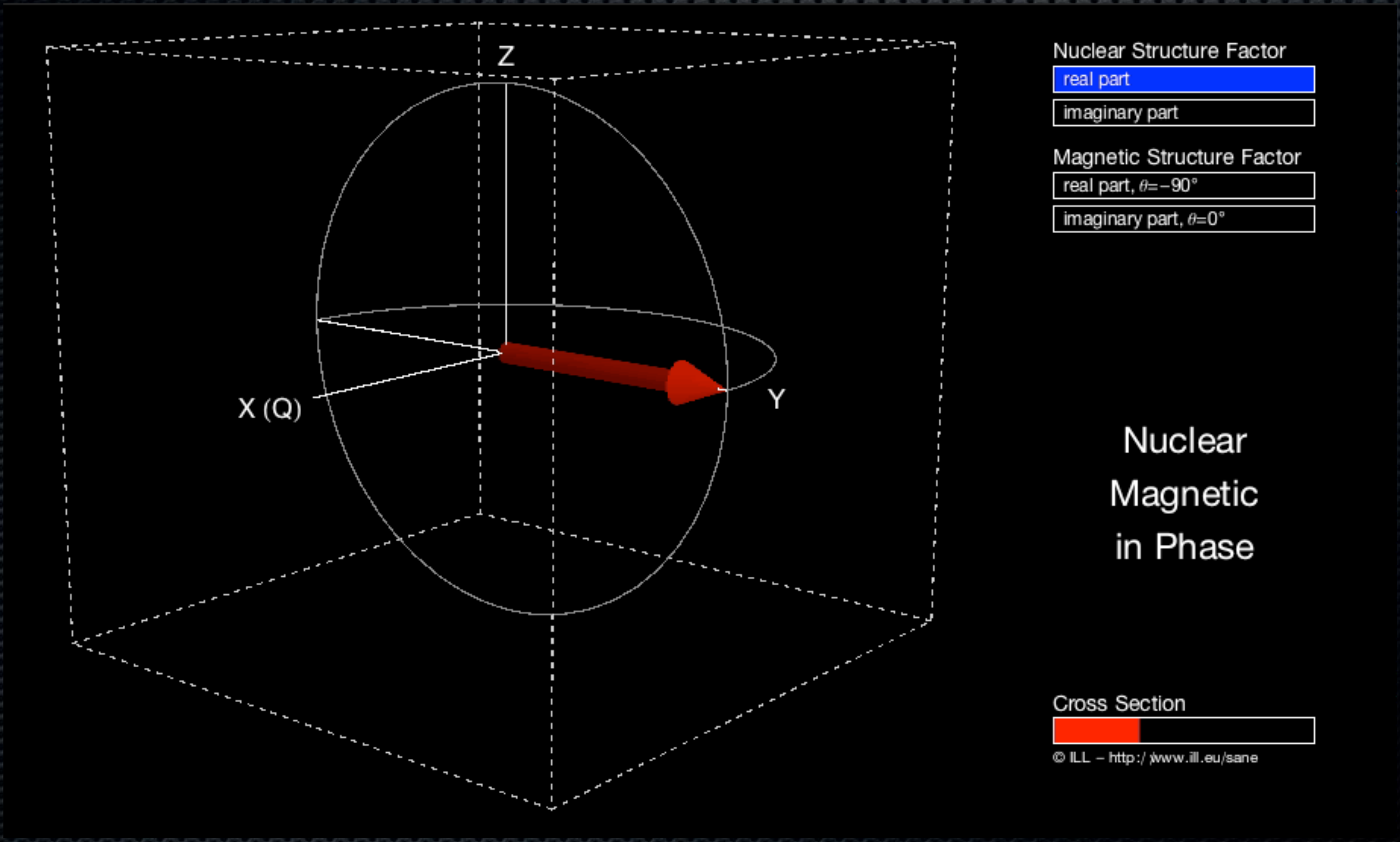
Spherical neutron polarimetry

Antiferromagnetic single crystals: $\tau=0$



Spherical neutron polarimetry

Antiferromagnetic single crystals: $\tau=0$



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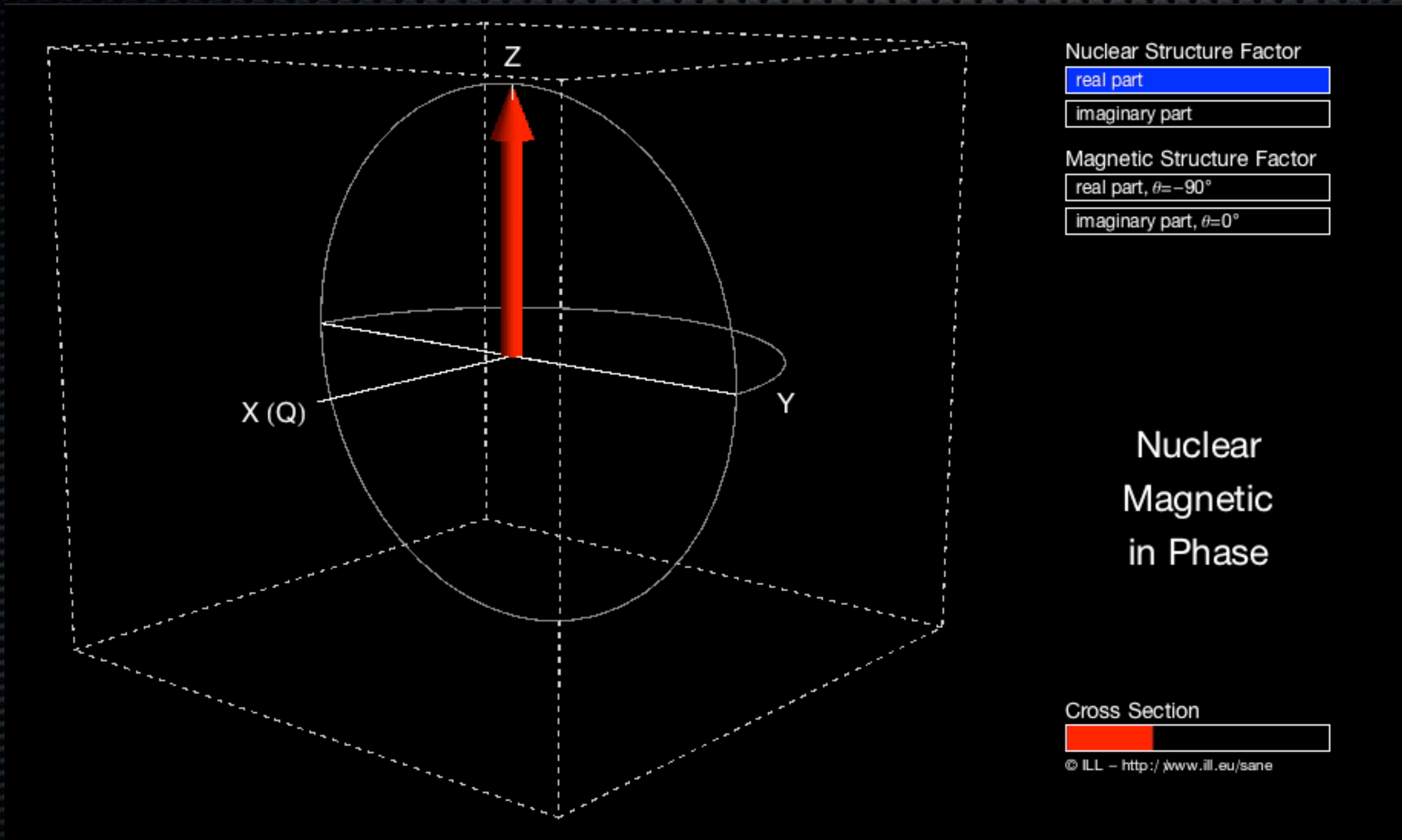

Spherical neutron polarimetry

Antiferromagnetic single crystals
with zero propagation vector

$$\frac{\partial\sigma}{\partial\Omega} = NN^* + \vec{M}_\perp \cdot \vec{M}_\perp^* + \imath\vec{P}_i \cdot (\vec{M}_\perp^* \wedge \vec{M}_\perp) + 2\vec{P}_i \cdot \Re(N^*\vec{M}_\perp)$$

When $\vec{M}_\perp$ is imaginary, the polarisation rotates around $\vec{M}_\perp$ by an angle depending on $\|\vec{M}_\perp\|/N$.

$$\vec{P}_f \frac{\partial\sigma}{\partial\Omega} = \vec{P}_i NN^* - \vec{P}_i(\vec{M}_\perp \cdot \vec{M}_\perp^*) + 2\Re(\vec{M}_\perp(\vec{P}_i \cdot \vec{M}_\perp^*)) + \imath(\vec{M}_\perp \wedge \vec{M}_\perp^*) + 2\Re(N^*\vec{M}_\perp) + 2\vec{P}_i \wedge \Im(N^*\vec{M}_\perp)$$

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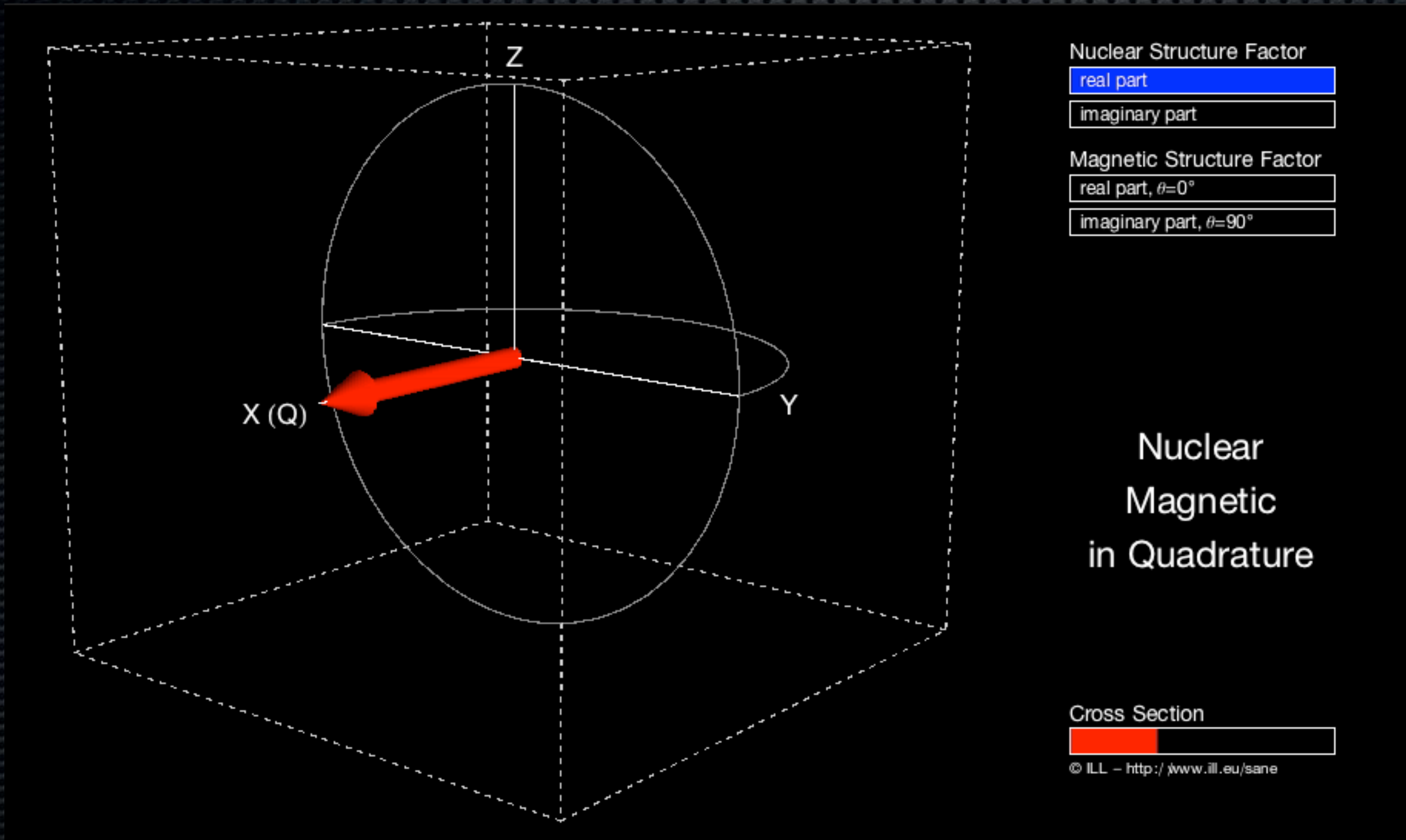

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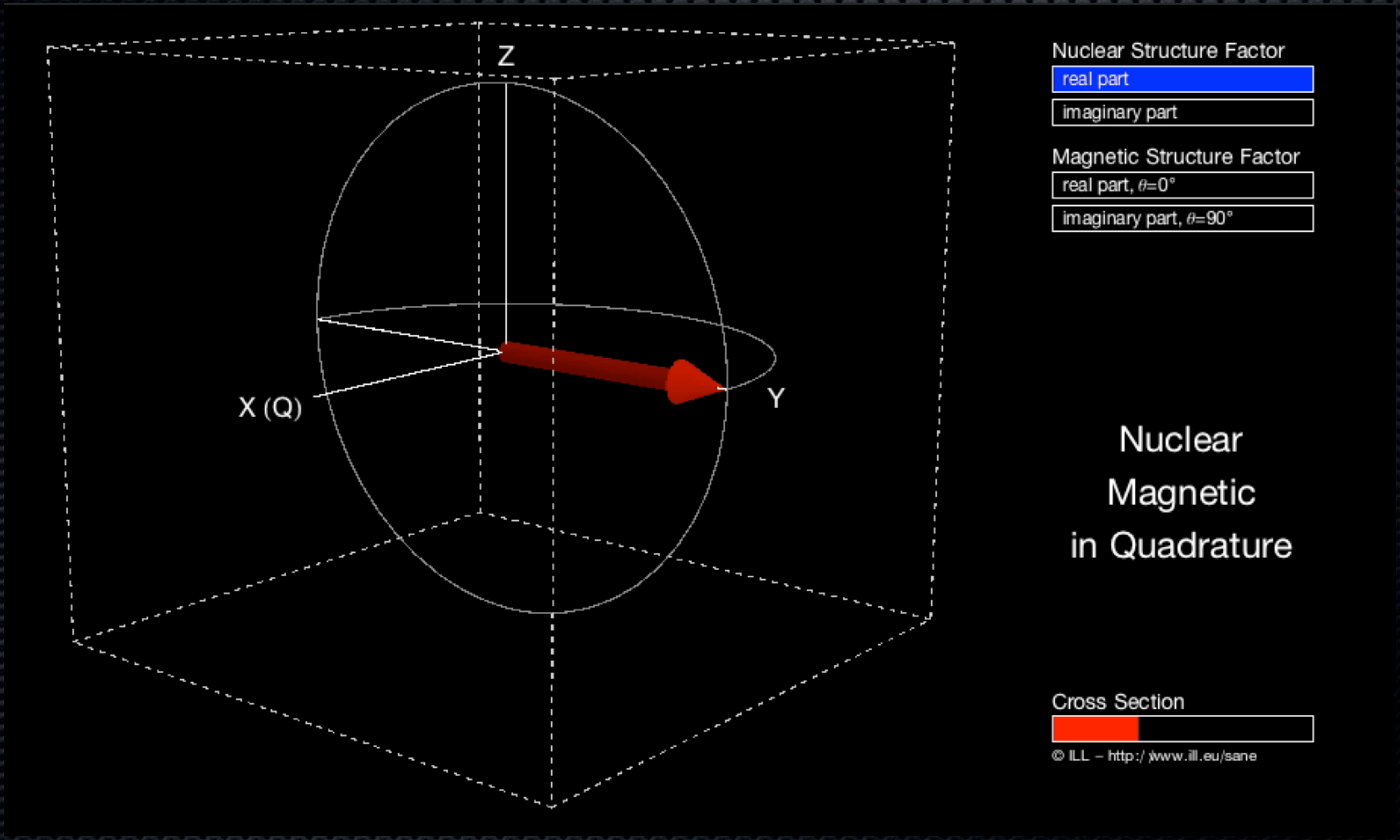

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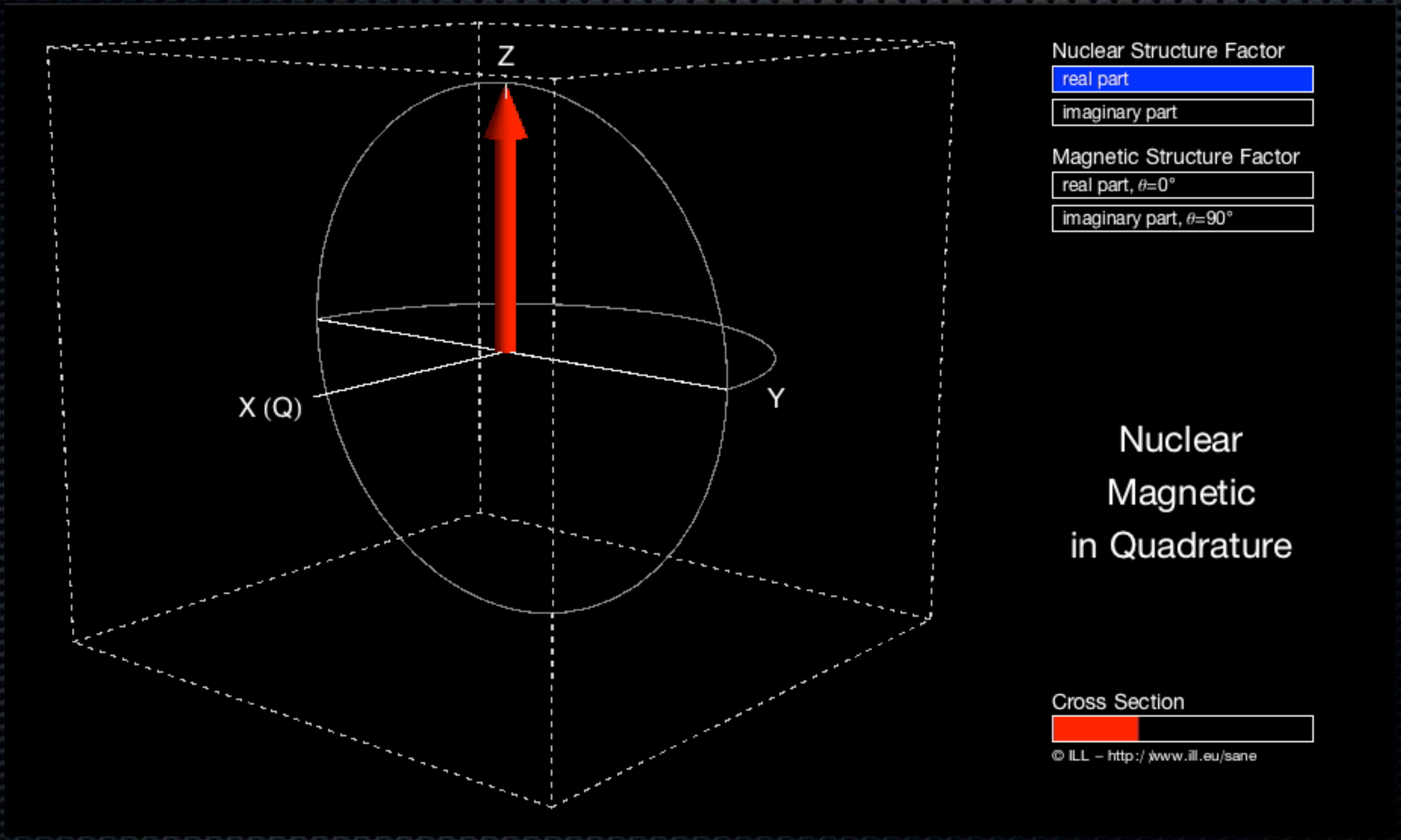
Antiferromagnetic single crystals: $\tau=0$



Spherical Neutron Polarimetry

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- **What happens to the polarisation upon scattering ?**
- Instrumentation — Cryopad and others
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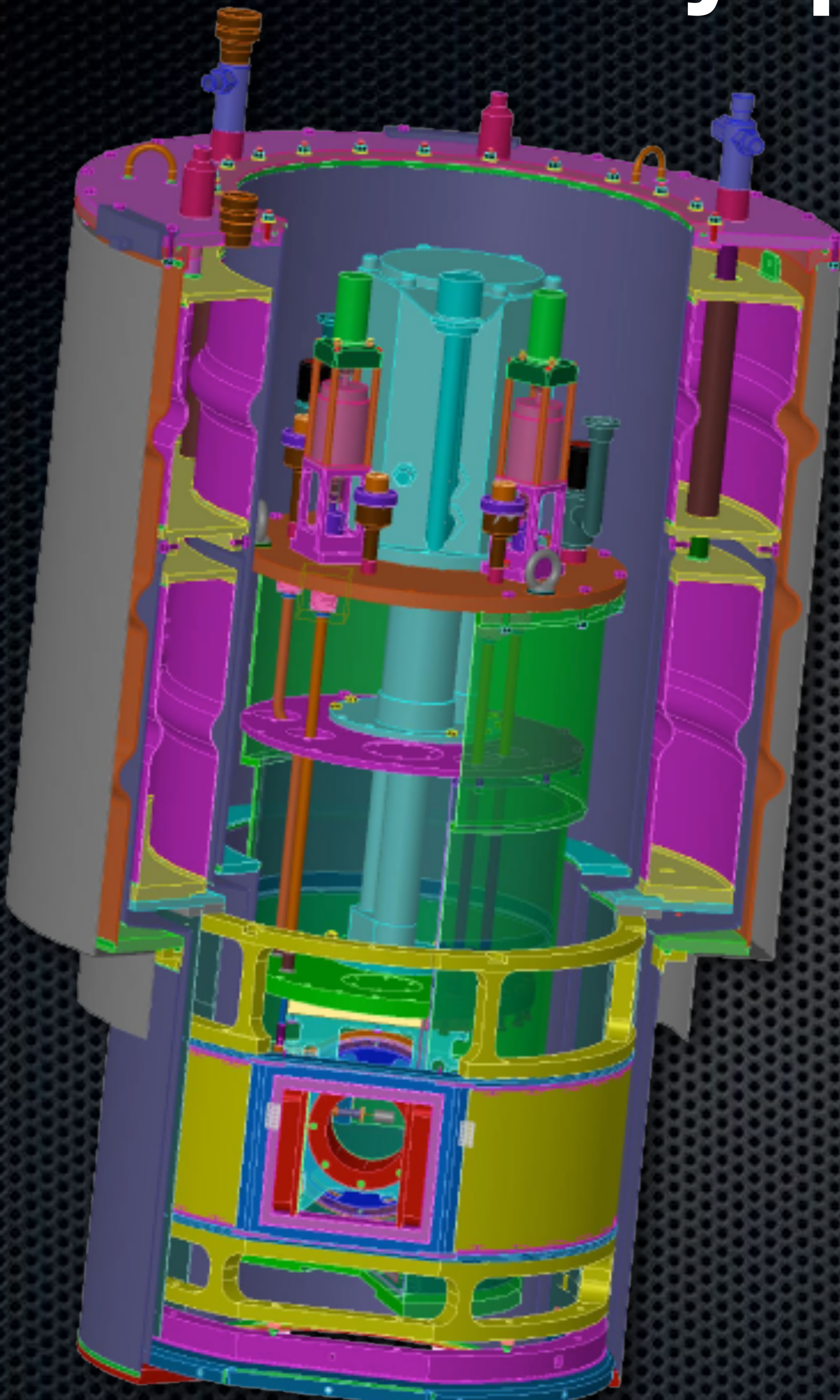
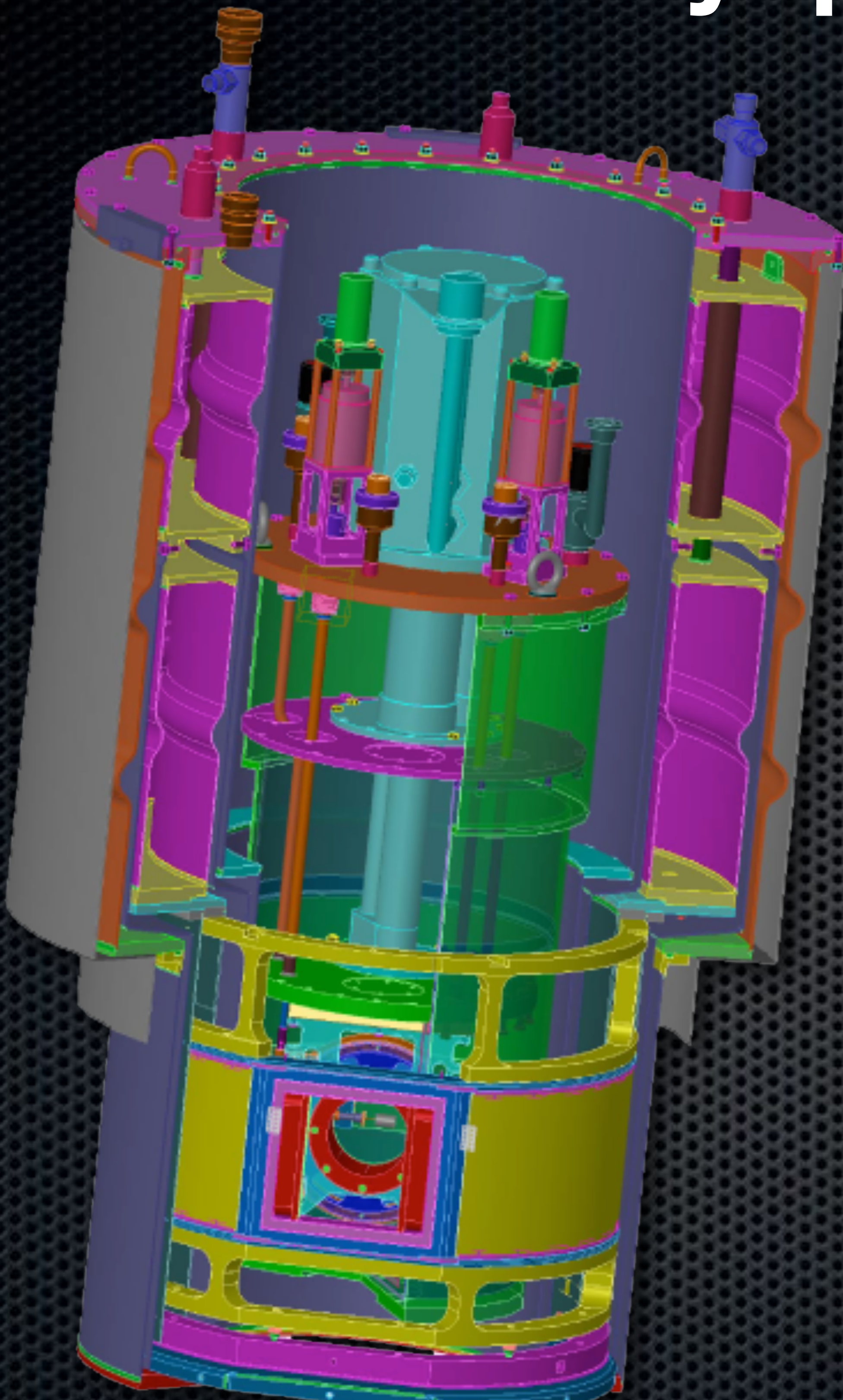
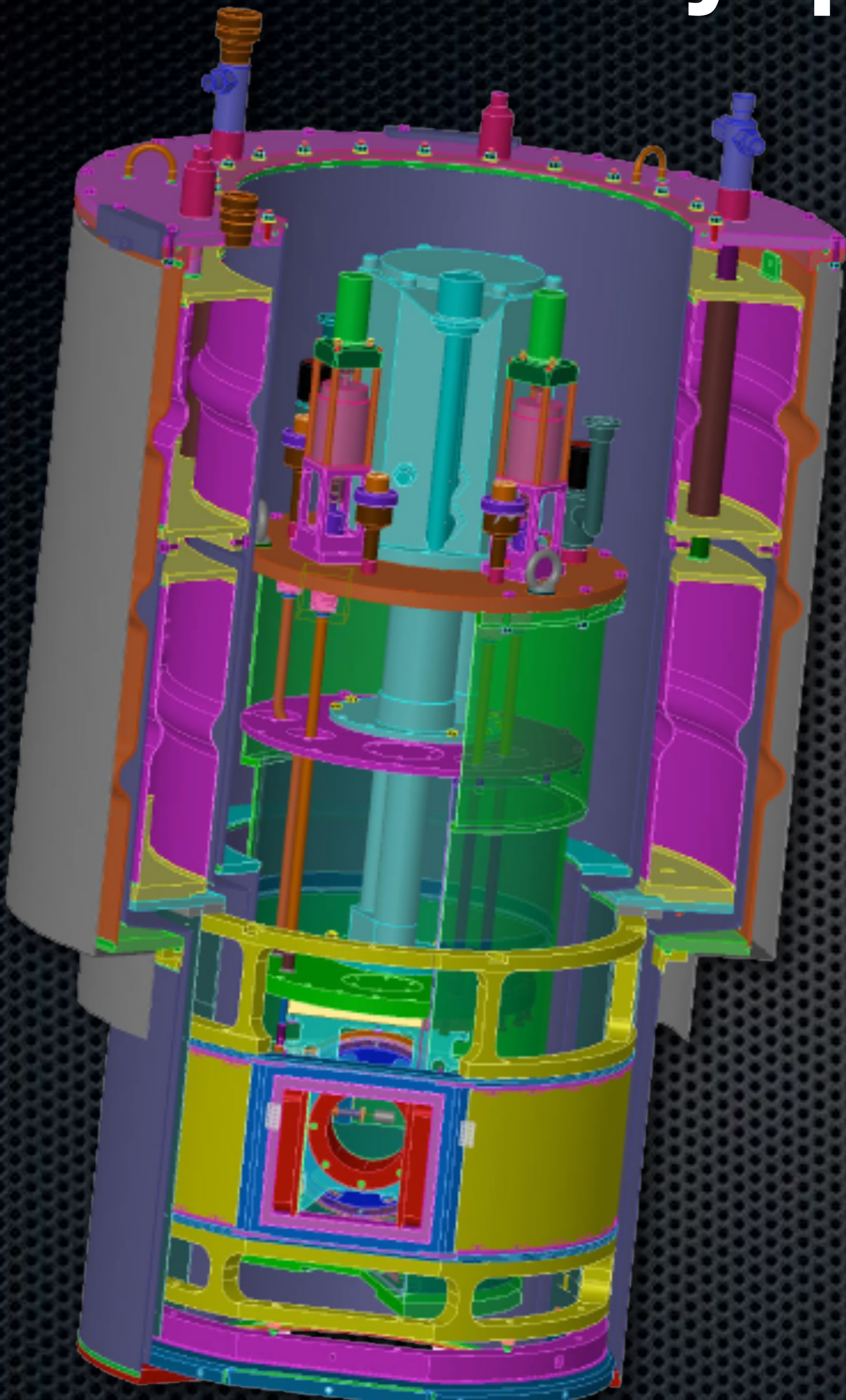
Cryopad in practice

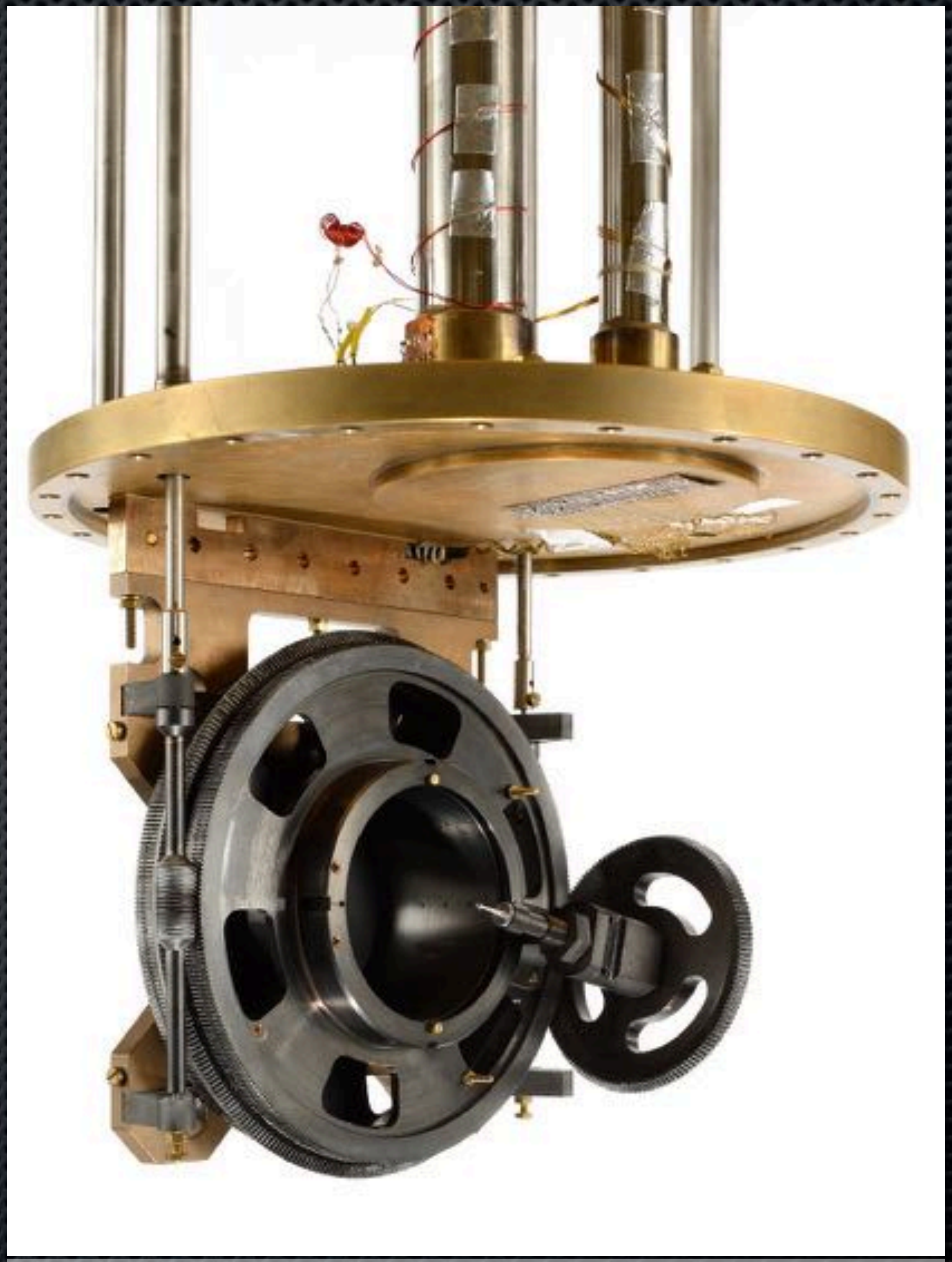


Cryopad in practice

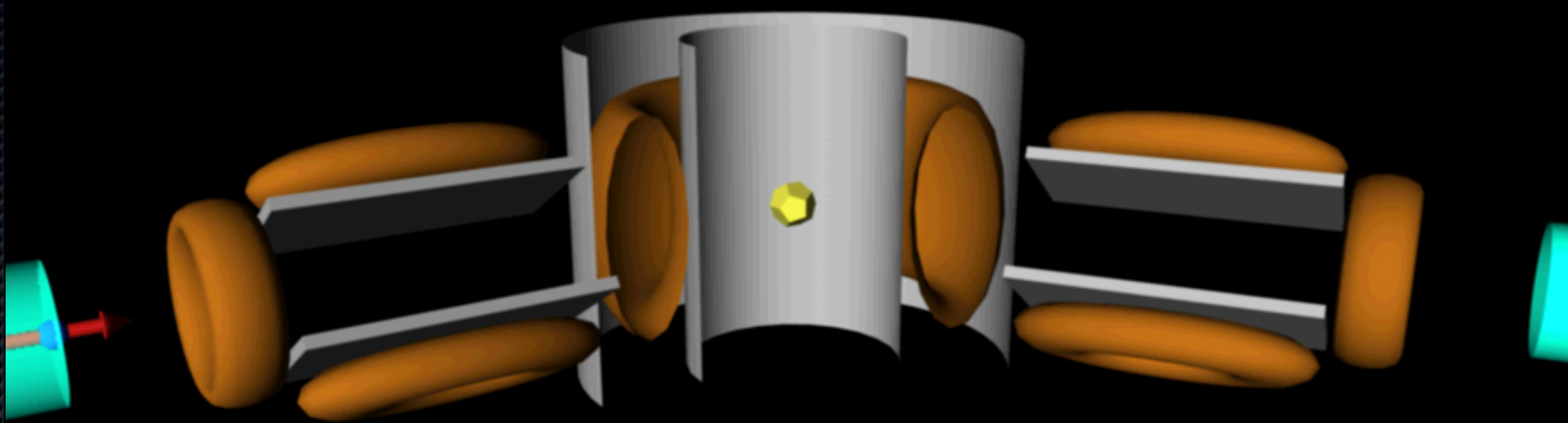


Cryopad in practice



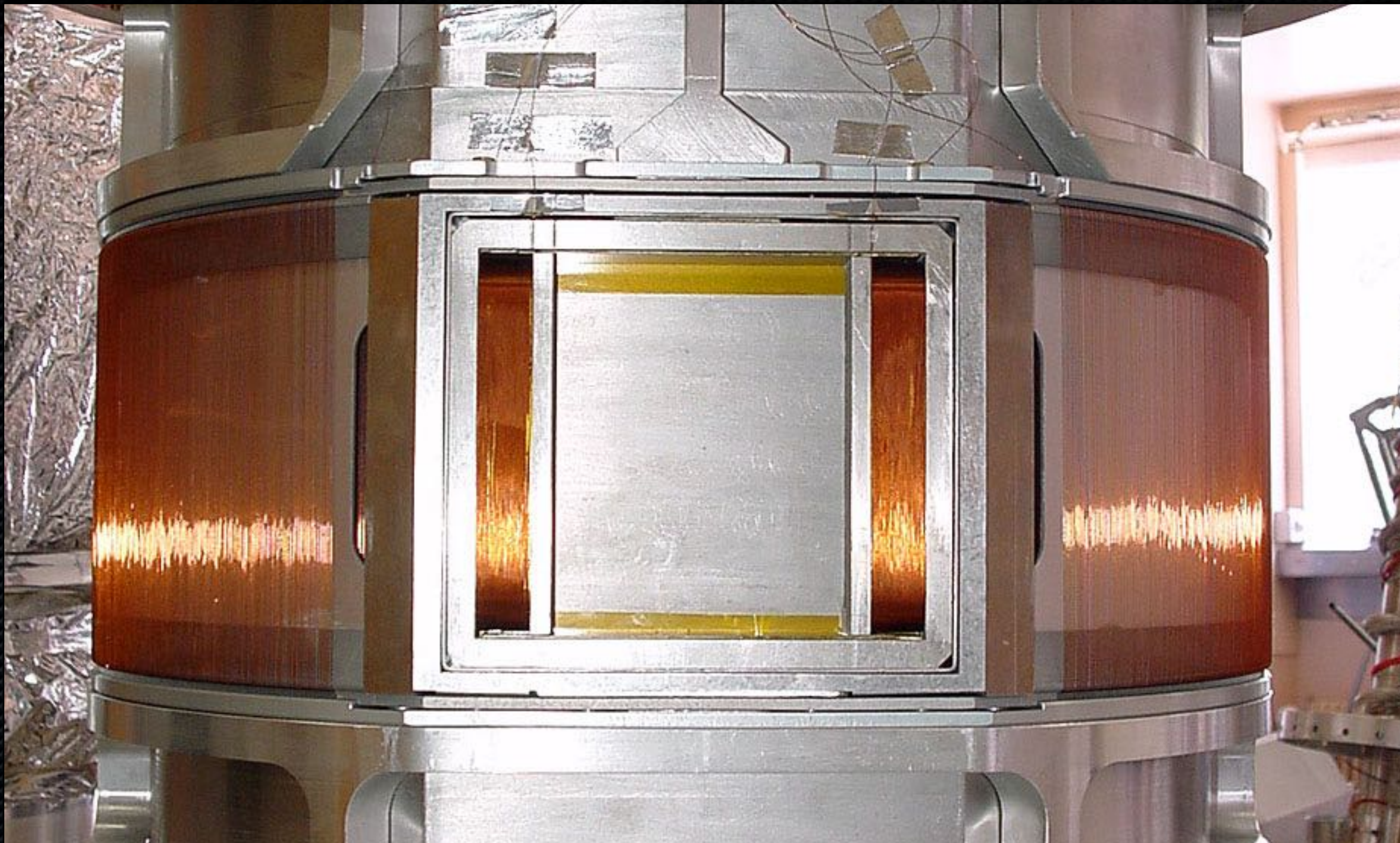


Cryopad in practice

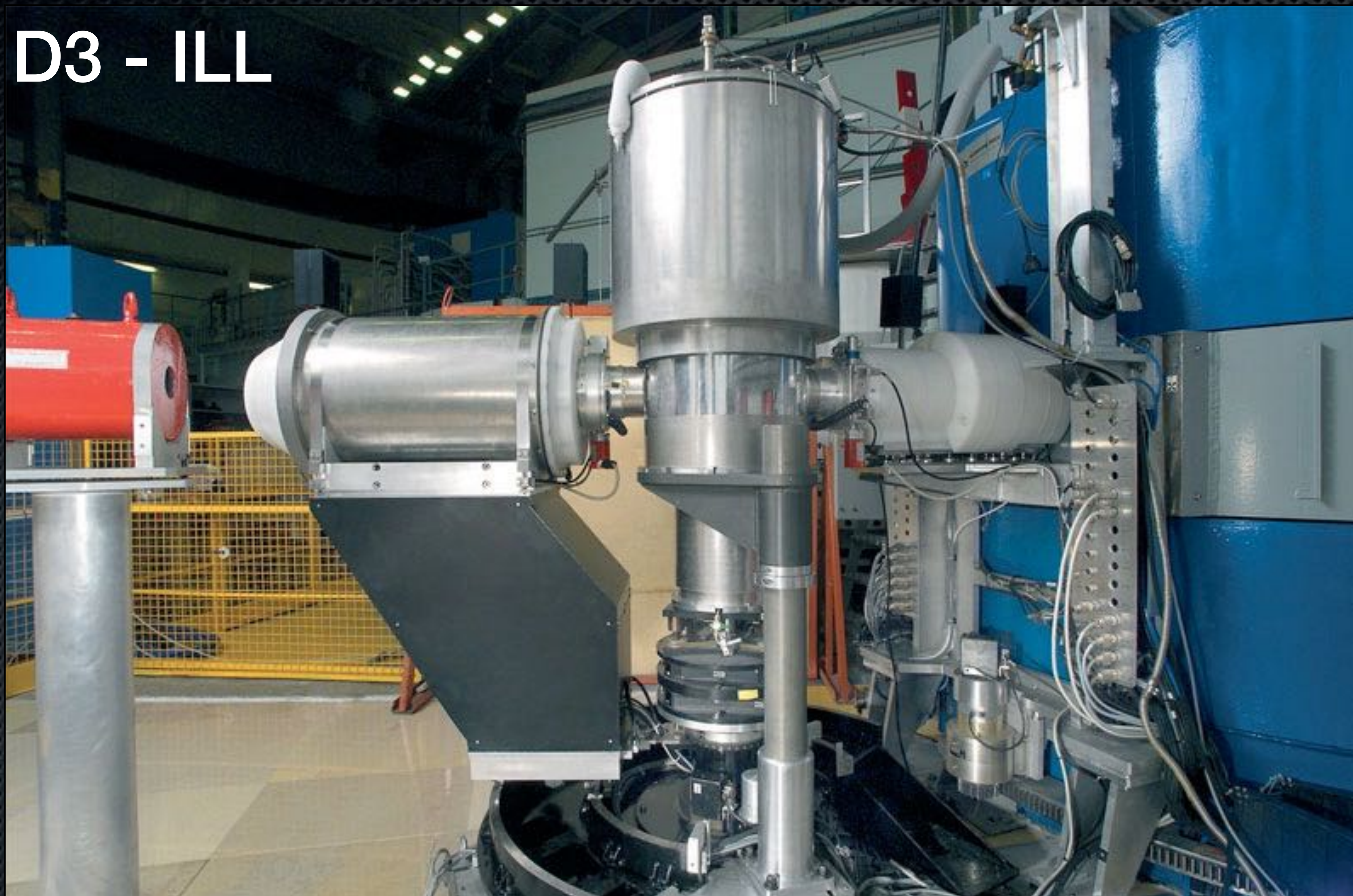


Cryopad - < 2mG in sample chamber

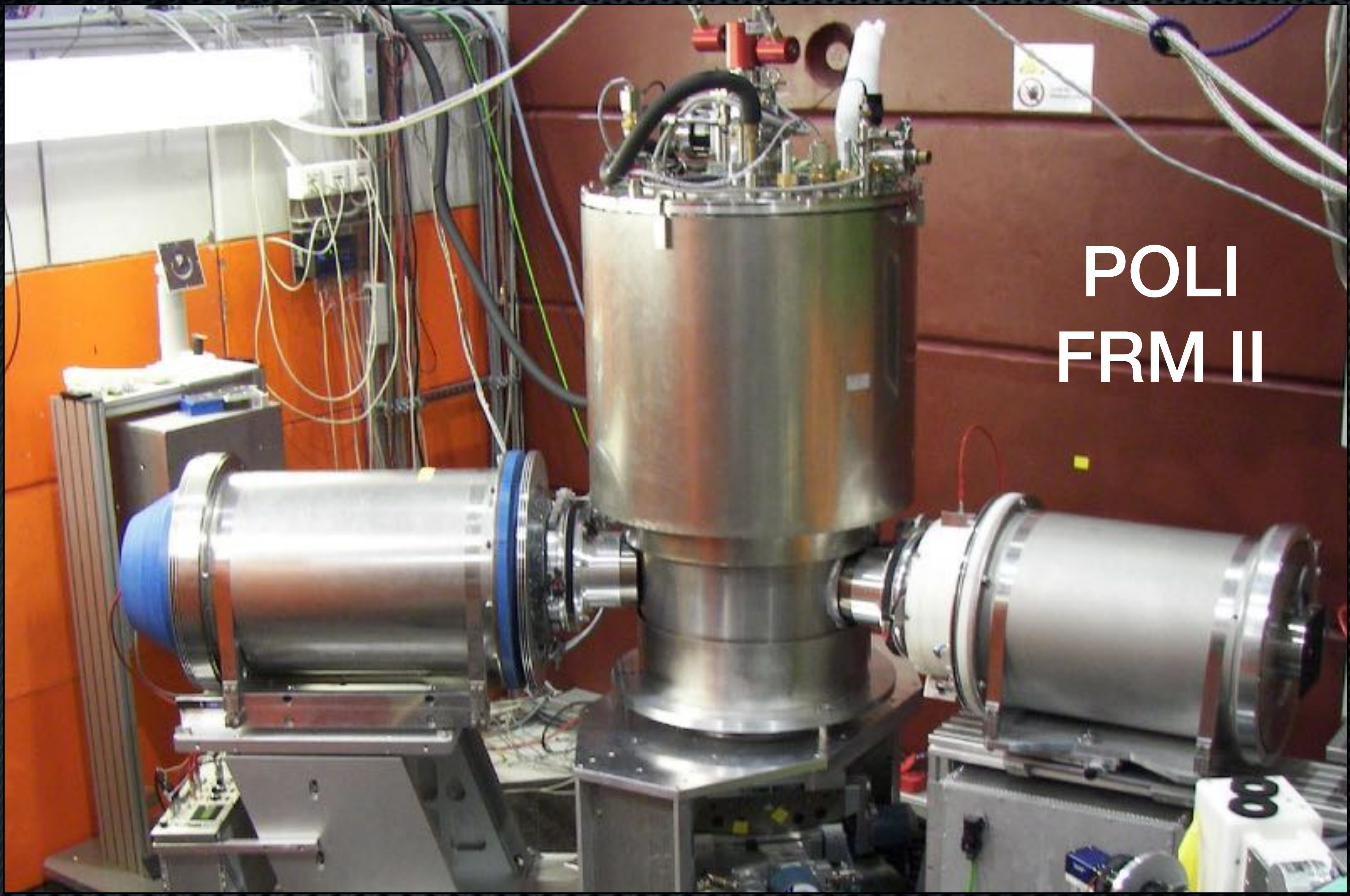
Cryopad in practice



Cryopad in practice

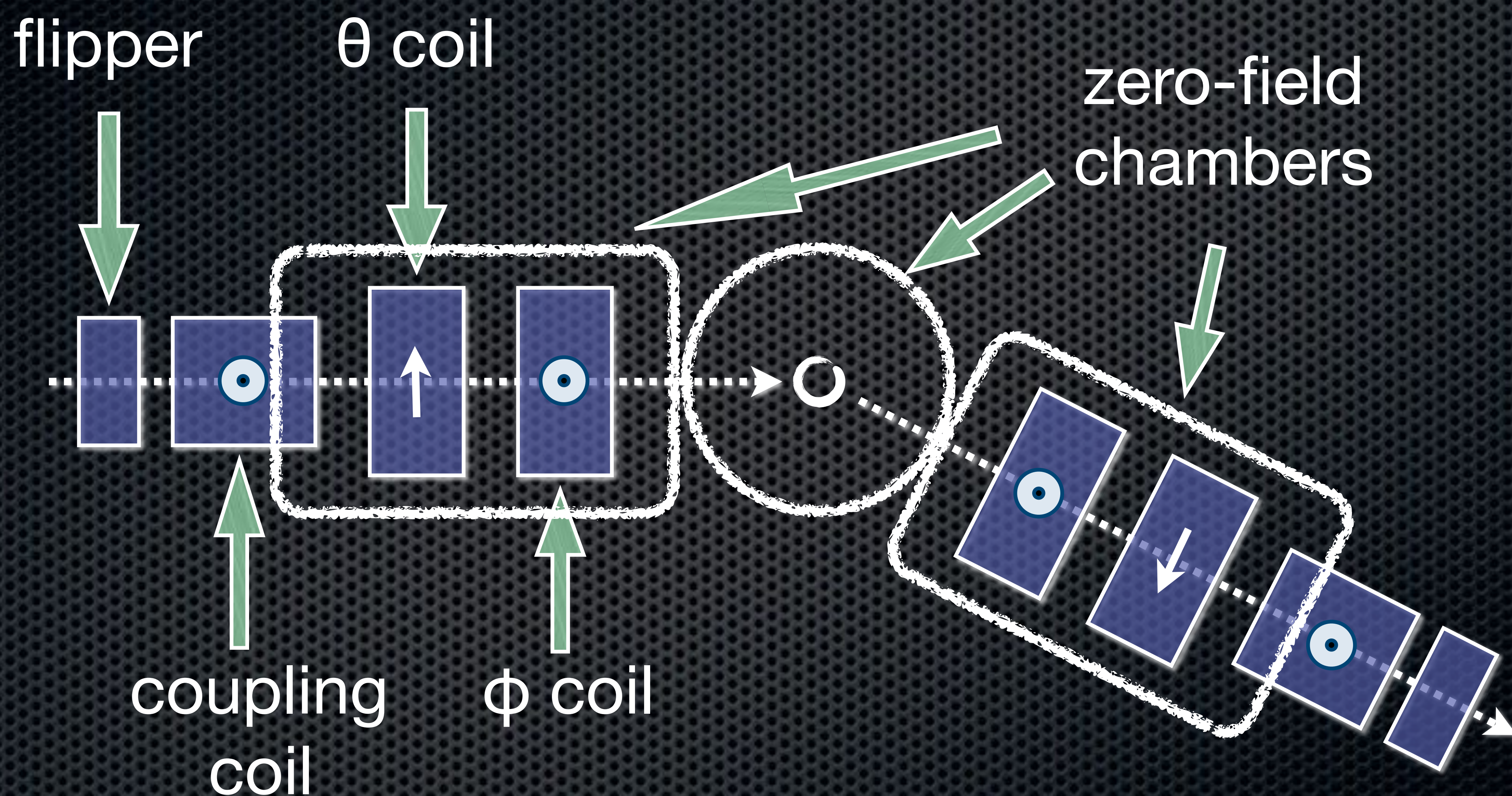


Cryopad in practice



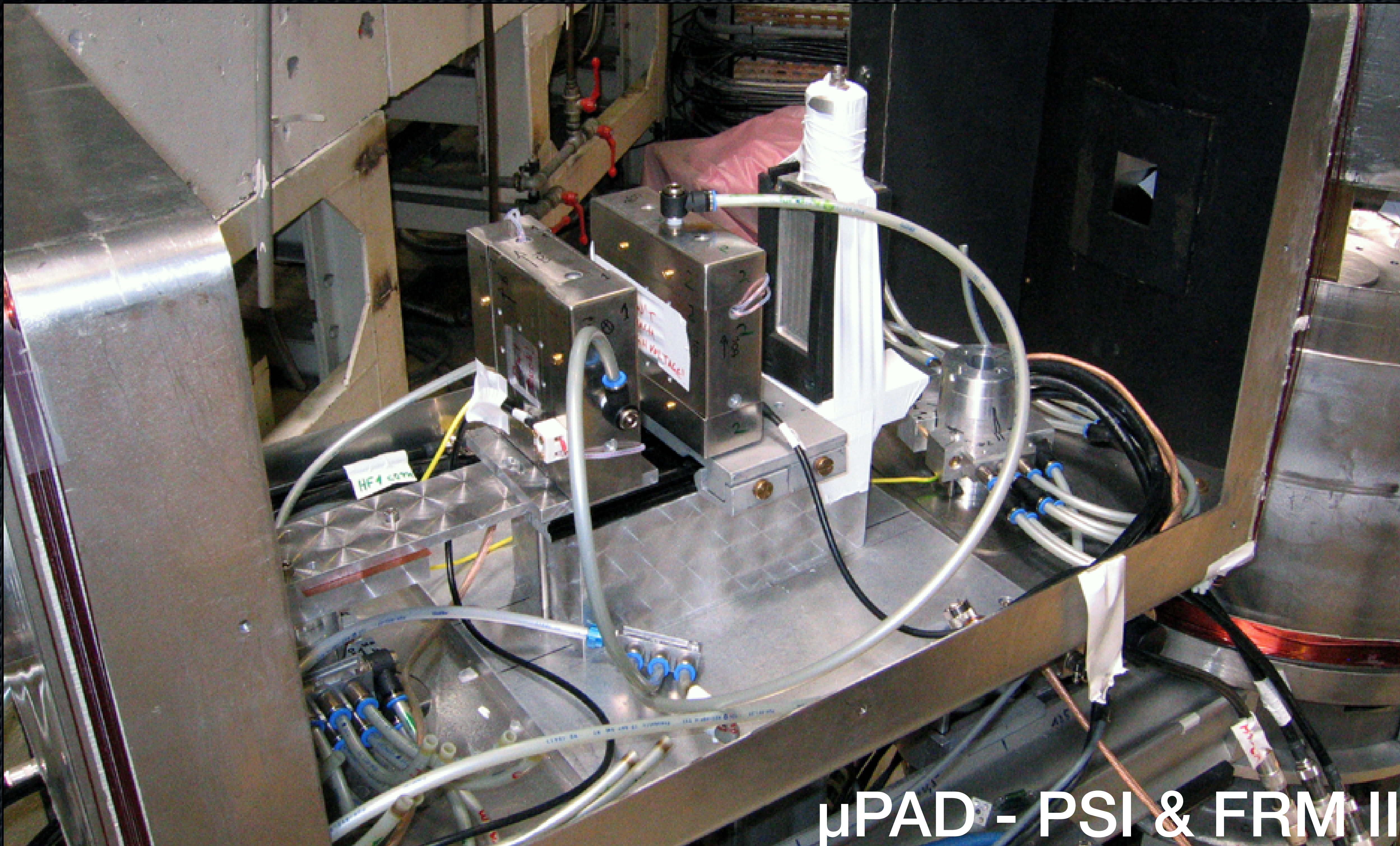
Cryopad in practice

µPad in practice

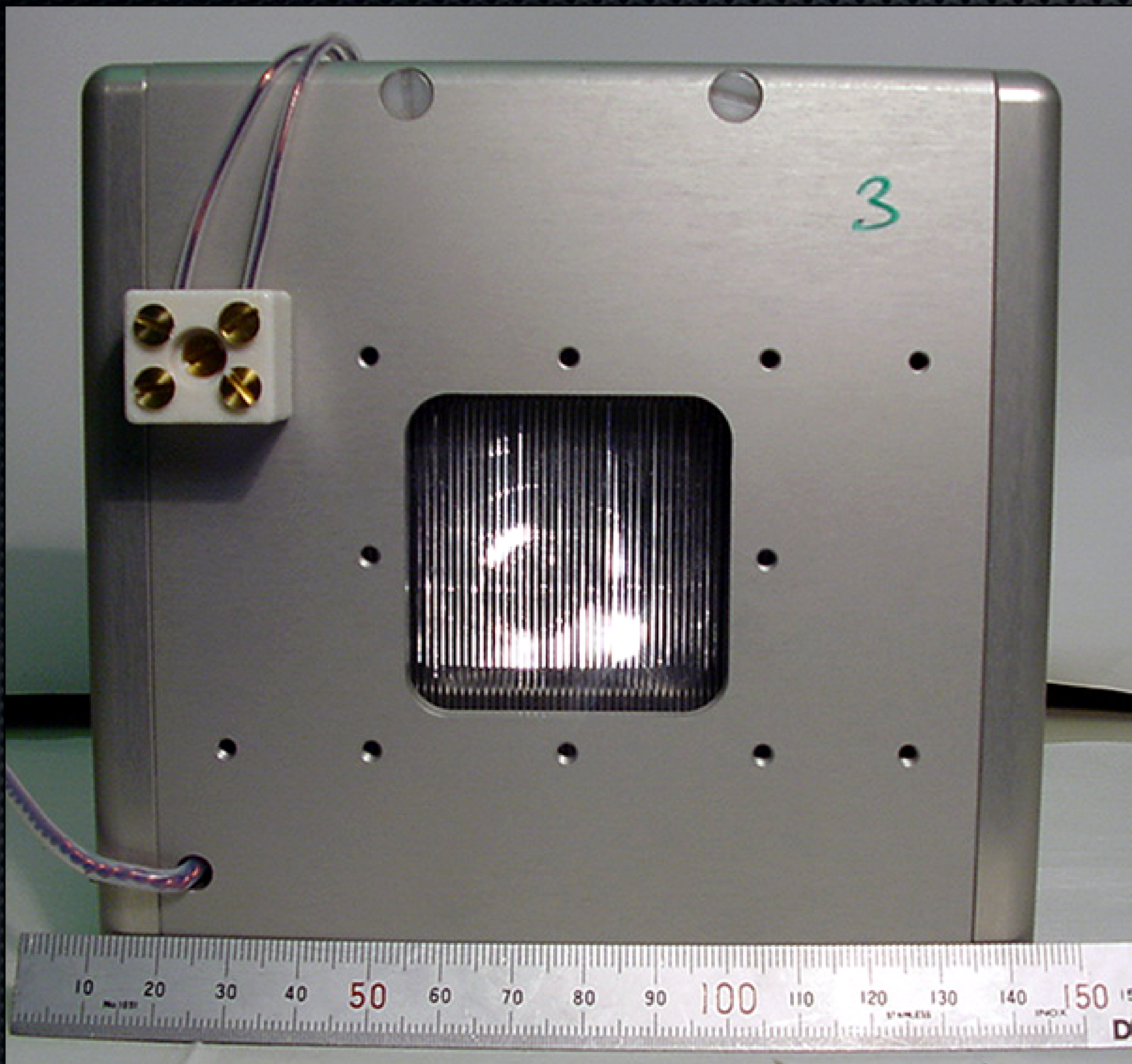


µPAD - PSI & FRM II

µPad in practice



µPad in practice



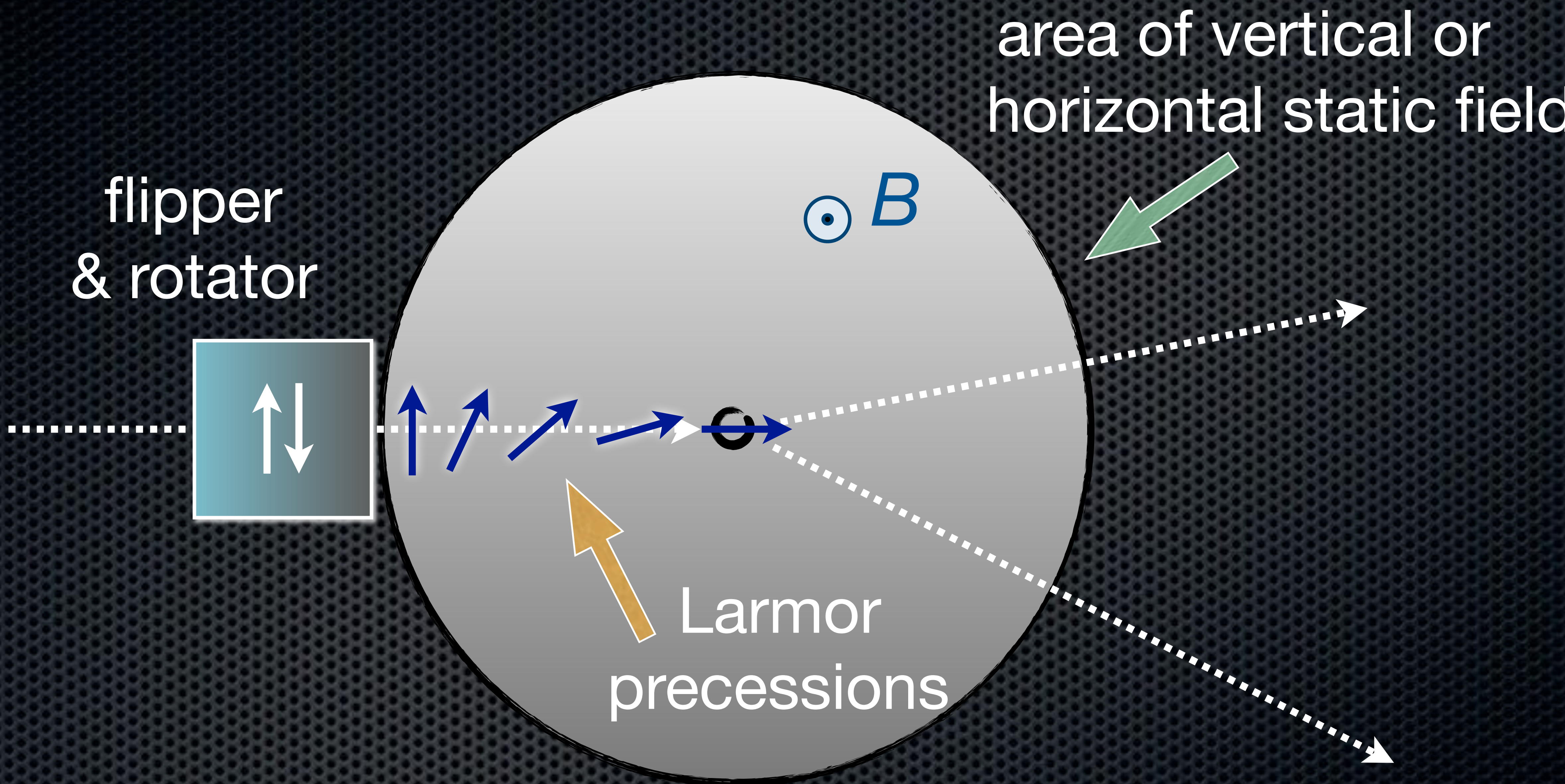
works but…

problem of leakage at high field i.e. for short wavelengths

problem with zero-field chamber at long wavelength because of field environment

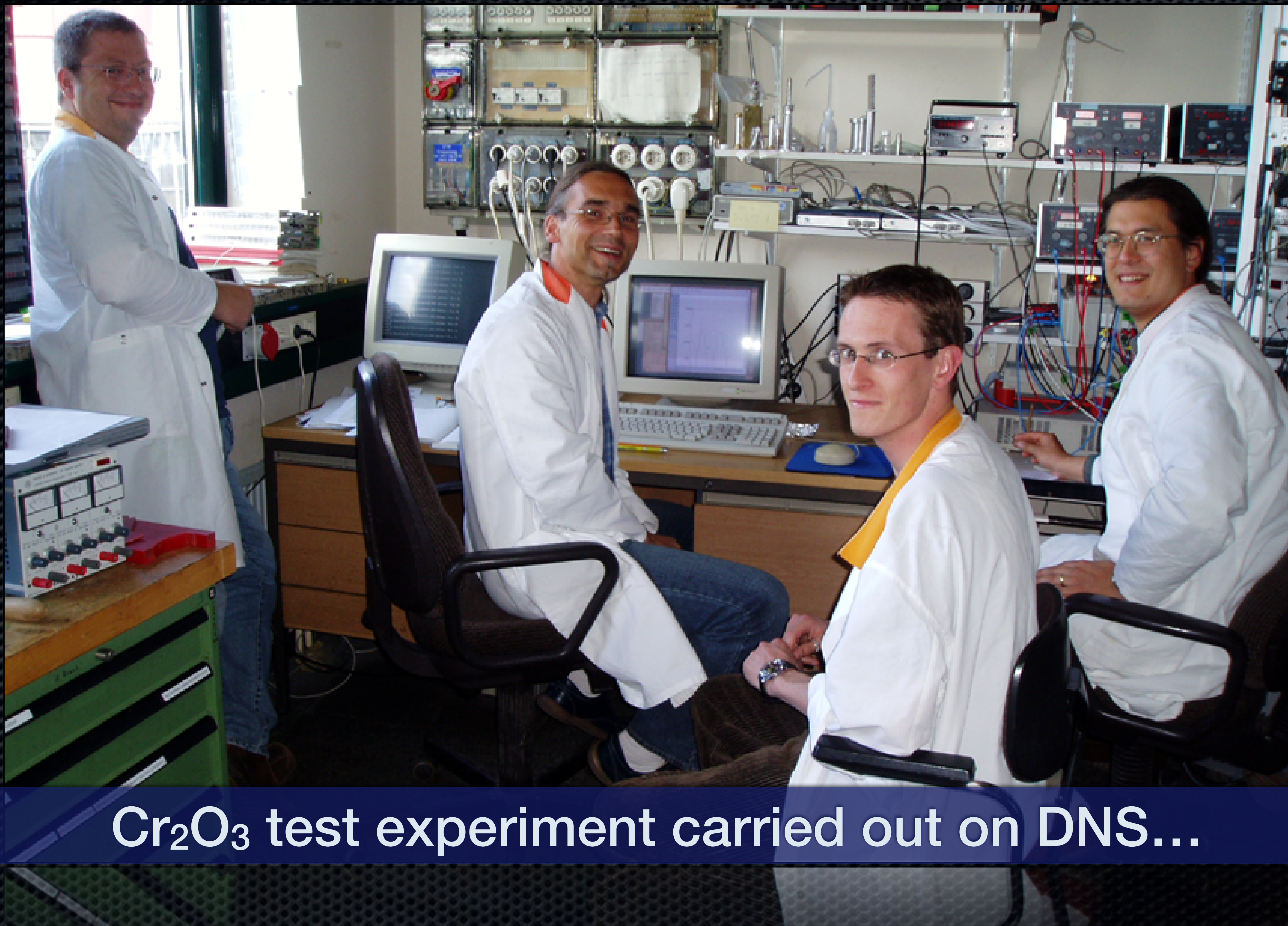
µ-metal "pumps" external fields

dnsPad in practice



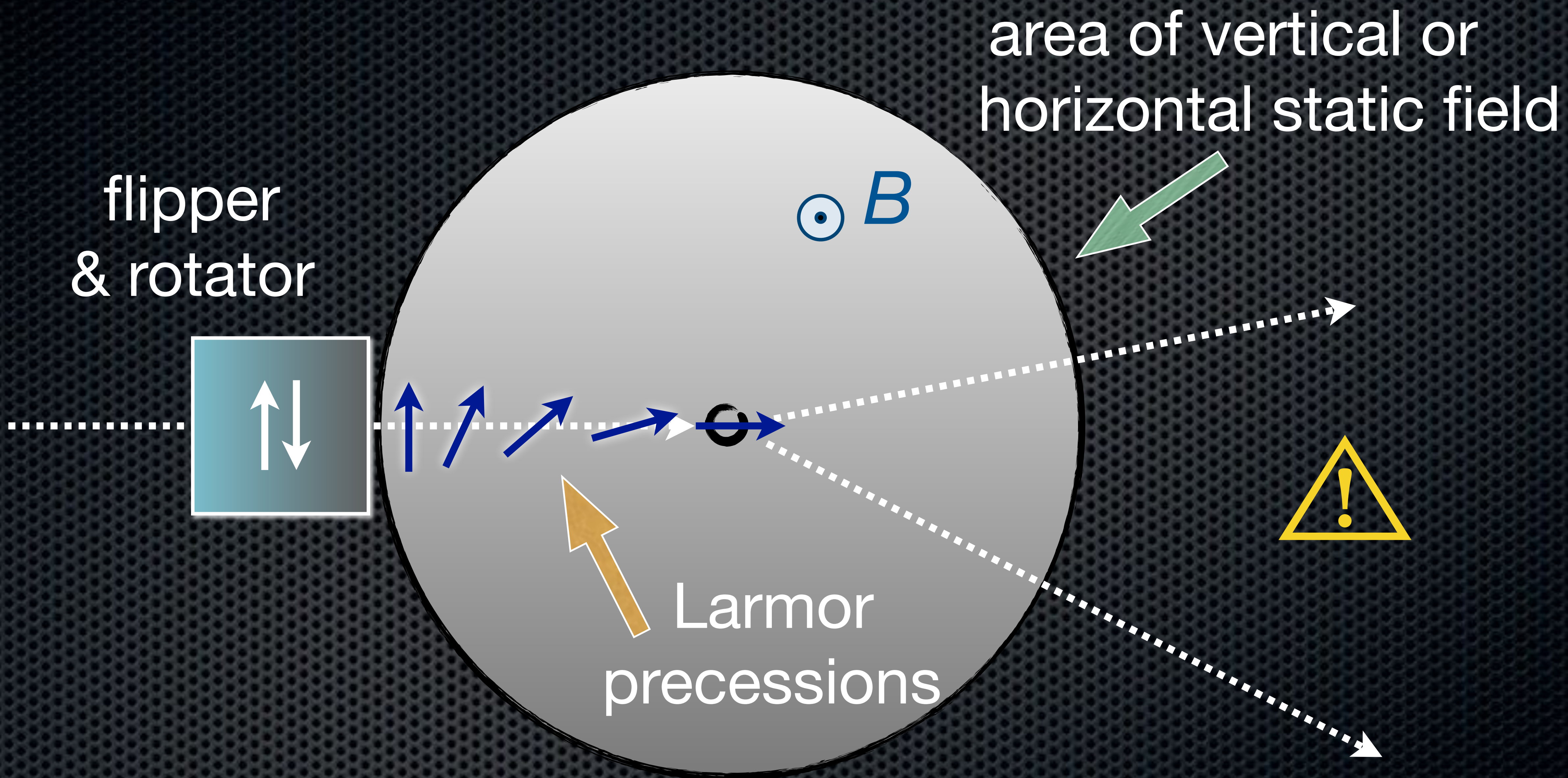
The incident direction of polarisation is controlled with the field applied around the sample area.

dnsPad in practice



Cr_2O_3 test experiment carried out on DNS…

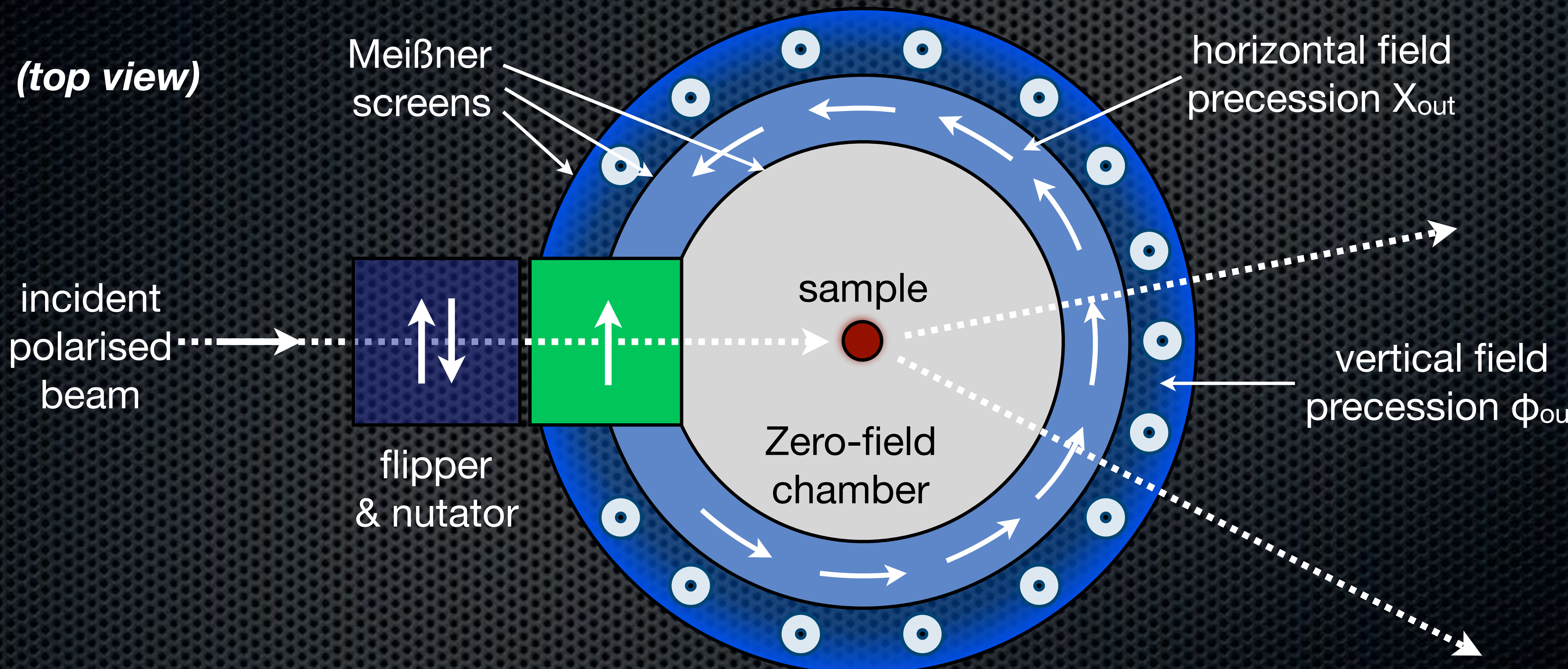
dnsPad in practice



The applied field decreases the resolution with which the orientation of the polarisation is set.

Spherical neutron polarimetry

in Time of Flight mode?



Solution proposed with a ^{3}He spin filter as analyser…

Spherical Neutron Polarimetry

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Some simple examples

- Procedure

1) Calculate the magnetic structure vectors analytically
2) Choose an orientation of the single crystal
3) Identify the families of Bragg reflections
4) Reduce your system of equations (using extinctions)
5) Select Bragg reflections of real interest
6) Measure

Some simple examples

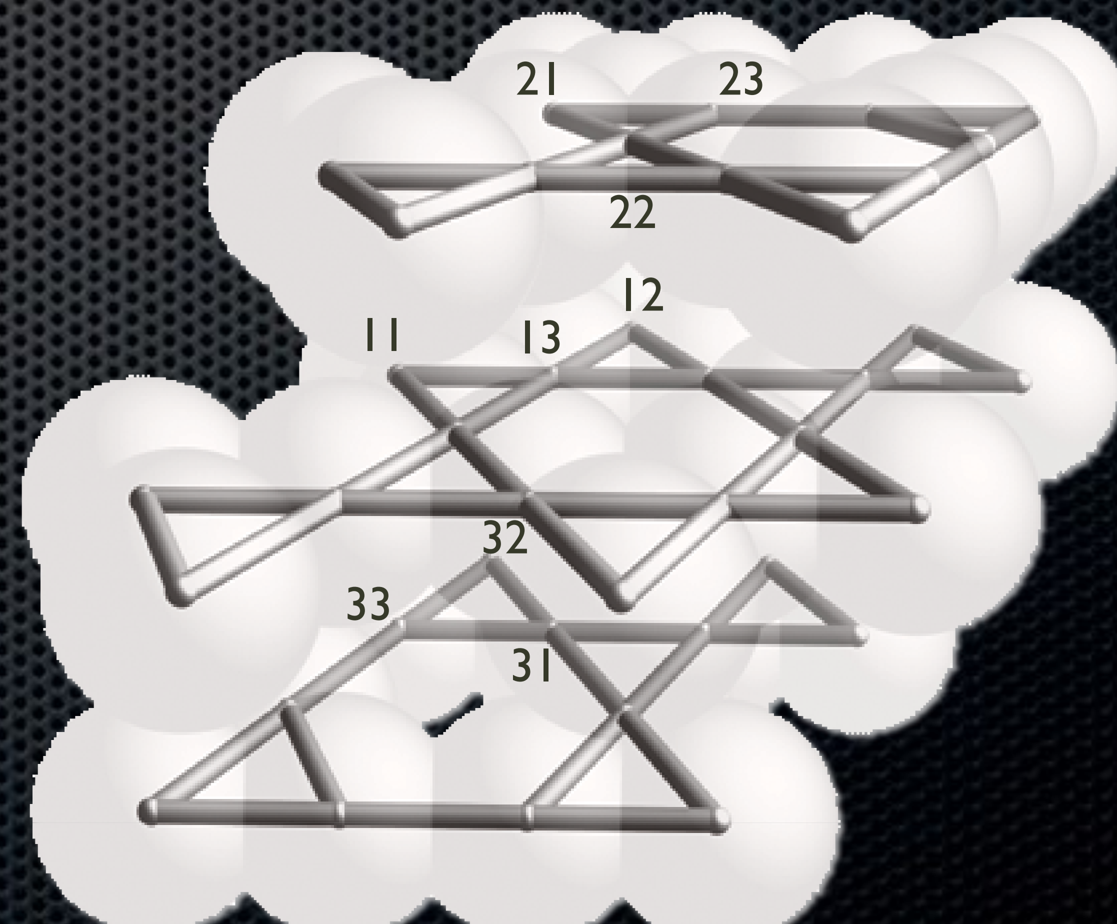
AF single crystals: $\tau \neq 0$

$KFe_3(SO_4)_2(OH)_6$ is a model Kagomé antiferromagnet with which to study the behaviour of highly frustrated systems.

The magnetic Fe atoms occupy one crystallographic site 9(d) and are distributed in 3 Kagomé planes.

space group $R\bar{3}m$

Fe atoms in $(\frac{1}{2}, 0, \frac{1}{2})$



Some simple examples

AF single crystals: τ ≠ 0

2 magnetic arrangements have been proposed from (unpolarised) powder diffraction experiments.

5 families of reflections are forbidden (extinct) :

$$\vec{M}(0,0,\tfrac{3}{2}+3\ell)=0 \quad \vec{M}(0,1,\tfrac{3}{2}+3\ell)=0 \quad \vec{M}(1,0,\tfrac{3}{2}+3\ell)=0$$

$$\vec{M}(1,1,\tfrac{5}{2}+3\ell)=0 \quad \vec{M}(1,1,\tfrac{1}{2}+3\ell)=0$$

so we deduce :

$$-3i(\vec{m}_{11}+\vec{m}_{12}+\vec{m}_{13})=0$$

$$i\vec{m}_{11}-i\vec{m}_{12}+i\vec{m}_{13}+e^{\frac{5i\pi}{6}}\vec{m}_{21}+e^{-\frac{i\pi}{6}}\vec{m}_{22}+e^{\frac{5i\pi}{6}}\vec{m}_{23}+e^{\frac{i\pi}{6}}\vec{m}_{31}+e^{-\frac{5i\pi}{6}}\vec{m}_{32}+e^{\frac{i\pi}{6}}\vec{m}_{33}=0$$

$$-i\vec{m}_{11}+i\vec{m}_{12}+i\vec{m}_{13}+e^{-\frac{5i\pi}{6}}\vec{m}_{21}+e^{\frac{i\pi}{6}}\vec{m}_{22}+e^{\frac{i\pi}{6}}\vec{m}_{23}+e^{-\frac{i\pi}{6}}\vec{m}_{31}+e^{\frac{5i\pi}{6}}\vec{m}_{32}+e^{\frac{5i\pi}{6}}\vec{m}_{33}=0$$

$$-i\vec{m}_{11}-i\vec{m}_{12}+i\vec{m}_{13}+e^{-\frac{i\pi}{6}}\vec{m}_{21}+e^{-\frac{i\pi}{6}}\vec{m}_{22}+e^{\frac{5i\pi}{6}}\vec{m}_{23}+e^{-\frac{5i\pi}{6}}\vec{m}_{31}+e^{-\frac{5i\pi}{6}}\vec{m}_{32}+e^{\frac{i\pi}{6}}\vec{m}_{33}=0$$

$$-i\vec{m}_{11}-i\vec{m}_{12}+i\vec{m}_{13}+e^{-\frac{5i\pi}{6}}\vec{m}_{21}+e^{-\frac{5i\pi}{6}}\vec{m}_{22}+e^{\frac{i\pi}{6}}\vec{m}_{23}+e^{-\frac{i\pi}{6}}\vec{m}_{31}+e^{-\frac{i\pi}{6}}\vec{m}_{32}+e^{\frac{5i\pi}{6}}\vec{m}_{33}=0$$

Some simple examples

AF single crystals: τ ≠ 0

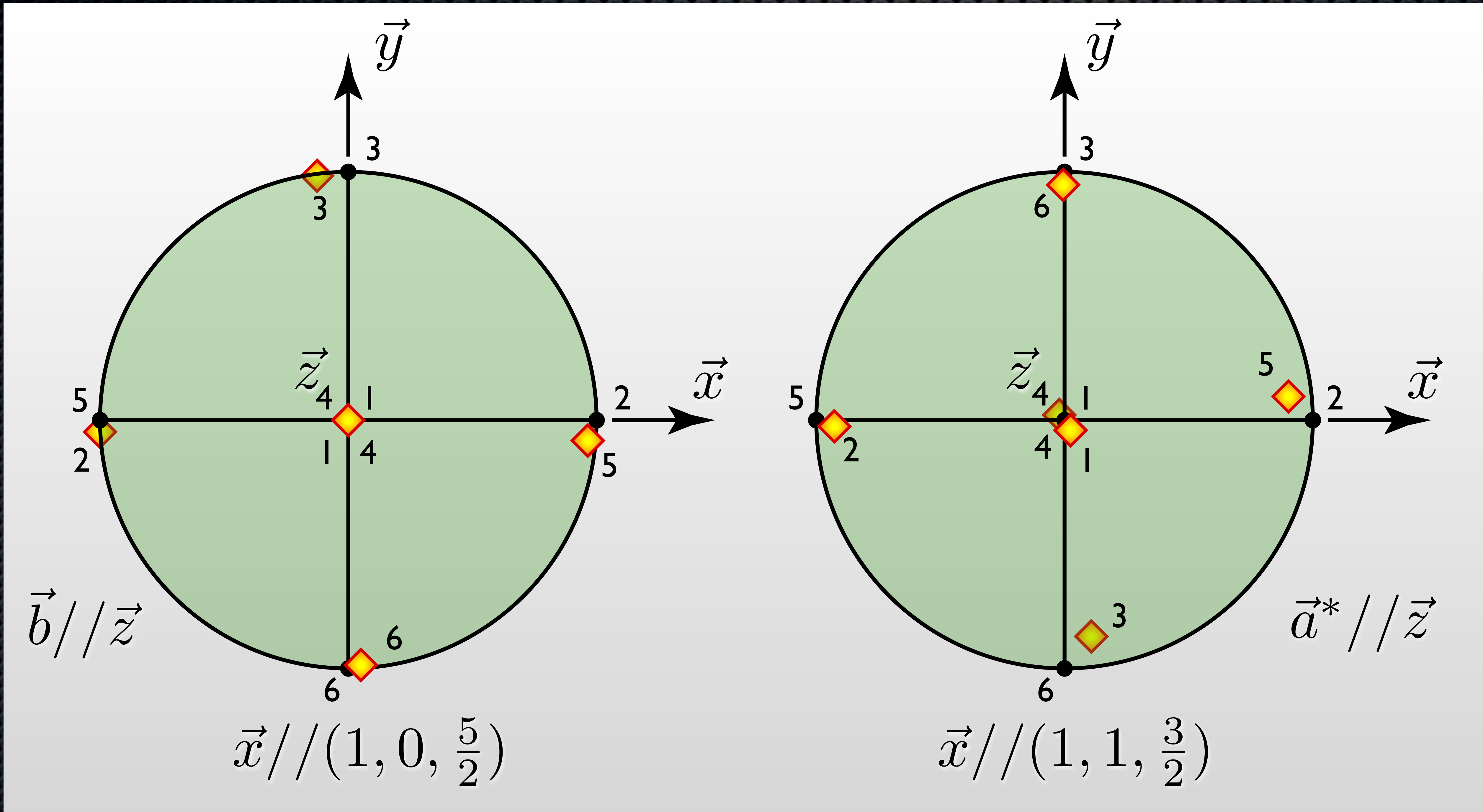
$$
\begin{cases}
\vec{m}_{23} = \vec{m}_{33} \\
\vec{m}_{13} = -\vec{m}_{11} - \vec{m}_{12} \\
-2\vec{m}_{13} = \vec{m}_{23} + \vec{m}_{33} \\
\vec{m}_{21} - \vec{m}_{22} = \vec{m}_{31} - \vec{m}_{32} \\
\vec{m}_{21} + \vec{m}_{22} - \vec{m}_{23} = \vec{m}_{31} + \vec{m}_{32} - \vec{m}_{33} \\
-2\left(\vec{m}_{11} - \vec{m}_{12}\right) = \left(\vec{m}_{21} - \vec{m}_{22}\right) + \left(\vec{m}_{31} - \vec{m}_{32}\right) \\
-2\left(\vec{m}_{11} + \vec{m}_{12} - \vec{m}_{13}\right) = \left(\vec{m}_{21} + \vec{m}_{22} - \vec{m}_{23}\right) + \left(\vec{m}_{31} + \vec{m}_{32} - \vec{m}_{33}\right)
\end{cases}
$$

⬇

$$
\begin{cases}
\vec{m}_{13} = -\vec{m}_{11} - \vec{m}_{12} \\
\vec{m}_{21} = -\vec{m}_{11}, \vec{m}_{22} = -\vec{m}_{12}, \vec{m}_{23} = \vec{m}_{11} + \vec{m}_{12} \\
\vec{m}_{31} = -\vec{m}_{11}, \vec{m}_{32} = -\vec{m}_{12}, \vec{m}_{33} = \vec{m}_{11} + \vec{m}_{12}
\end{cases}
$$

Some simple examples

AF single crystals: $\tau \neq 0$



Some simple examples

AF single crystals: τ ≠ 0

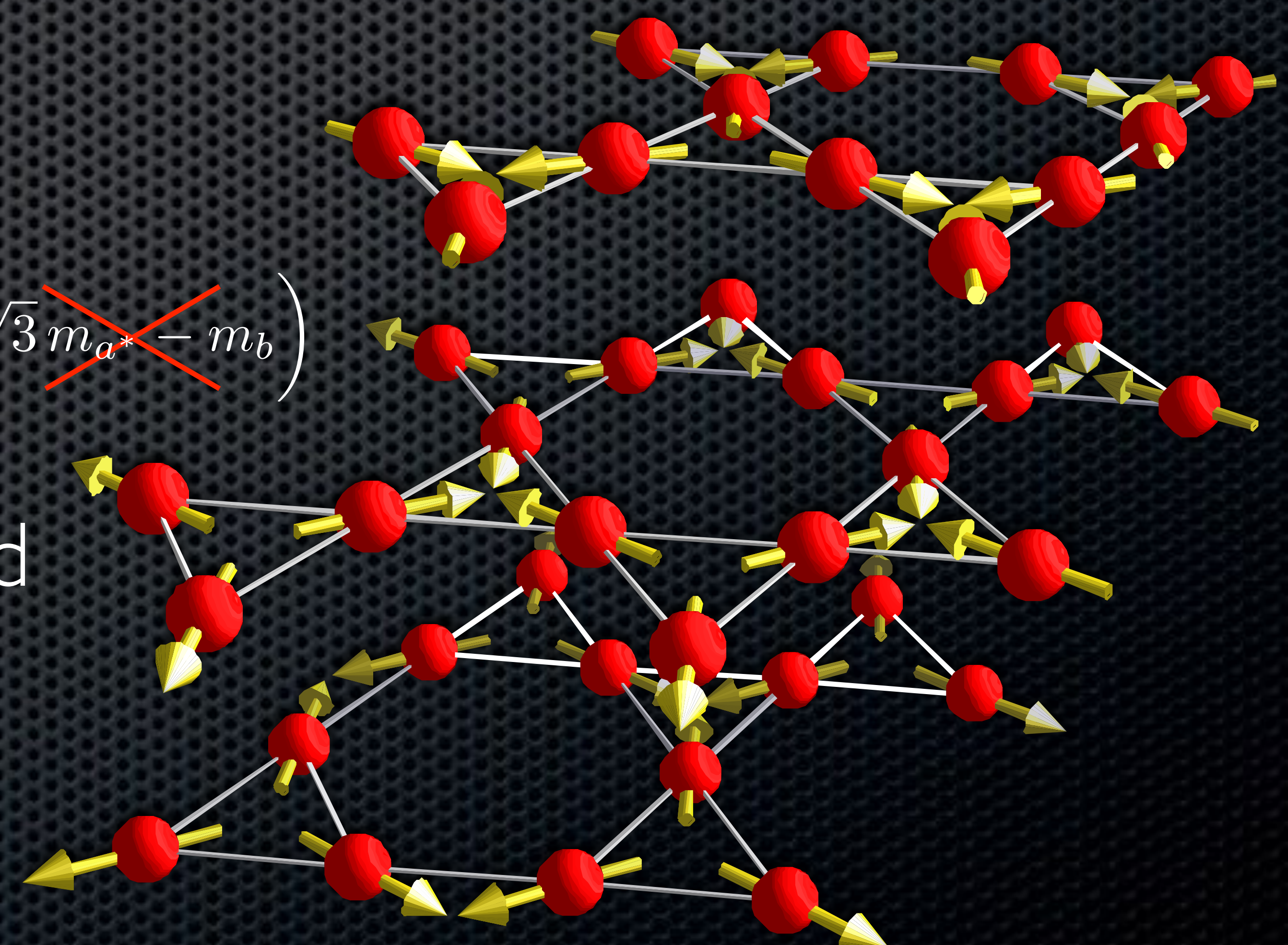
- No depolarisation of the beam : there is no magnetic domains and therefore no expected component along $\vec{c}$.

- The polarisation rotates around $\vec{y}$ for the reflection [1 0 5/2] :

$$\vec{\mathcal{M}}_{\perp,10\frac{5}{2}} \propto \left(0, -\frac{1}{\sqrt{27}}\left(5m_{a^*} + 5\sqrt{3}\,m_b + 4m_{c^*}\right), \sqrt{3}\,m_{a^*} - m_b\right)$$

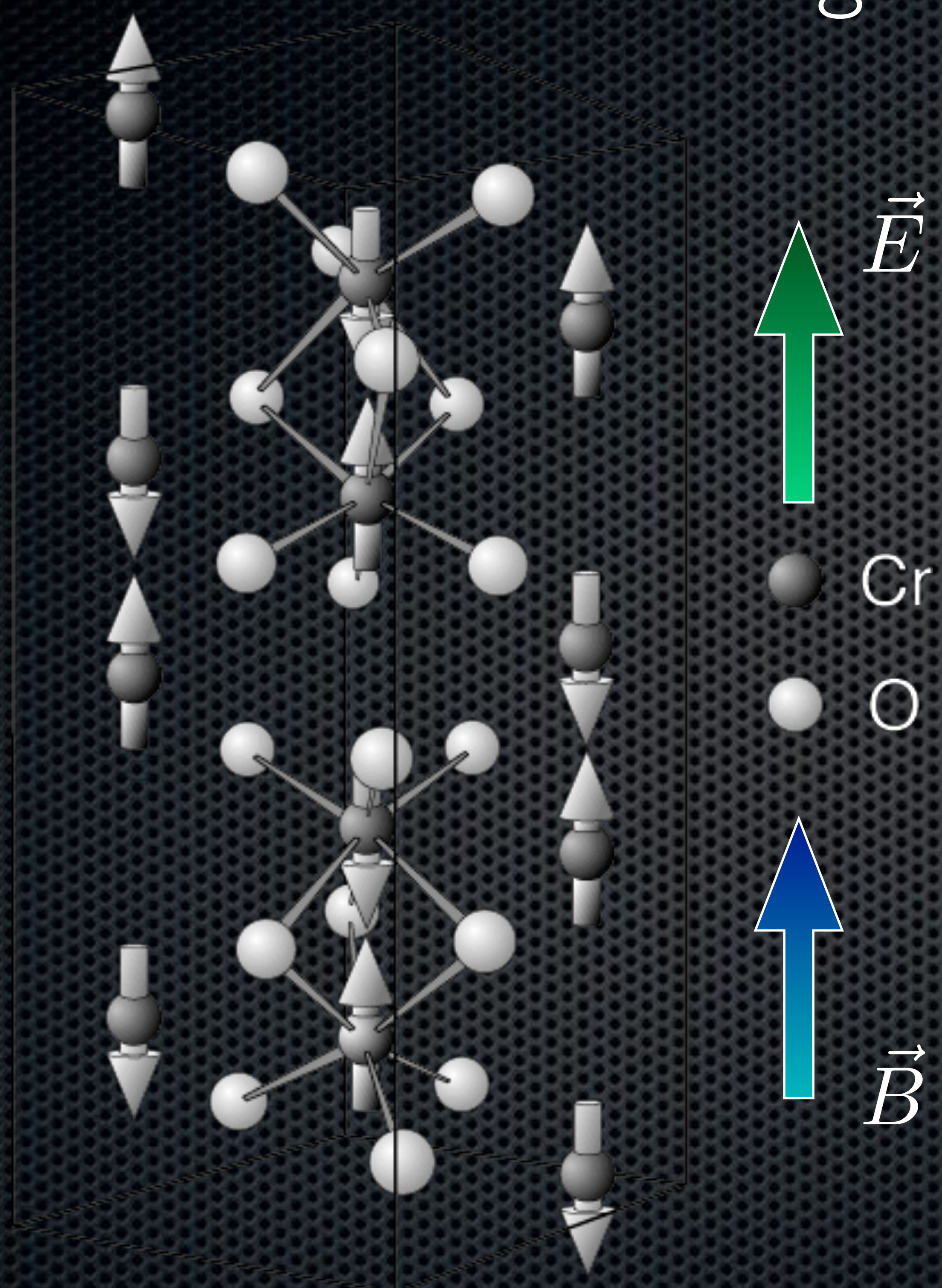
- The polarisation rotates around $\vec{z}$ for the reflection [1 1 3/2] :

$$\vec{\mathcal{M}}_{\perp,11\frac{3}{2}} \propto \left(0, -\frac{2}{\sqrt{13}}m_{c^*}, m_{a^*}\right)$$



Some simple examples

AF single crystals: $\tau = 0$

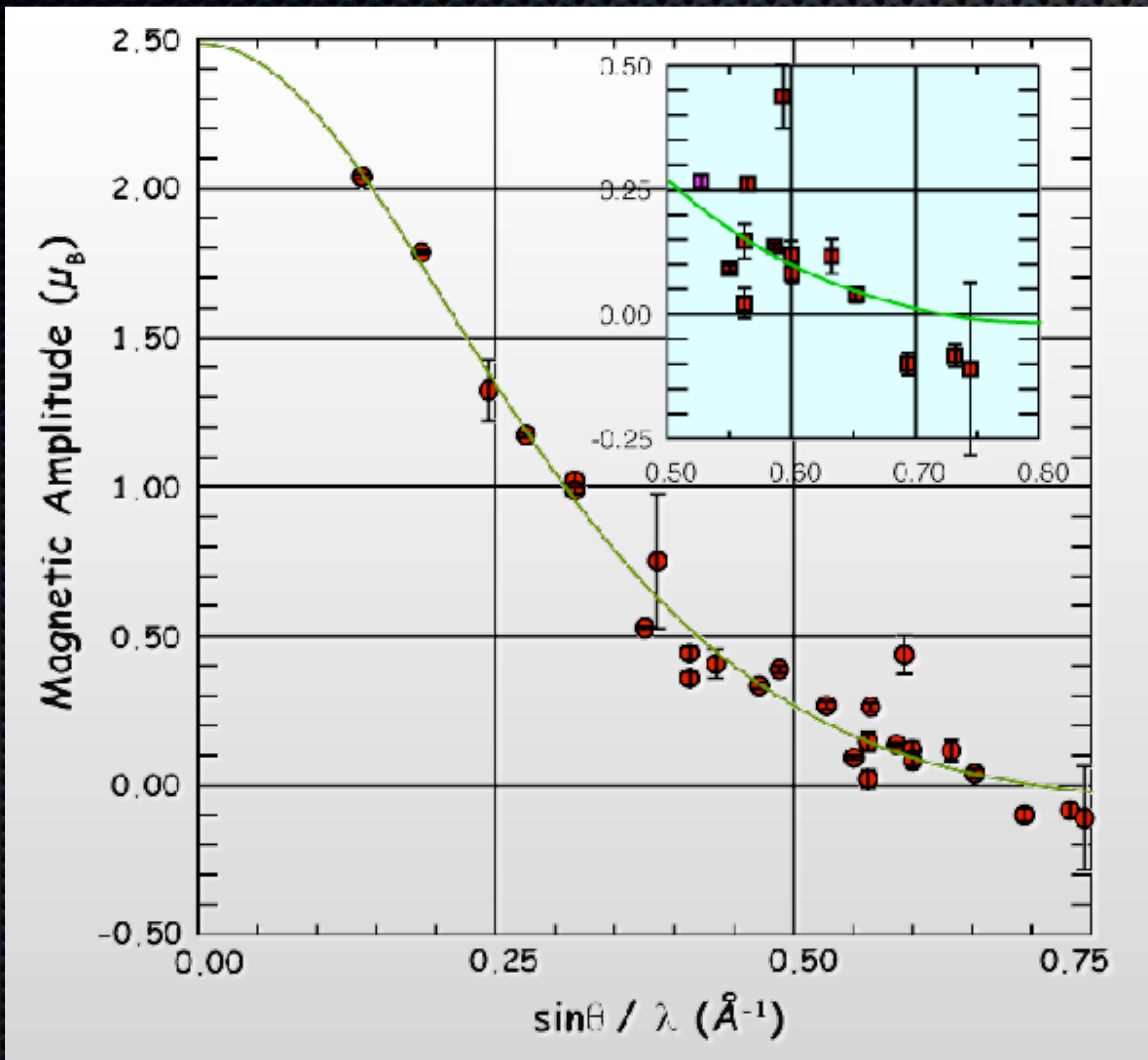


- Cr_2O_3 is a magneto-electric compound belonging to the class of antiferromagnets in which nuclear and magnetic scattering occur in the same reflections and are in quadrature :

$$N(\vec{Q}) \in \Re, \vec{M}(\vec{Q}) \in \Im^3$$

Some simple examples

AF single crystals: $\tau = 0$



The polarisation rotates around the magnetic interaction vector:

$$\vec{P}_f = Rot(\vec{O}y, \alpha) \cdot \vec{P}_i$$

$$\frac{||\vec{M}_\perp||}{|N|} = \sqrt{\frac{1 - \cos(\alpha)}{1 + \cos(\alpha)}}$$

$$\vec{O}x // \vec{Q}, \vec{O}z \text{ vertical}$$

$$\vec{O}y = \vec{O}z \wedge \vec{O}x$$

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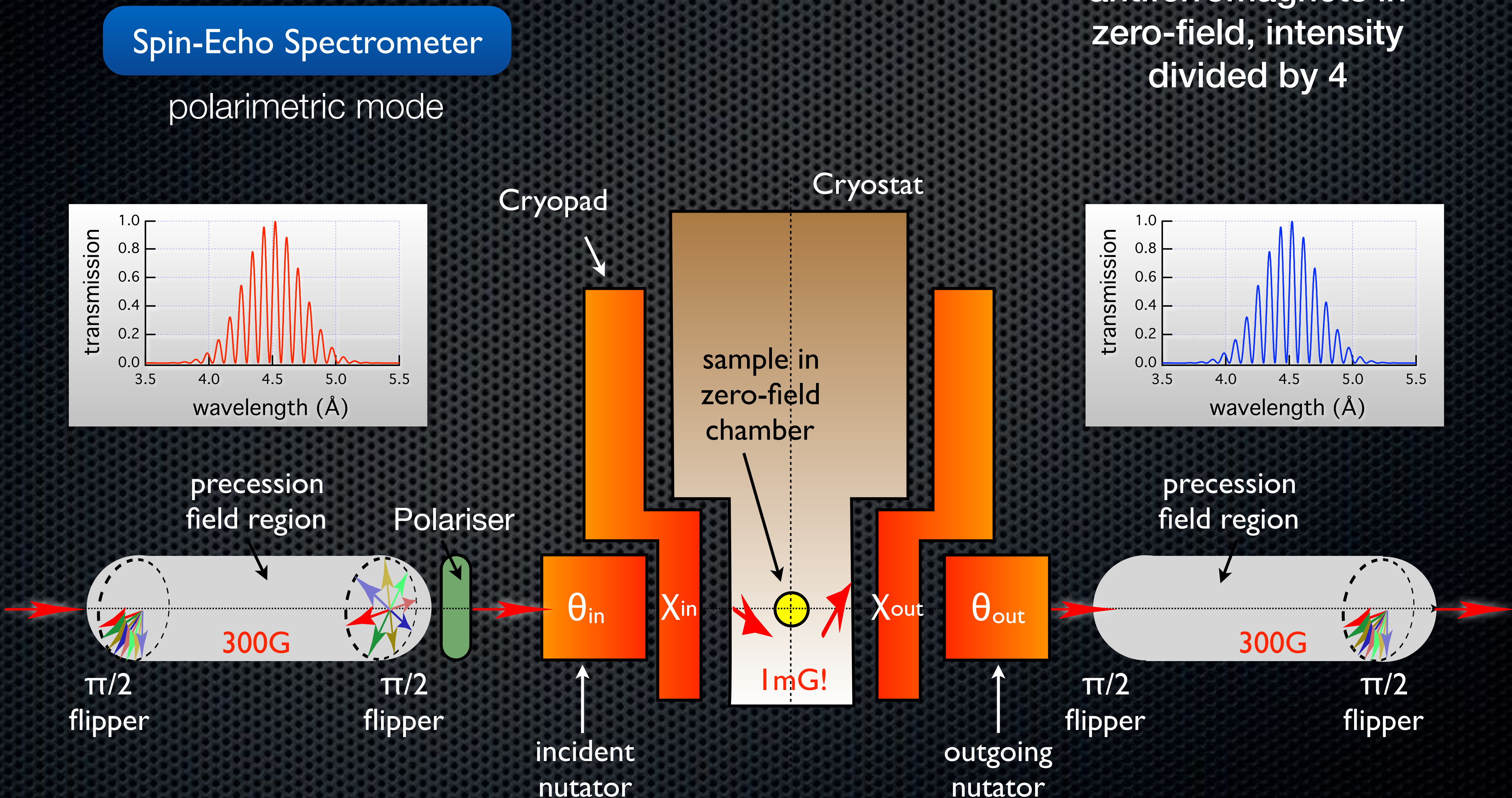
Polarimetric Neutron Spin Echo

antiferromagnets in zero-field, intensity divided by 4



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Many thanks
for your attention

and to the colleagues and friends who contributed to these works.