HZB Helmholtz Zentrum Berlin

Semi-Classical View of Larmor Precession

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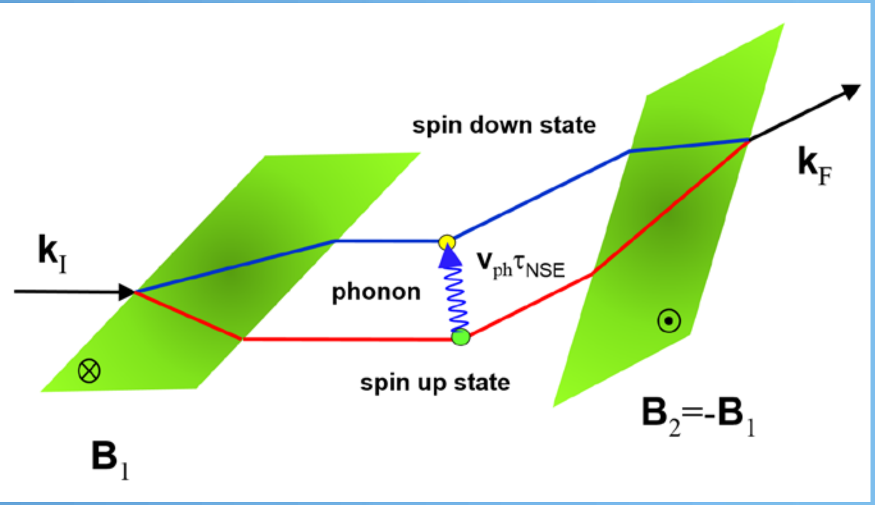

Key Question(s)

Is a semi-classical ray-tracing description of Larmor precession instrumentation useful?

How is Larmor precession mapped into the semi-classical model?

Is there any predictive power to the semi-classical model?

(NOT: new predictions beyond classical Larmor precession model)

Outline

Quantum-Mechanical Description of Larmor Precession

- Magnetic Moments in a Magnetic Field
- Spin ½ Particles
- Velocities of Spin States in a Magnetic Field

Semi-Classical Ray-Tracing Model of Spinor Component Propagation

- Quasi-Elastic Scattering
- SESANS
- Inelastic Scattering
- Elastic Scattering - Larmor Diffraction

MIEZE Larmor Diffraction

- MIEZE with Tilted RF Flippers
- MIEZE Larmor Diffraction

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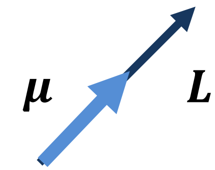
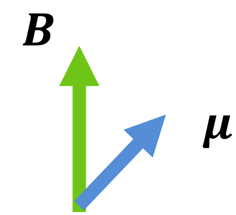
Magnetic Moments in a Magnetic Field

Magnetic moments have associated angular (orbital) momentum $\boldsymbol{L}$:

$$\boldsymbol{\mu} = \frac{e}{2mc}\boldsymbol{L} = \gamma \boldsymbol{L}$$

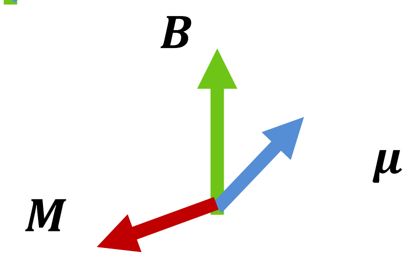


γ: gyromagnetic ratio; electron: 1.76 x 10^{11} rad/s/T



Magnetic moments in a magnetic field have energy:

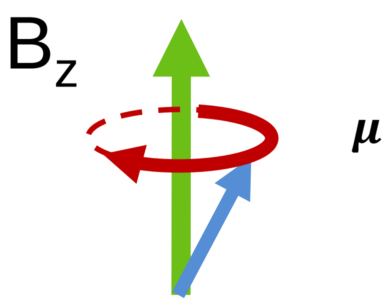
$$H = -\boldsymbol{\mu} \cdot \boldsymbol{B} = -\left(\frac{e}{2mc}\boldsymbol{L}\right) \cdot \boldsymbol{B}$$



Magnetic moments in a magnetic field experience torque :

$$\boldsymbol{M} = \boldsymbol{\mu} \times \boldsymbol{B}$$

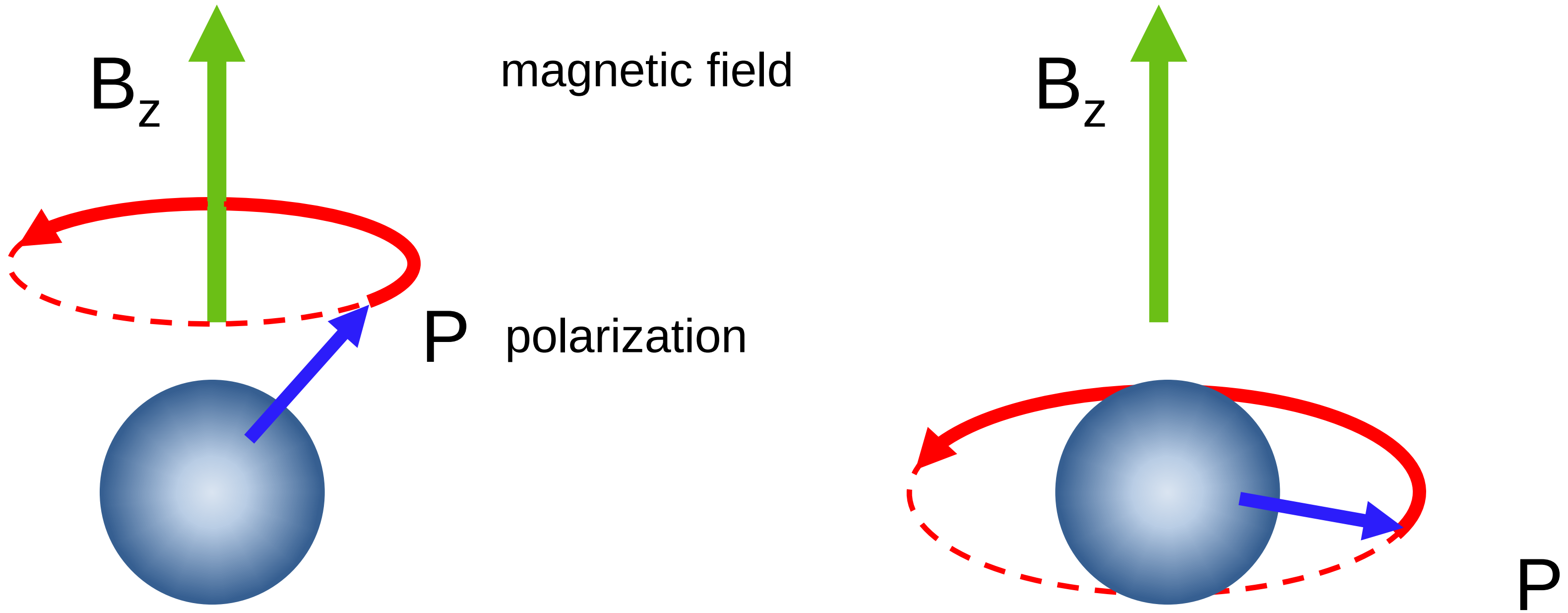
Momentum conservation prevents that magnetic moments align with $\boldsymbol{B}$



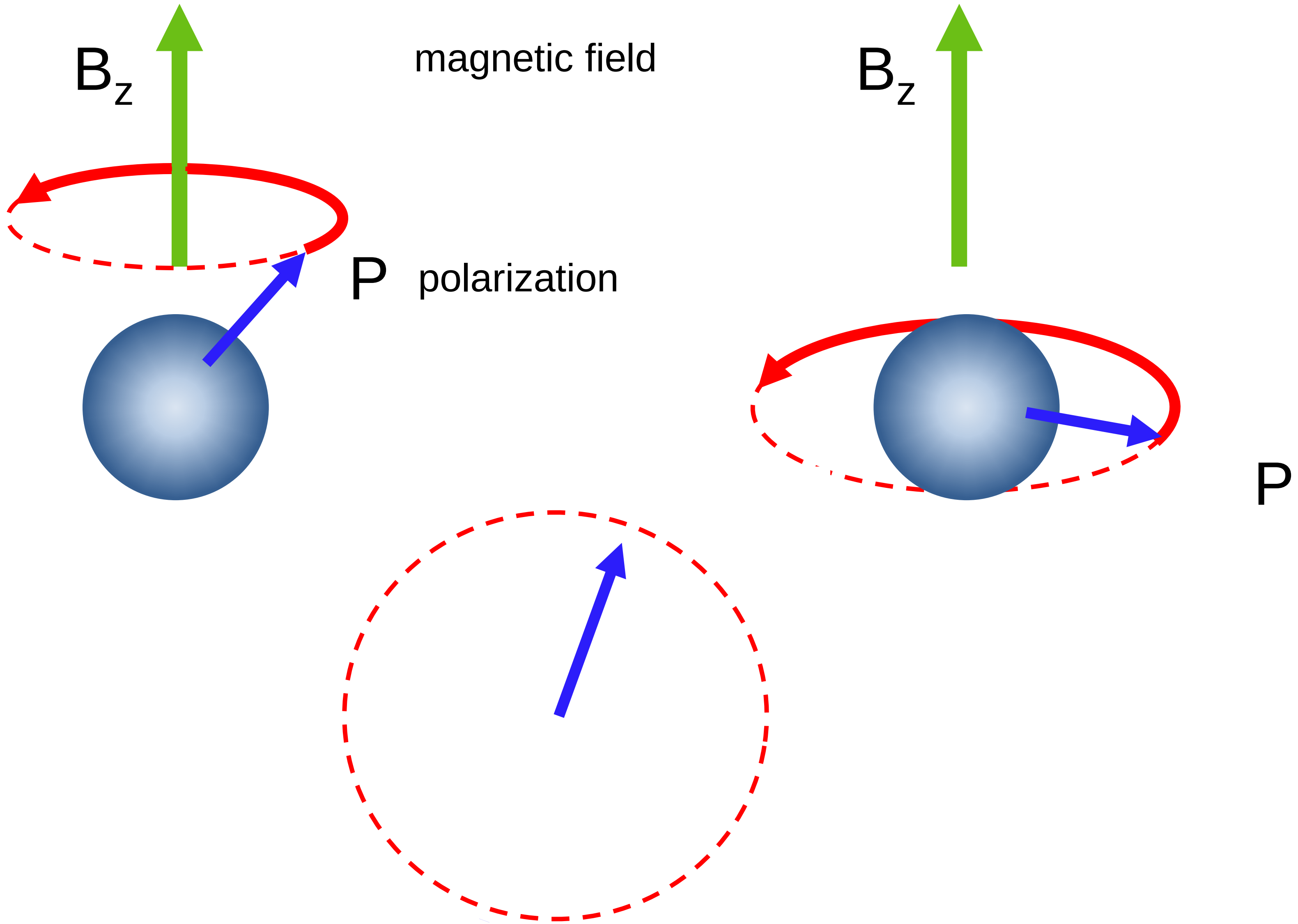
$$\boldsymbol{M} = \frac{d\boldsymbol{L}}{dt}$$

$$\omega_{Larmor} = \frac{d\varphi}{dt}$$ (Larmor precession)

Principle of Neutron Spin-Echo: Larmor Precession



Principle of Neutron Spin-Echo: Larmor Precession



Magnetic Moments in a Magnetic Field

Magnetic moments interacting with a gradient of a magnetic field will experience a force

$$\boldsymbol{F} = \boldsymbol{\nabla}(\boldsymbol{\mu} \cdot \boldsymbol{B}) = \mu_z \frac{\partial B_z}{\partial z} \boldsymbol{e}_z$$

Since

$$\mu_z = \gamma L_z = \gamma \hbar m_l \text{ and } -l \leq m_l \leq +l$$

and quantum number l for angular momentum can only have integer values:

beam splitting into $(2l + 1)$ beams expected !

but Stern-Gerlach experiment (1922) says:
particles can have intrinsic momentum (spin) with quantum number $s = {}^1/_2$

neutron is a spin $s = {}^1/_2$ particle

Spin ½ Particles

Spin has an associated magnetic moment:

$$\boldsymbol{\mu} = \gamma \boldsymbol{S}$$

γ: gyromagnetic ratio; neutron: -183.2 rad MHz/T = -18.32 rad kHz/Gauss

Larmor frequency: $\omega_L = \frac{\gamma S B}{S} = \gamma B$

Space of angular momentum states for spin $s = {}^1/_2$ is 2D $(-s \le m_s \le +s)$:

$$|s, m_s\rangle = \left|\frac{1}{2}, +\frac{1}{2}\right\rangle = |\uparrow\rangle \qquad |s, m_s\rangle = \left|\frac{1}{2}, -\frac{1}{2}\right\rangle = |\downarrow\rangle$$

General state (spinor) is a linear combination of basis set of states:

$$\alpha|\uparrow\rangle + \beta|\downarrow\rangle = \binom{\alpha}{\beta} \qquad \alpha^2 + \beta^2 = 1$$

Quantum state of a spin-1/2 particle is described by a 2-dimensional vector

Pauli Spin Matrices

QM: observables correspond to operators: here: spin operator $\hat{\boldsymbol{S}} = \left(\hat{S}_x, \hat{S}_y, \hat{S}_z\right)$

results of a measurement given by eigenvalues of $\hat{\boldsymbol{S}}$ acting on state (spinor):

$$\hat{\boldsymbol{S}} = \frac{\hbar}{\mathbf{2}}\boldsymbol{\sigma}$$

if the *z*-axis is chosen as quantization axis then

$$\hat{\boldsymbol{S}} = \frac{\hbar}{\mathbf{2}}\boldsymbol{\sigma} = \left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right)$$

$\boldsymbol{\sigma}$: Pauli spin matrices $\boldsymbol{\sigma}$

the states $|\uparrow\rangle_z = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|\downarrow\rangle_z = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are eigenstates of $\hat{S}_z$

the states $|\uparrow\rangle_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $|\downarrow\rangle_x = \frac{1}{\sqrt{2}}\begin{pmatrix} +1 \\ -1 \end{pmatrix}$ are eigenstates of $\hat{S}_x$

quantization axis is field direction (*z*-direction) !

a state prepared to initially be “polarized“ along the x direction is $|\uparrow\rangle_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Velocities of Spin States in a Magnetic Field

quantization axis is field direction (*z*-direction) $B_z = B_1$!

a state prepared to initially be “polarized“ along the x direction is $|\uparrow\rangle_x = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

This is one state !

In a measurement half of the beam corresponds to spin up components,
half of the beam corresponds to spin down components
in the Larmor precession apparatus there is however no measurement
and spin up and spin down states are not eigenstates!

Components of the wavefunction have different potential energies $E_{pot} = \mp\frac{\hbar}{2}\gamma B_1$

By applying magnetic field total energy is conserved

kinetic energy of the components must change leading to a relative phase shift
which can be visualized as classical trajectories:
spin down component decelerates, spin up componente accelerates!

$$v_{\pm} = v_1 \pm \frac{\hbar\omega_z}{mv_1} \qquad \omega_z = \frac{\mu B_1}{\hbar} = \frac{1}{2}\omega_1$$

Phases of Spin Components in a Magnetic Field

Phase of a plane wave:

$$\varphi = kx - \omega t$$

different spinor components have different phases ***when leaving the precession field region in front of the sample***

$$\varphi_{1\pm} = k_\pm L_1 - \frac{E_1}{\hbar} t_\pm$$

with

$$k_\pm = \frac{m}{\hbar} v_\pm \qquad t_\pm = \frac{L_1}{v_\pm} = \frac{L_1}{v_1} \mp \frac{\hbar \omega_z L_1}{m v_1^3} = t_1 \mp \frac{\tau_{NSE}}{2}$$

$$\varphi_{1\pm} = k_1 L_1 \pm \omega_z \frac{L_1}{v_1} - \frac{E_1}{\hbar}\left(t_1 \mp \frac{\tau_{NSE}}{2}\right)$$

different spinor components have different phases at the ***position of the sample***

$$t_{S\pm} = \frac{L_1}{v_\pm} + \frac{L_S}{v_1} \qquad \Delta t_S = t_{S-} - t_{S+} = \frac{\hbar \omega_1 L_1}{m v_1^3} = \tau_{NSE}$$

$$\Delta \varphi_S = \frac{2\omega_z L_1}{v_1} + \frac{E_1}{\hbar} \tau_{NSE}$$

Phase Difference and Larmor Precession Angle

phase difference

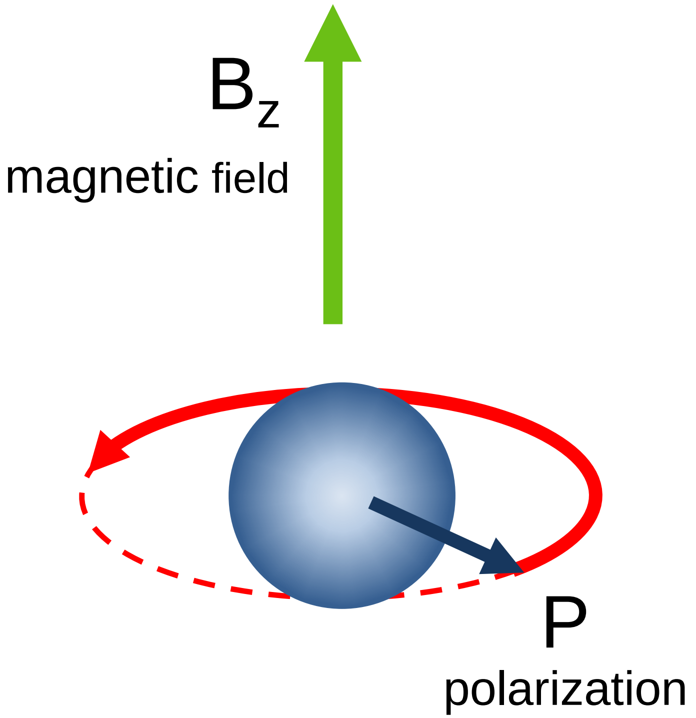
$$\Delta\varphi_S = \frac{2\omega_z L_1}{v_1} + \frac{E_1}{\hbar}\tau_{NSE}$$

the classical Larmor precession angle

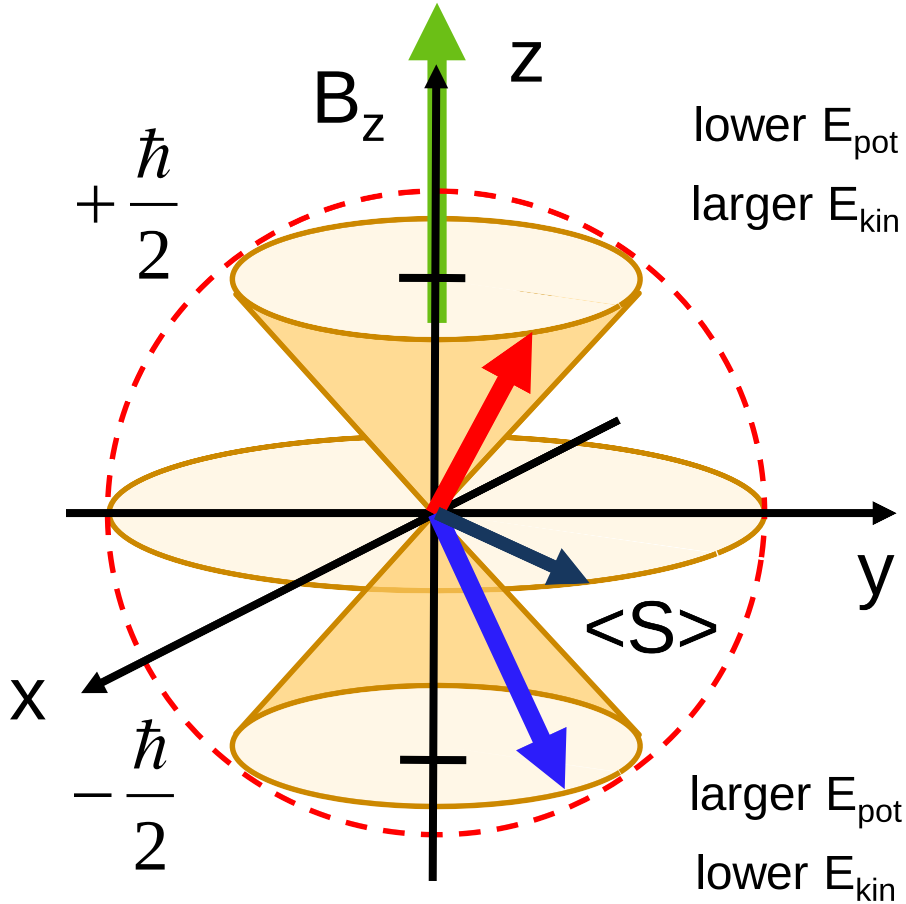
spin echo time dependence

Spin Precession of Neutrons

classical mechanics



quantum mechanics



In B-field: spin up state speeds up; spin down state slows down

Some Remarks

1) The semi-classical model shows phase differences in a graphical way.

2) The semi-classical model is based on plane wave states mapped to classical trajectories.
   This leads to a self-contradiction:
   a plane wave is infinitely extended in space;
   a classical trajectory is by definition well defined in space.

3) Phases are not observables.

4) No new physics by the semi-classical model which you could not obtain with classical Larmor precessing point-like particles

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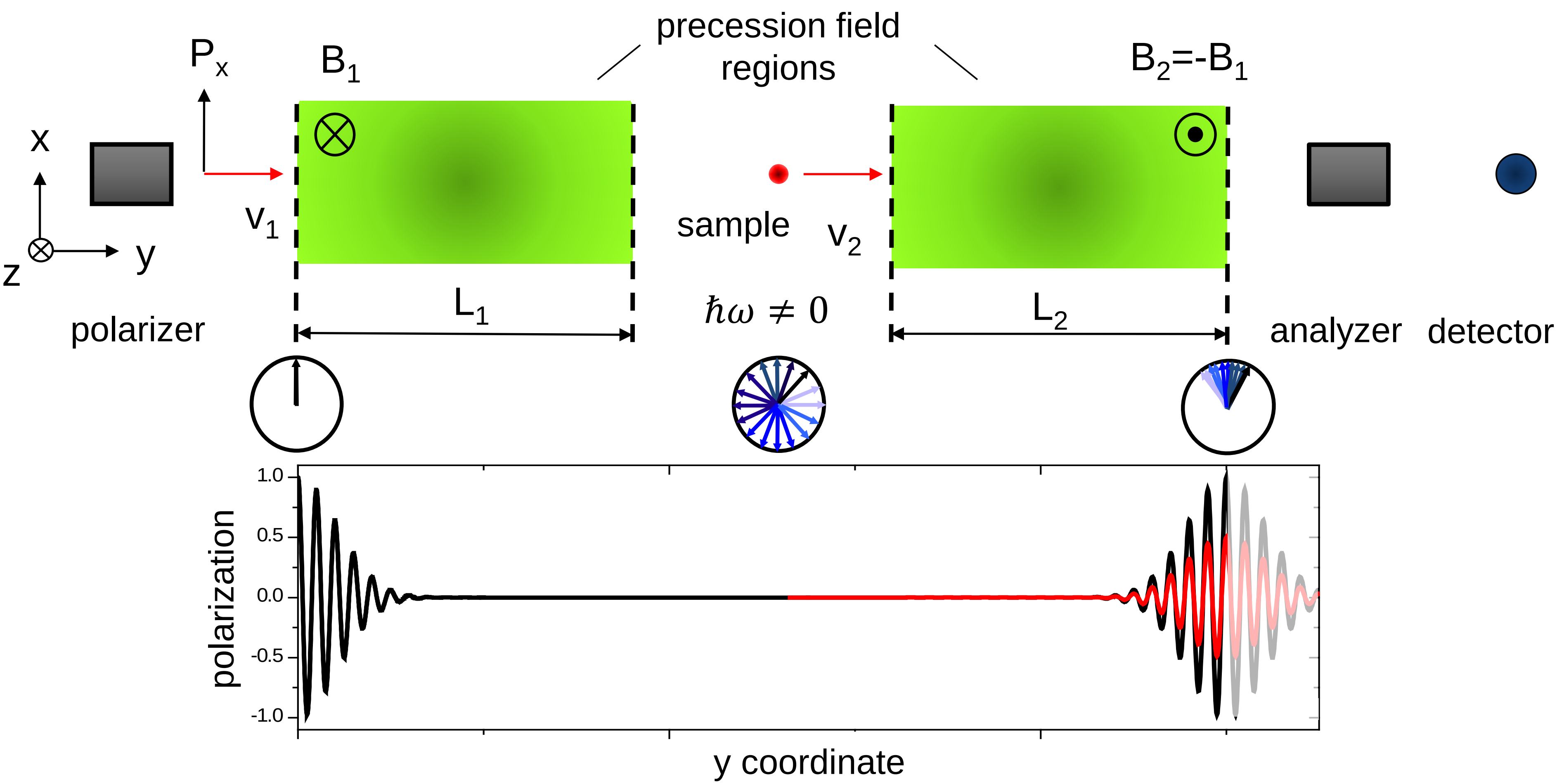
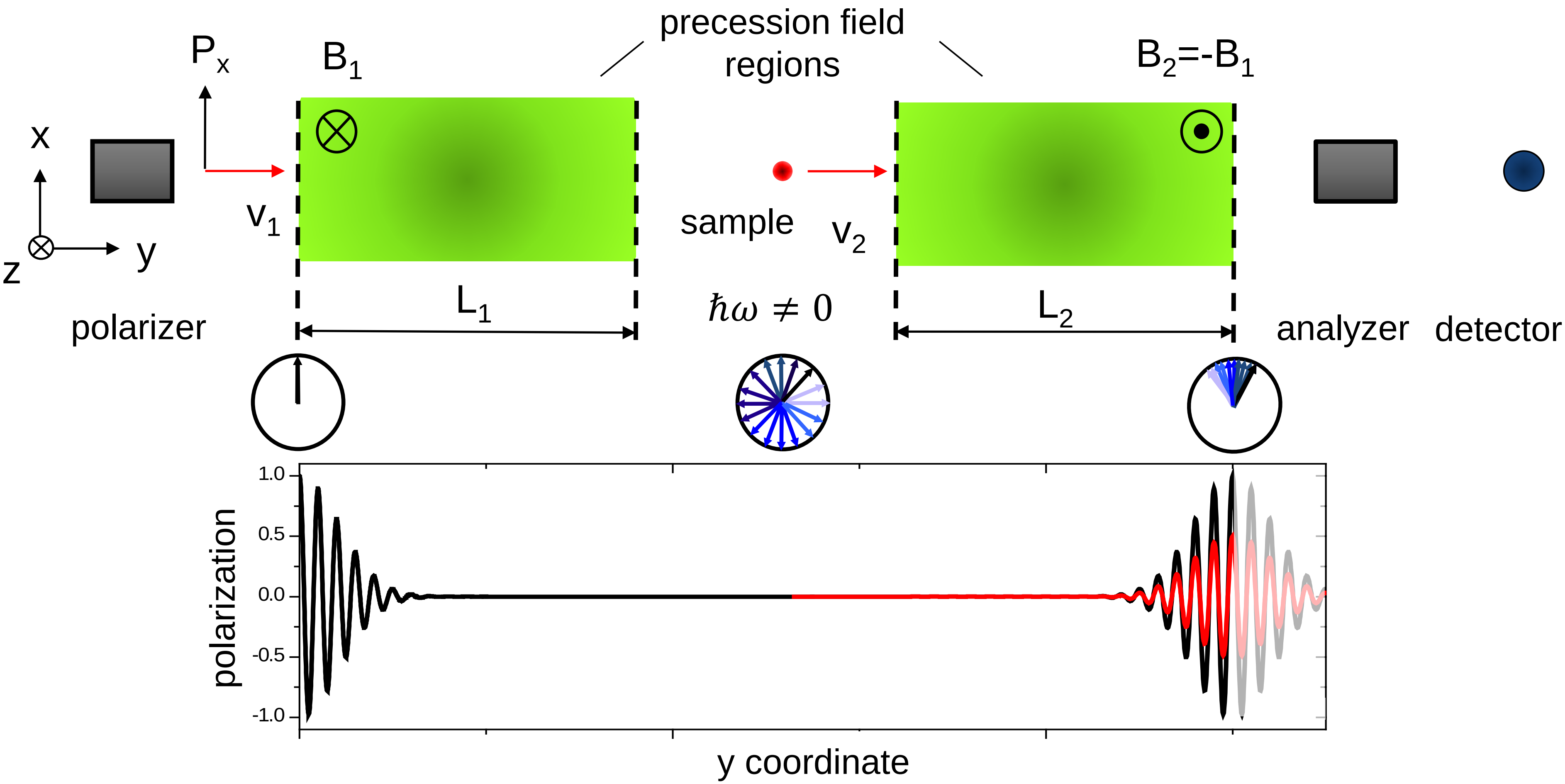
Quasi-elastic NSE: Classical Larmor Precession



Quasi-elastic NSE: Classical Larmor Precession



total phase at echo point

$$\phi = -\tau_{NSE} \frac{\Delta E}{\hbar}$$

NSE polarization

$$P(\tau_{NSE}) \propto \int S(\mathbf{Q}, \Delta\omega) \cos(\tau_{NSE} \Delta\omega) d\Delta\omega$$

Semi-Classical Ray-Tracing View of NSE

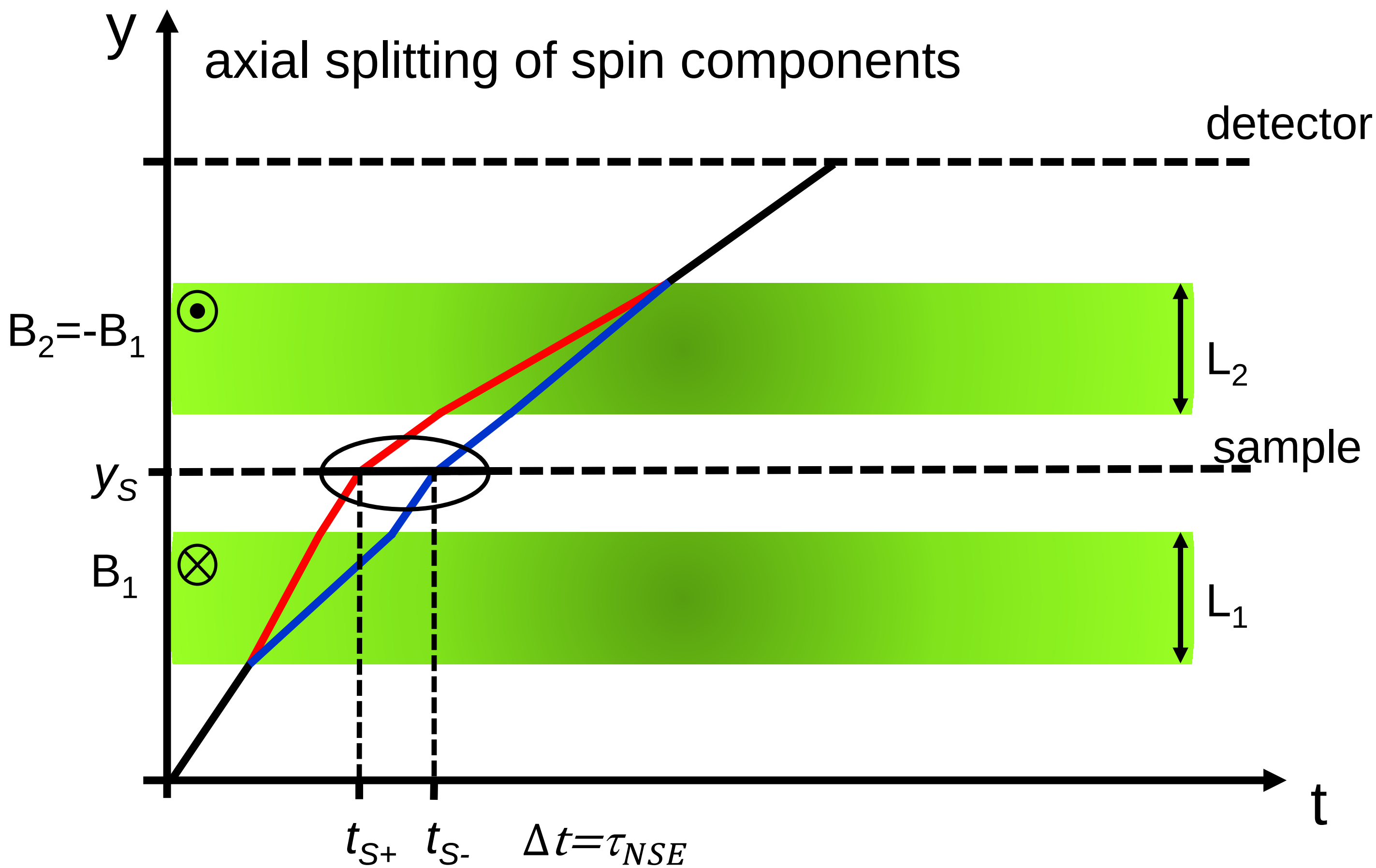
two spin components … with different velocities …. determined by the magnetic field

$$|+\rangle_x = \frac{1}{\sqrt{2}}\left(|+\rangle_z + |-\rangle_z\right) \qquad v_\pm = v_1 \pm \frac{\hbar\omega_z}{m v_1} \qquad \omega_z = \frac{\mu B_1}{\hbar}$$

Semi-Classical Ray-Tracing View of NSE

two spin components … with different velocities …. determined by the magnetic field

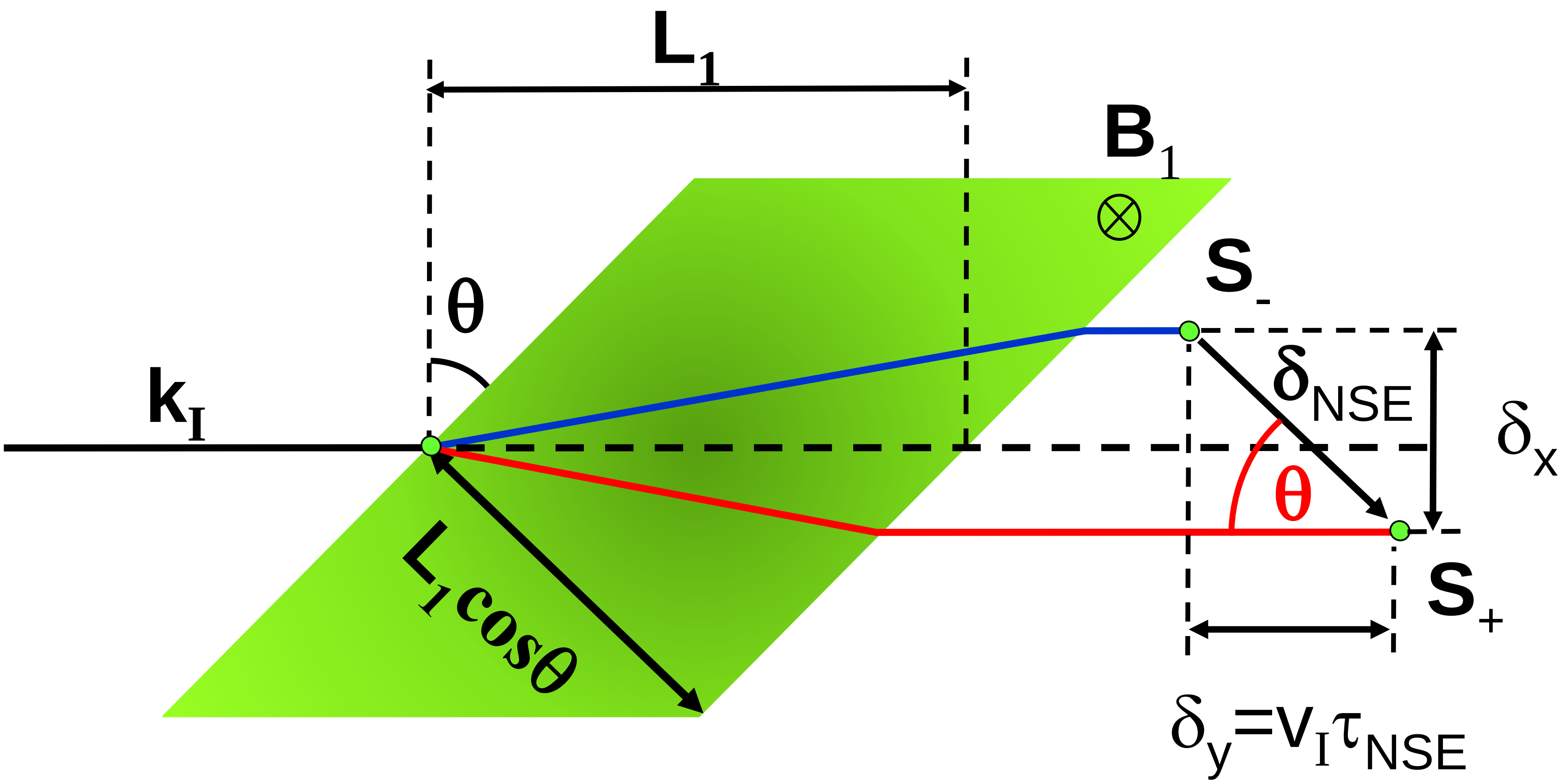
$$|+\rangle_x = \frac{1}{\sqrt{2}}\left(|+\rangle_z + |-\rangle_z\right) \qquad v_\pm = v_1 \pm \frac{\hbar\omega_z}{m v_1} \qquad \omega_z = \frac{\mu B_1}{\hbar}$$



R. Gähler et al., Physica B **229** 1 (1996)

Tilted Magnetic Fields: Lateral Spatial Separation

axial and lateral splitting of spin components



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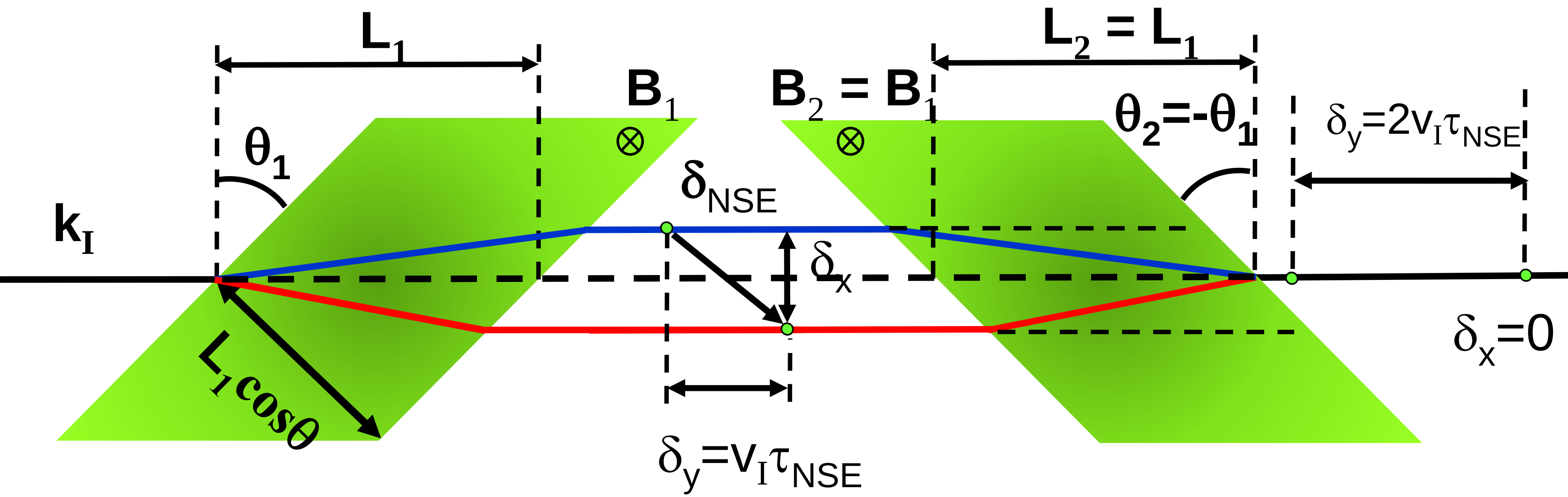
Refocussing of Spin Components

SESANS apparatus: no scattering at the sample !
a direct beam experiment with a SESANS setup

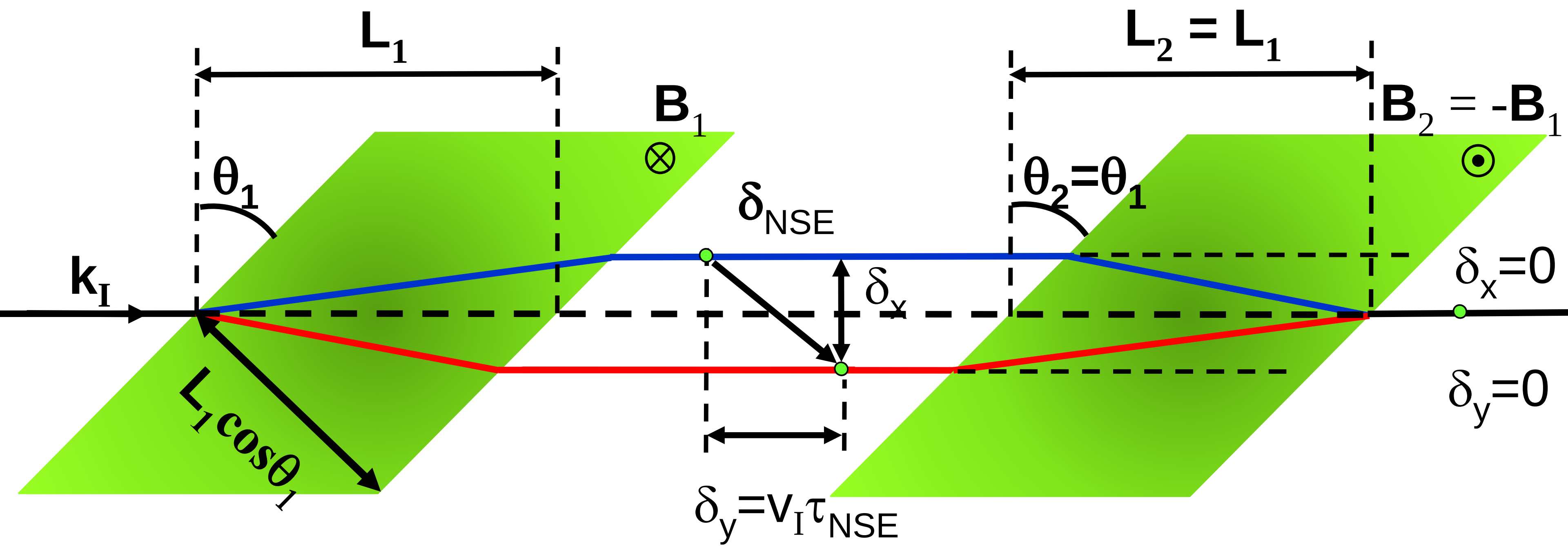


Refocusing condition only met in space, but still temporal difference!
Fail: does not lead to a spin echo signal

phase difference must be zero to get meaningful spin-echo signal
Here: phase difference depends linearly on v_I !

Refocussing of Spin Components: SESANS

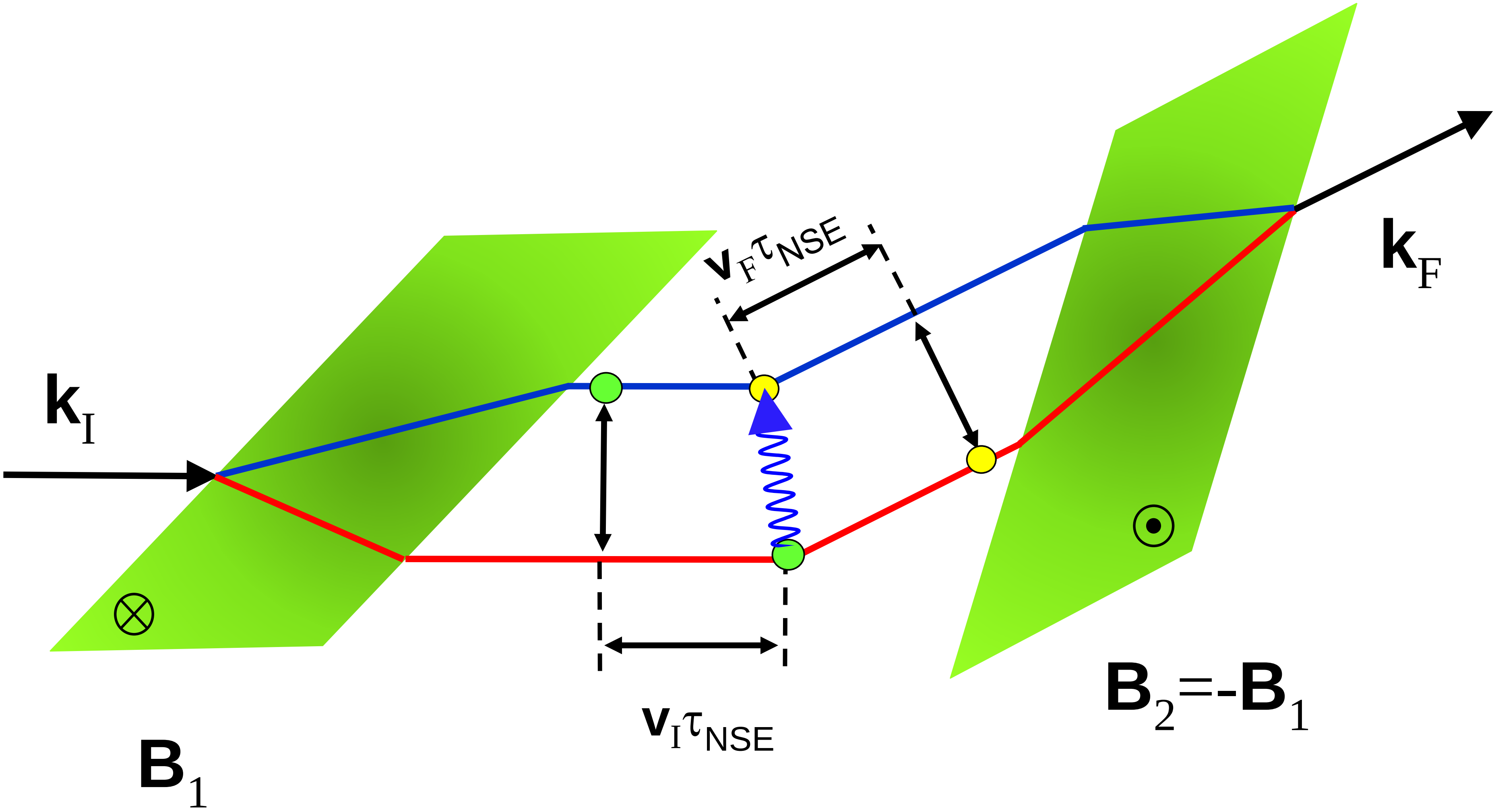
SESANS apparatus: no scattering at the sample !
a direct beam experiment with a SESANS setup



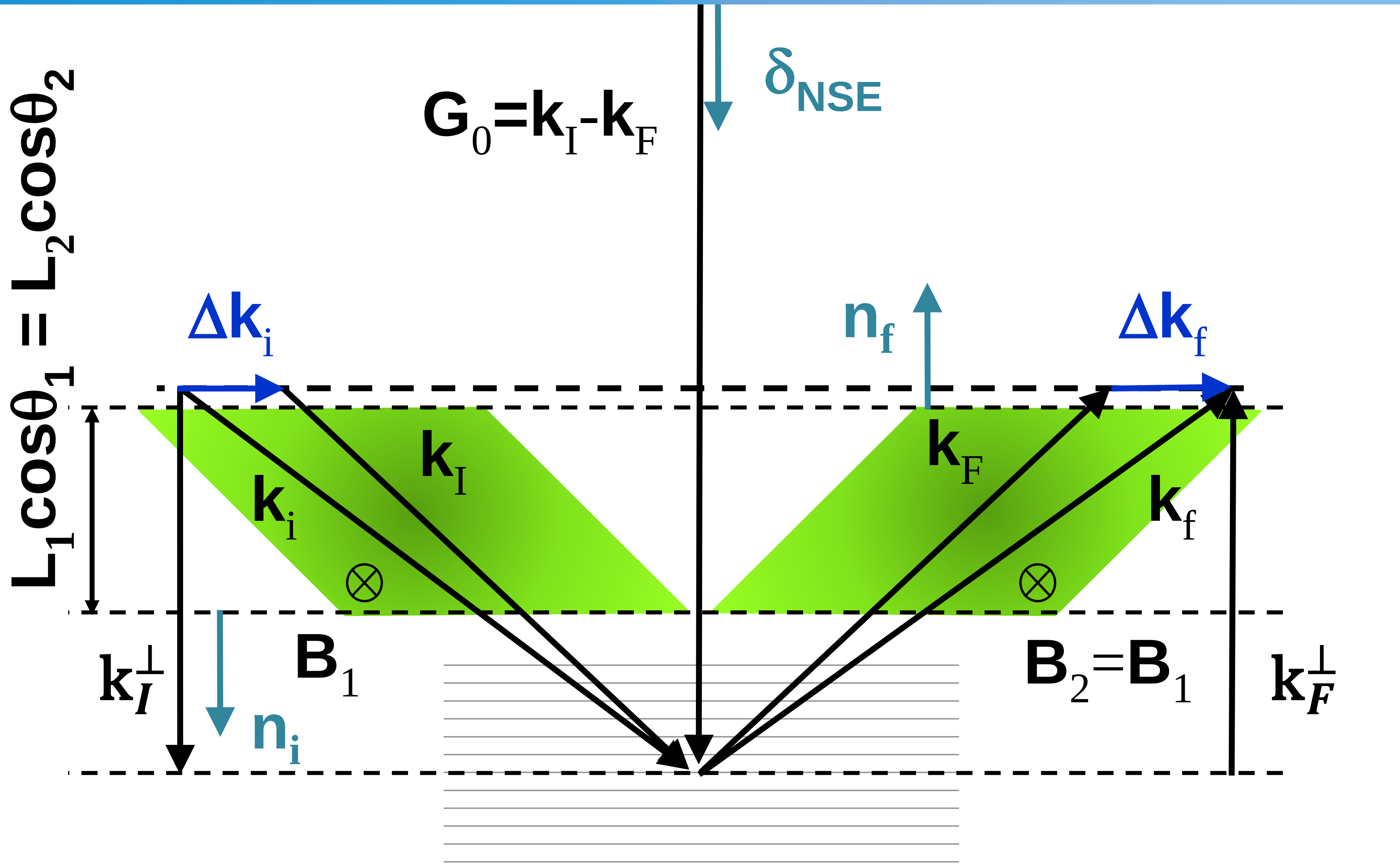
Construction rule (“Konstruktionsvorschrift”):
Meet the condition that phase difference is zero
-> will give you a meaningful spin-echo signal

corresponds to refocusing in space and time in the semi-classical ray-tracing model

Inelastic Scattering



Larmor Diffraction: Classical Larmor Precession



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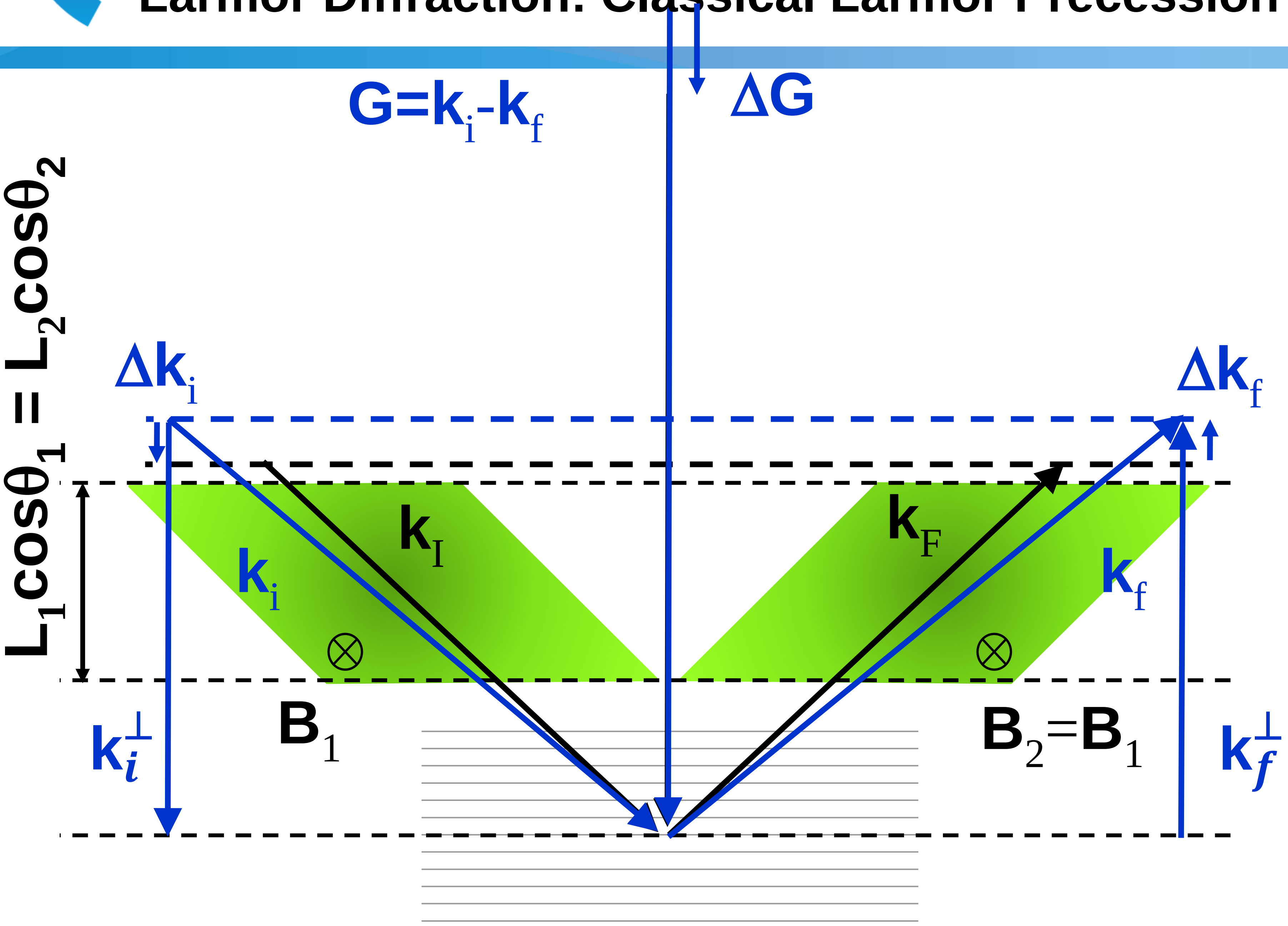

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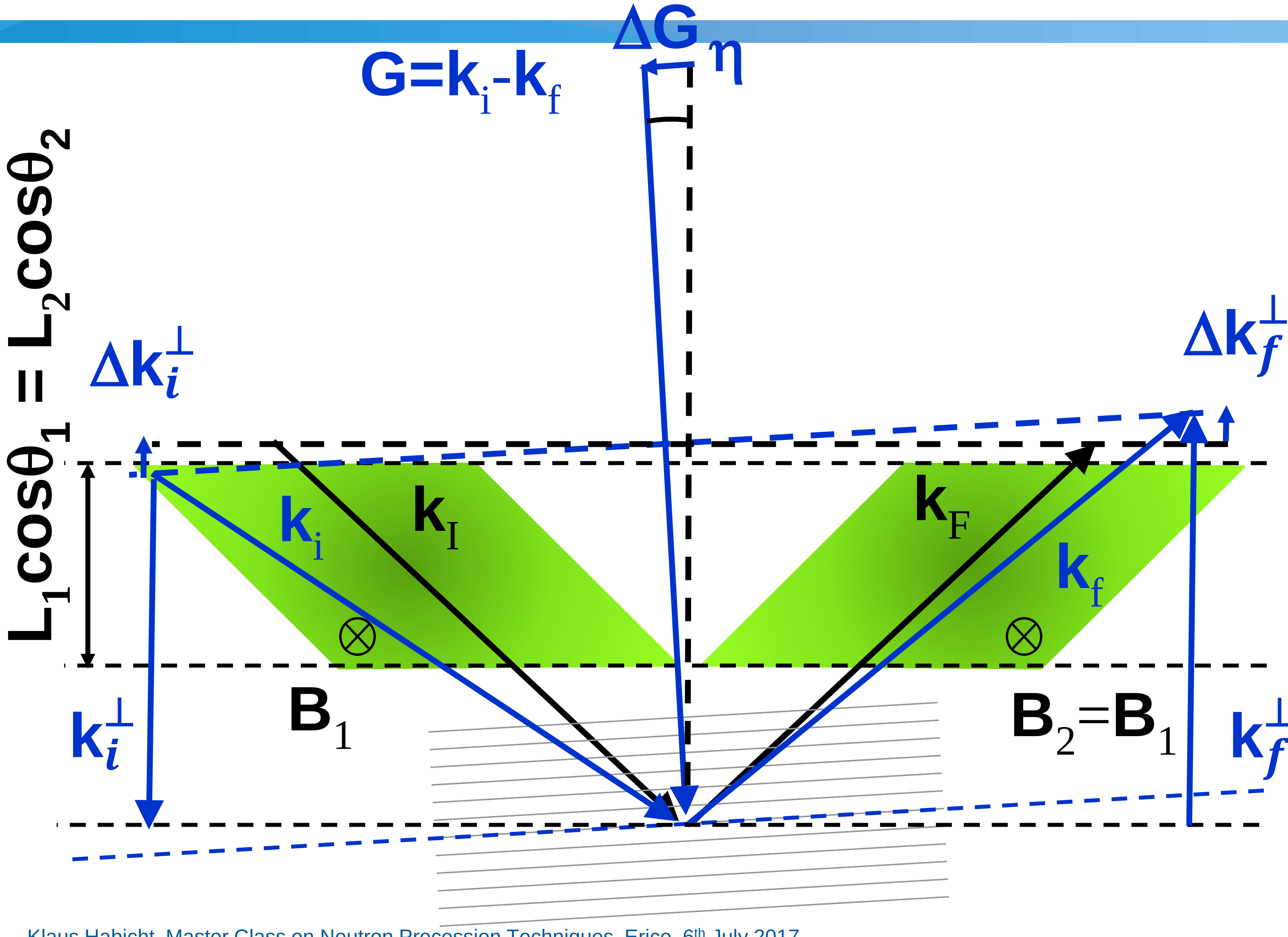
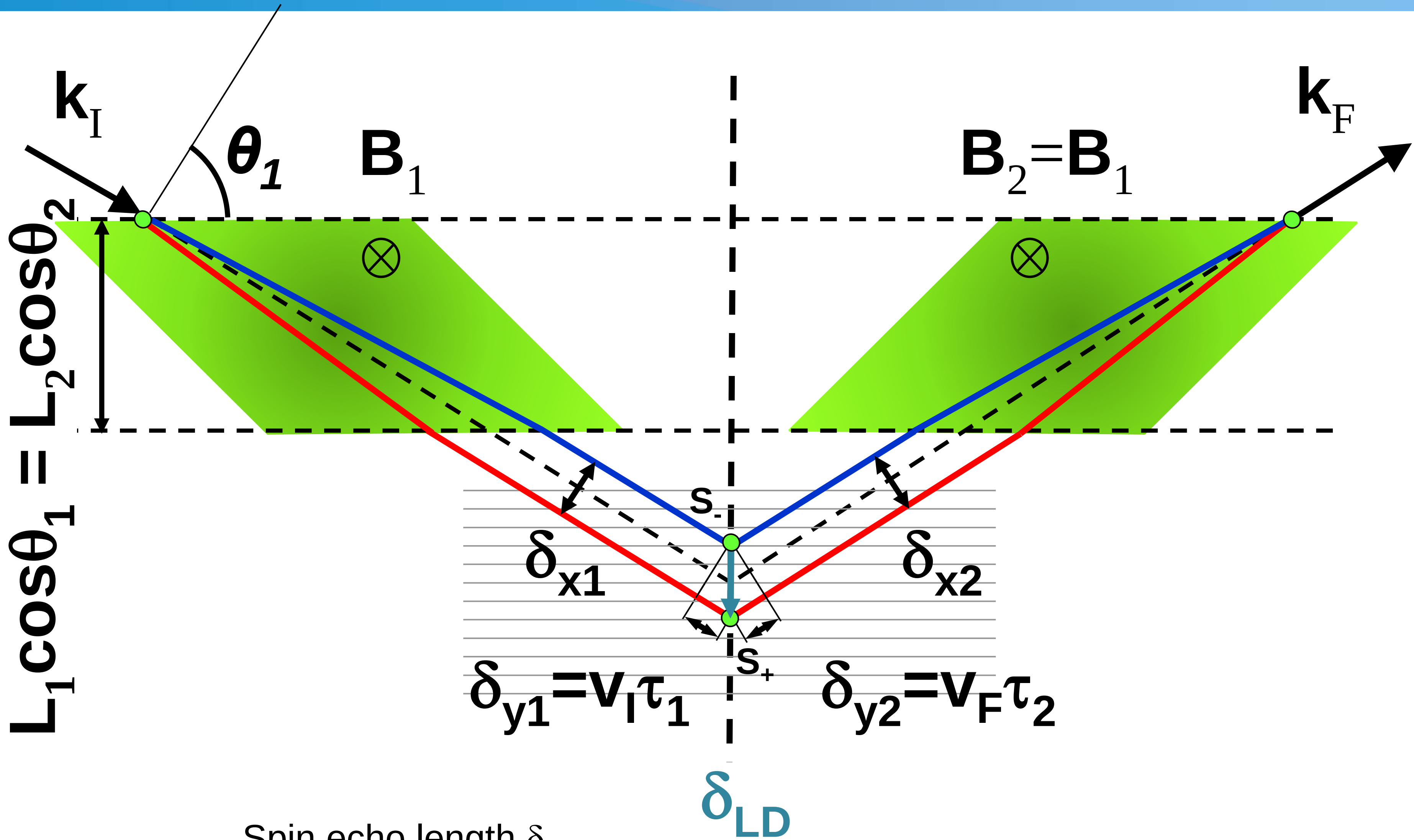
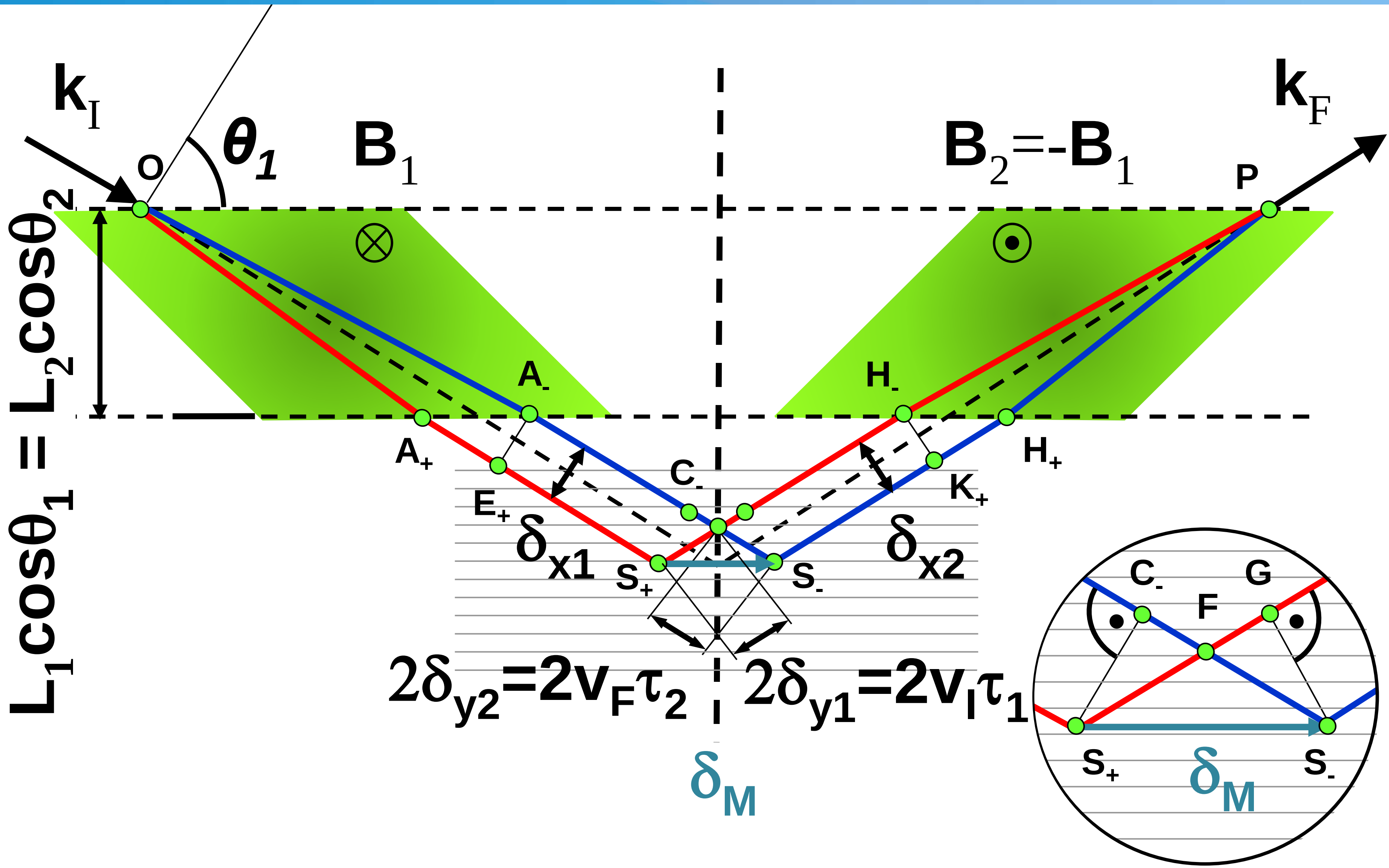


Semi-Classical View of Larmor Diffraction

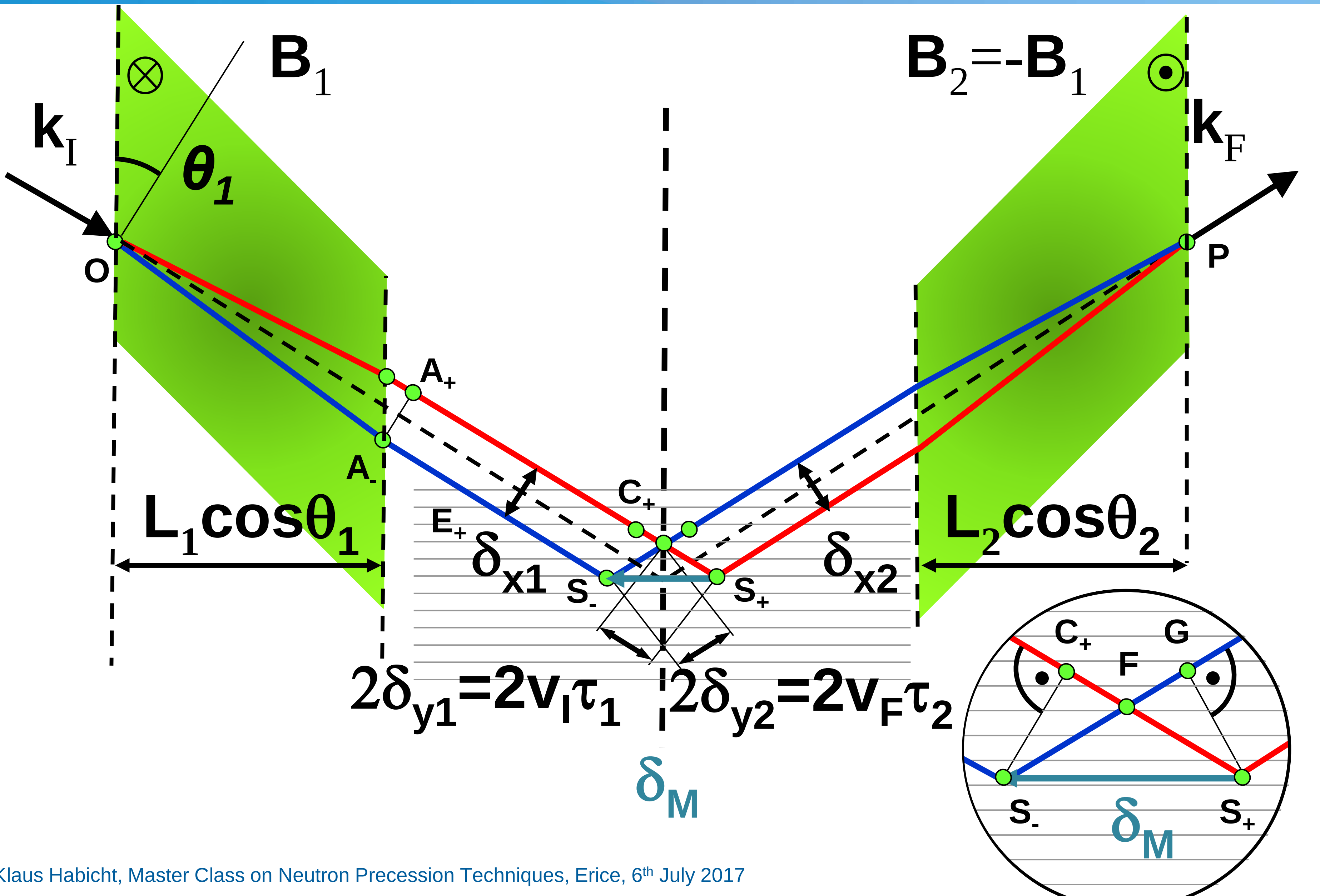


Spin echo length δ_{LD}

Mosaic-Sensitive Arrangement I



Mosaic-Sensitive Arrangement II



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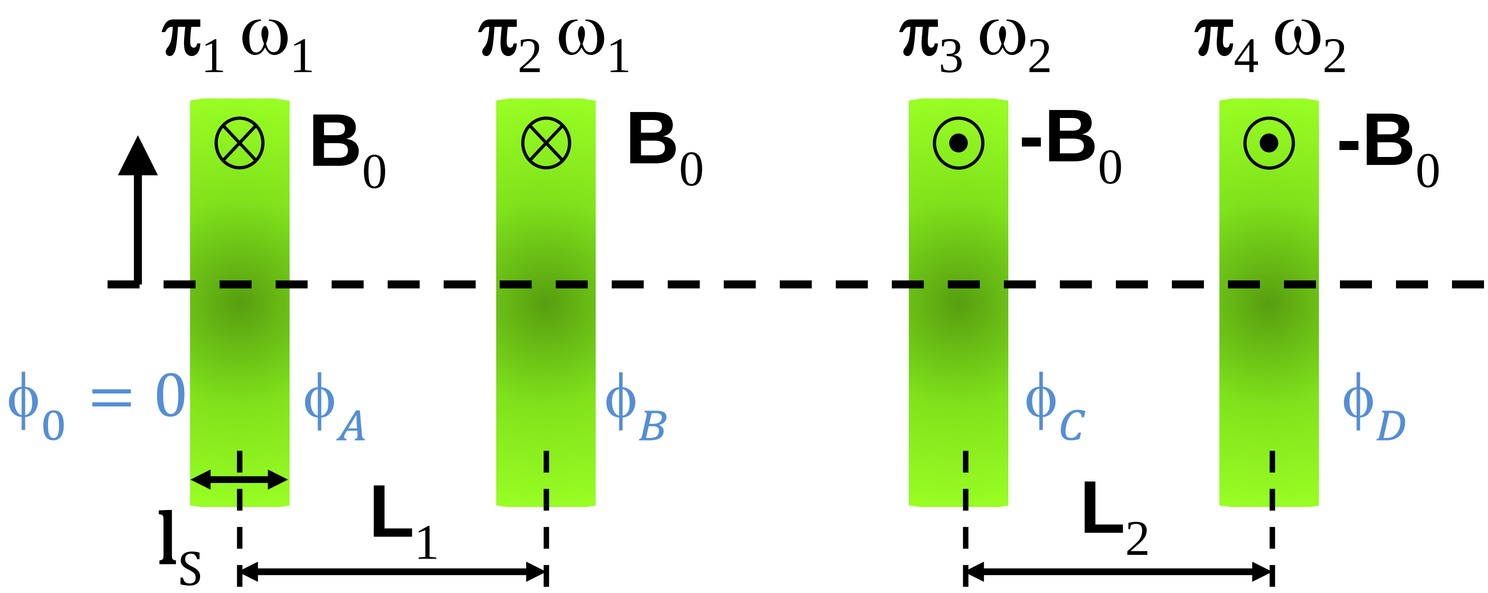
Equivalence of RF Flippers and Static Fields

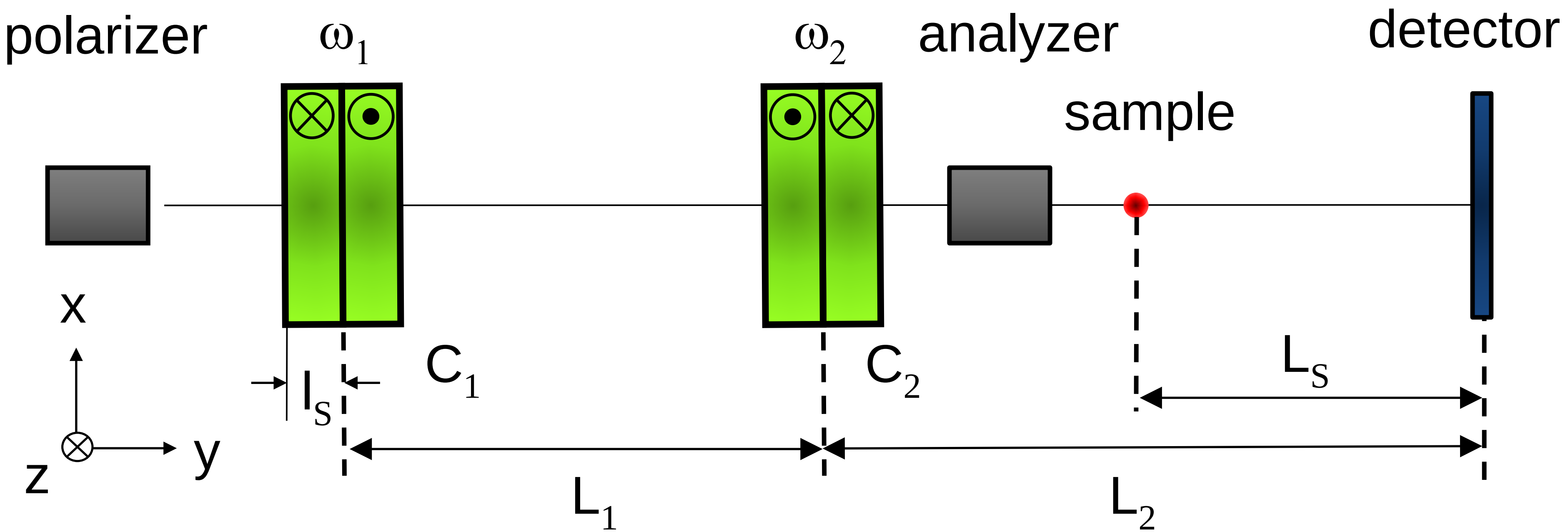
This arrangement of RF flippers ….



… corresponds to this arrangement of static fields

MIEZE Setup with RF Spin Flippers

standard MIEZE arrangement



satisfy MIEZE condition …

$$\frac{\omega_2 - \omega_1}{\omega_2} = \frac{L_1}{L_1 + L_2}$$

… to get time-modulated detector signal …

$$I(t_D) = \frac{I_0}{2}\left(1 + P_0 \cos \omega_M t_D\right) \int S(\mathbf{Q}, \omega) \cos \omega_M \tau_M d\omega$$

time-modulated signal is independent of the incident velocity distribution

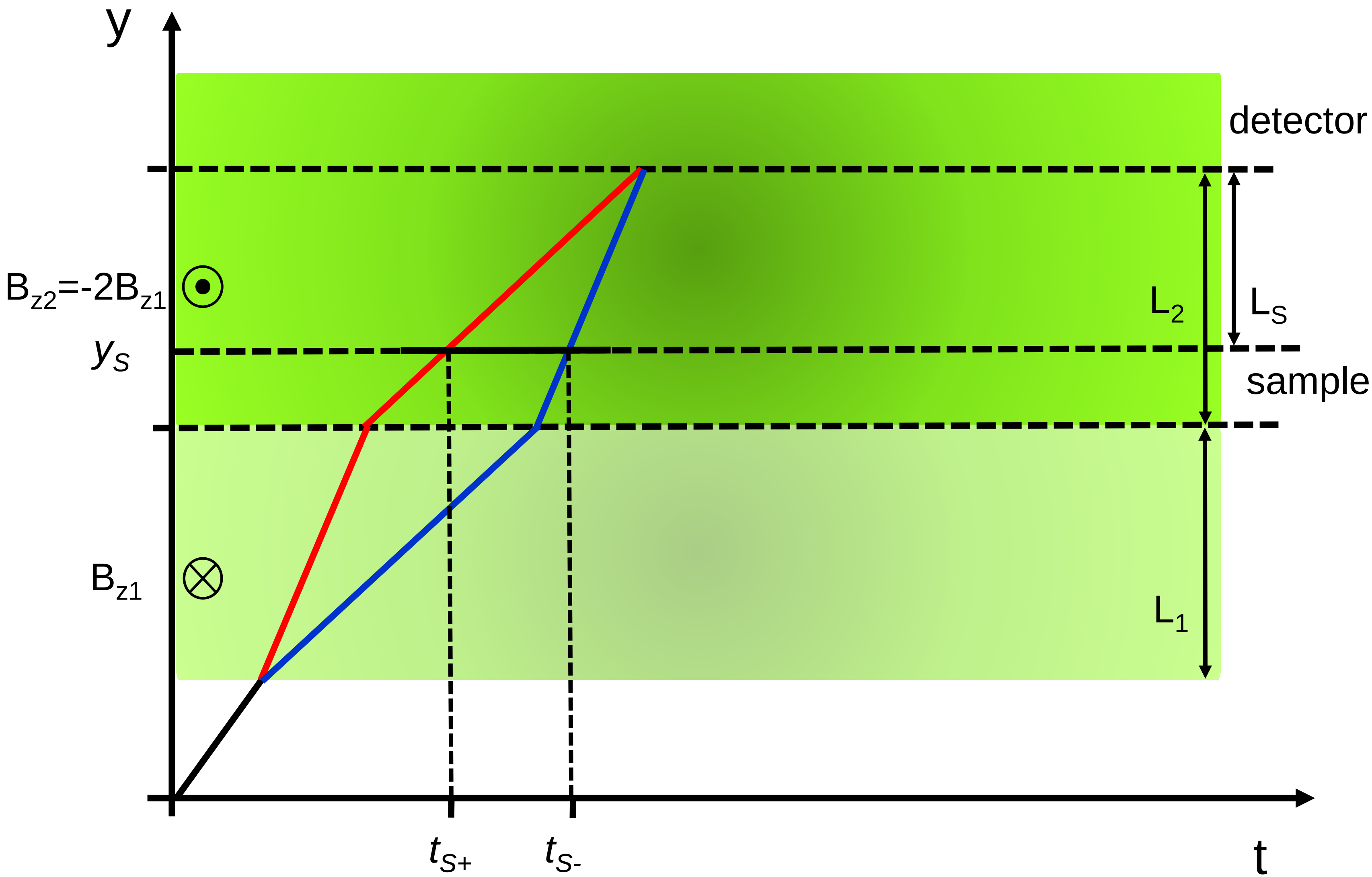
… with frequency ….

$$\omega_M = 4\left(\omega_2 - \omega_1\right)$$

… and MIEZE time ….

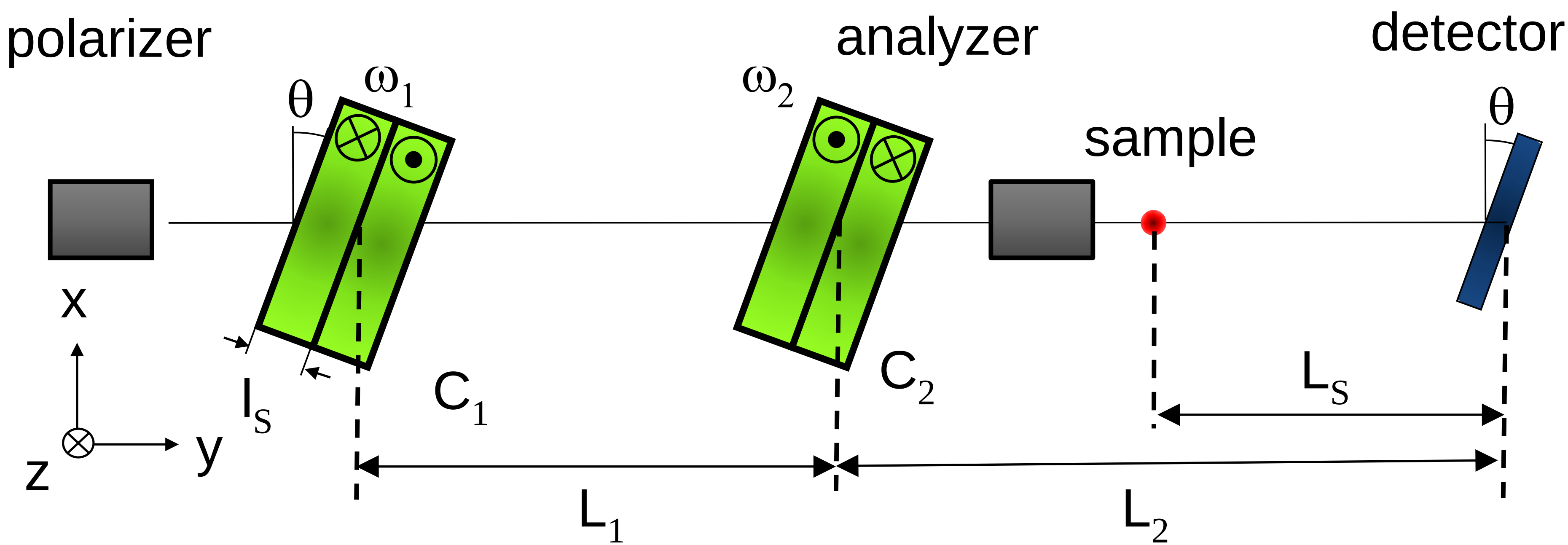
$$\tau_M \equiv \omega_M \frac{\hbar\omega}{m v_I^3} L_S$$

Semi-Classical Ray-Tracing View of MIEZE



MIEZE with Tilted RF-Flippers

tilt RF coils and detector to obtain MIEZE signal as usual



satisfy MIEZE condition …

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… to get time-modulated detector signal …

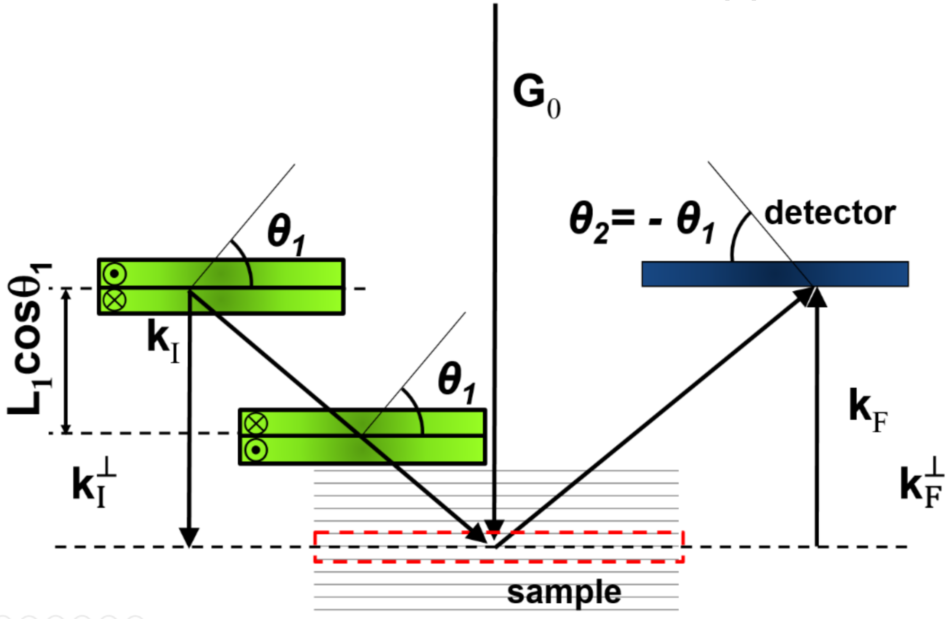
$$I(t_D) = \frac{I_0}{2}\left(1 + P_0 \cos\omega_M t_D\right)\int S(\mathbf{Q},\omega)\cos\omega_M \tau_M d\omega$$

time-modulated signal is independent of the incident velocity distribution

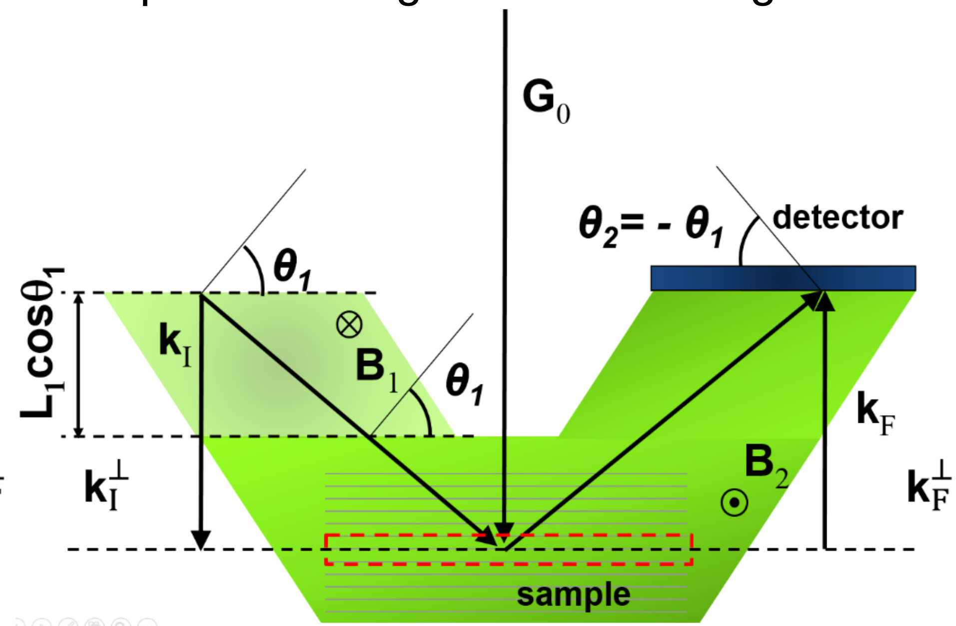
→ **MIEZE-SEMSANS or MIEZE Larmor Diffraction**

MIEZE Larmor Diffraction

Larmor diffraction with two RF flippers



equivalent magnetic field arrangement



satisfy MIEZE condition for Larmor diffraction…

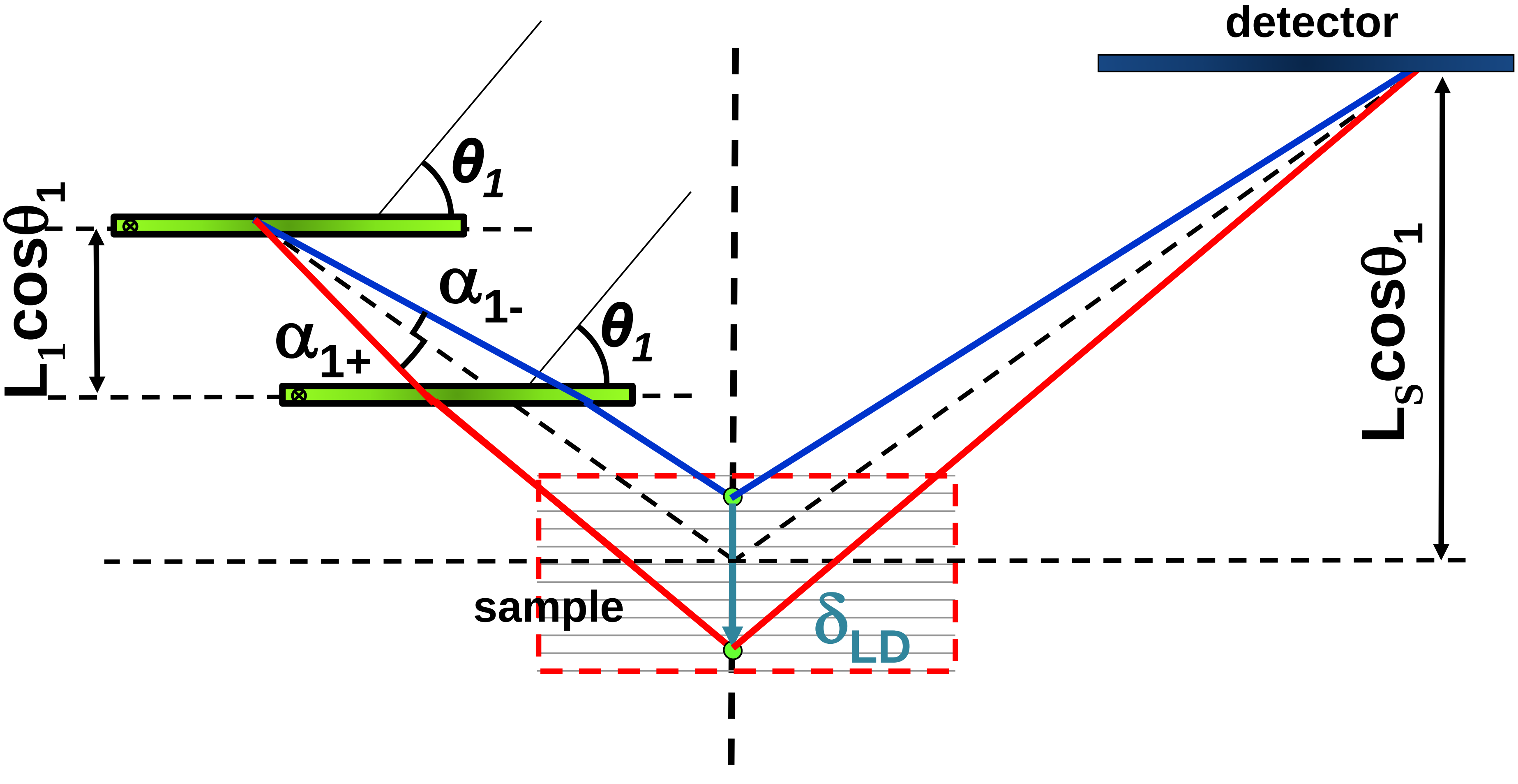
$$\frac{\omega_1}{\omega_2 - \omega_1} = \frac{L_2 - 2L_S}{L_1}$$

… to get Larmor precession angle depending on d-spacing only (independent of mosaic)

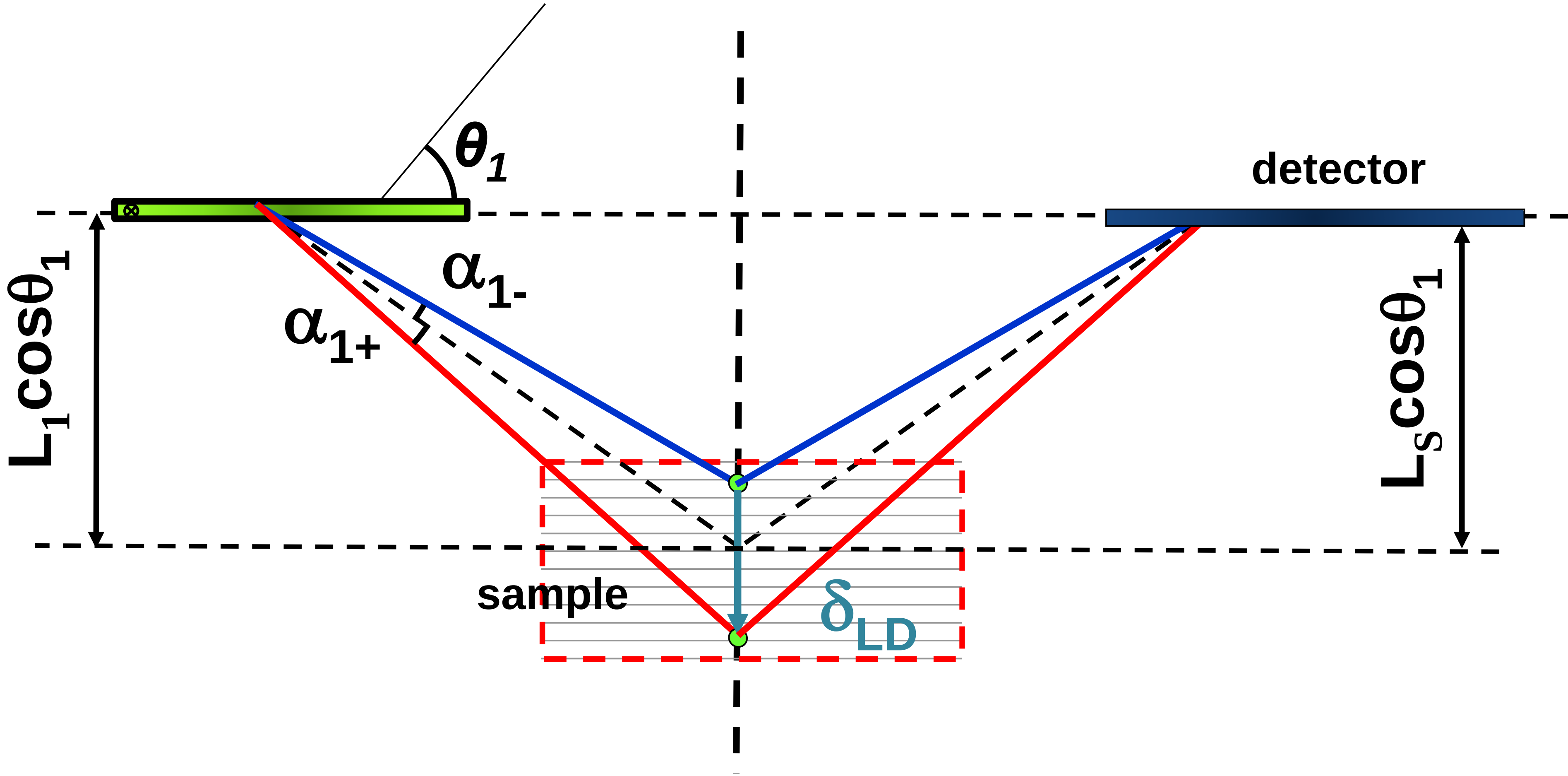
$$\phi_2 = -\omega_M \frac{m}{h} \left(L_1 + L_2 + 2L_S \right) \cos\theta d_0$$

contrast of time-modulated detector signal encodes distribution of d-spacing

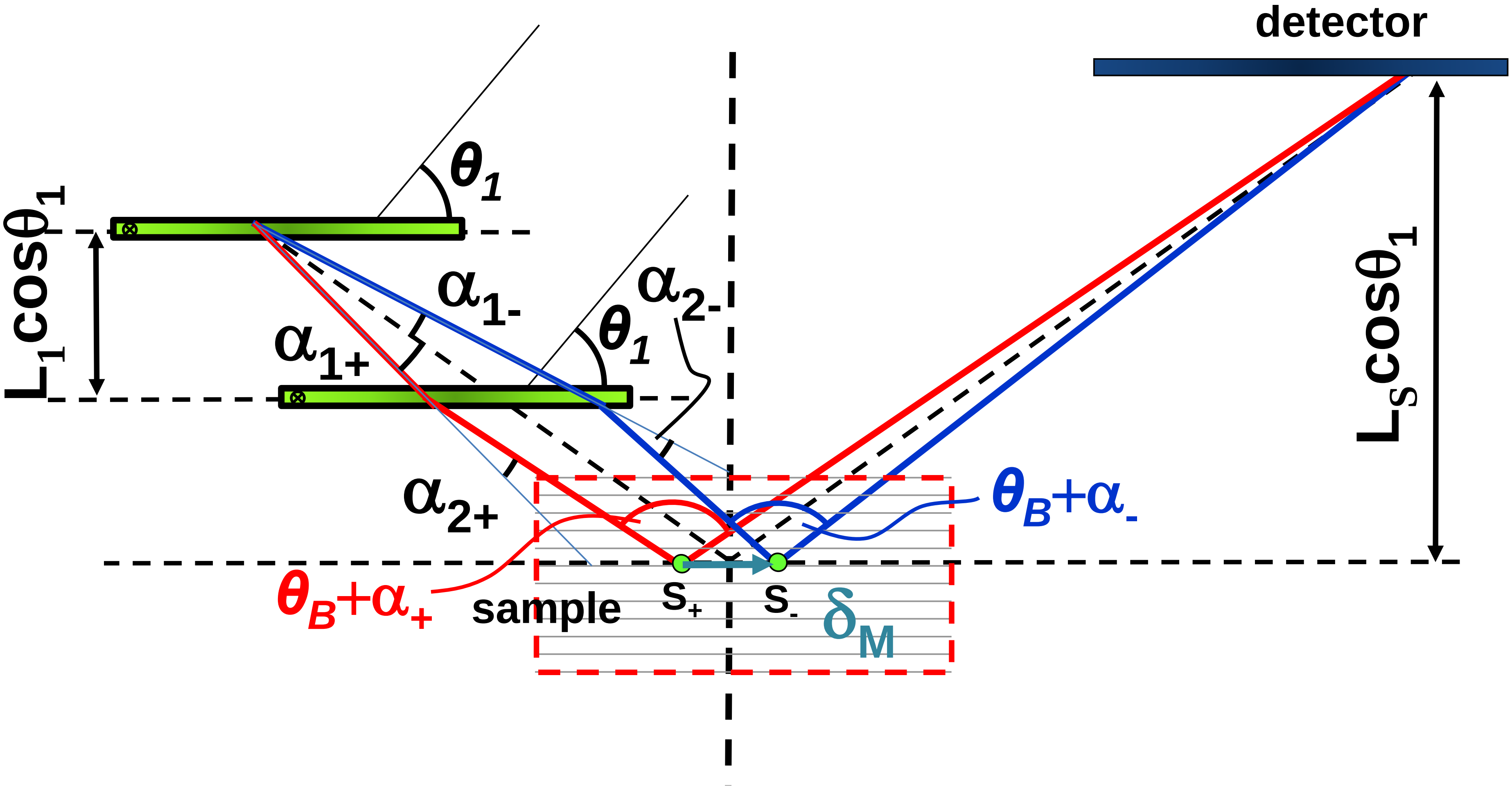
MIEZE Larmor Diffraction



MIEZE Larmor Diffraction with a Single RF Flipper



MIEZE with Sensitivity to Sample Mosaicity



Advantages of MIEZE Larmor Diffraction

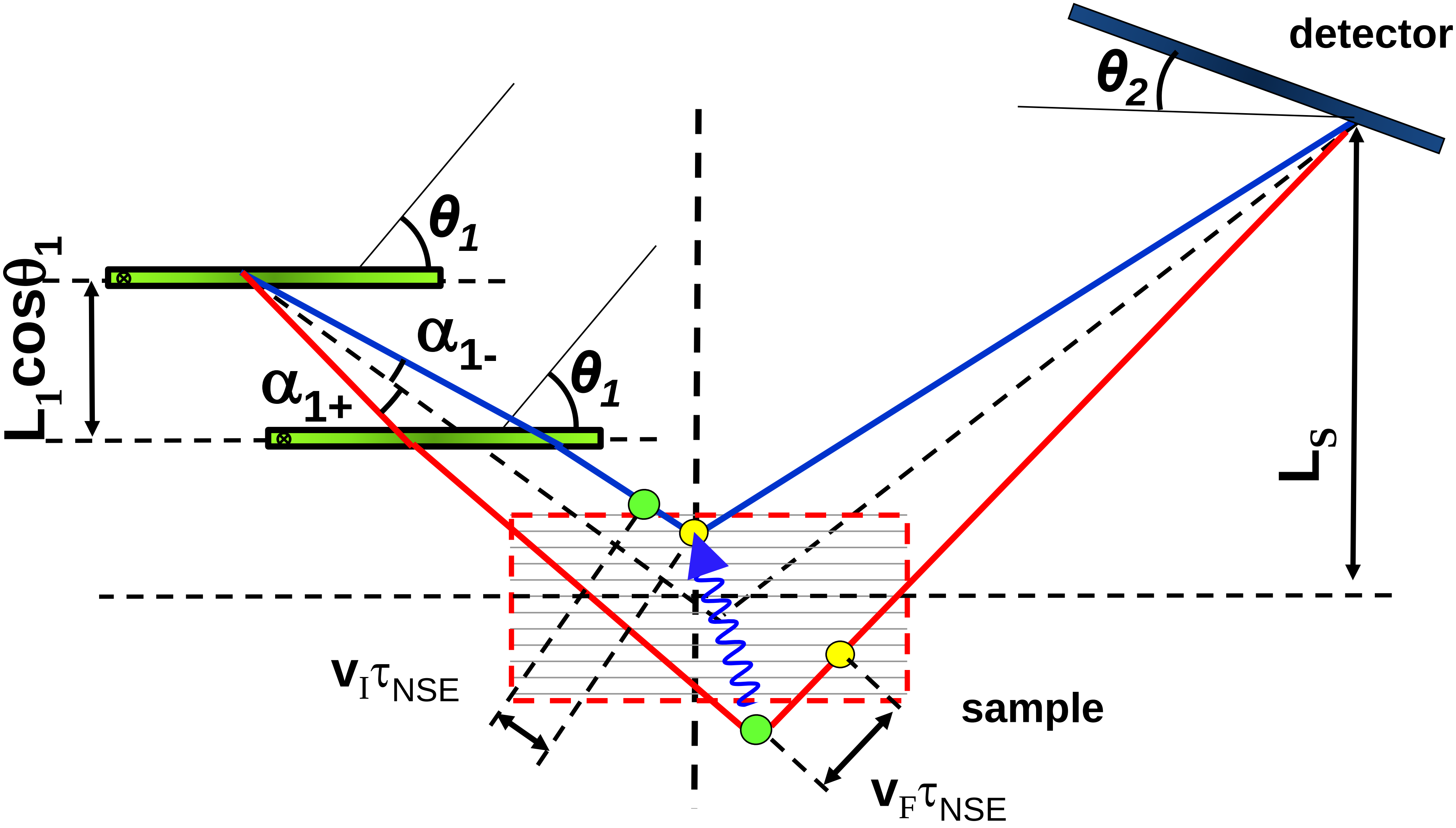
Advantages:

- **fairly simple field arrangement: only a single arm needed**
- **works even with a single RF flipper coil**
- **applicable to depolarizing samples**
- **ferromagnetic domains accessible**
- **loss of contrast due to finite beam divergence removed**

Drawback:

- **thin plate sample geometry needed**
- **thin detector needed**

Exercise: SE Focussing with Inelastic MIEZE



Semi-Classical View of Larmor Precession

Models can be useful to communicate ideas.

Models can be useful to make predictions.

Models are made to help you think.

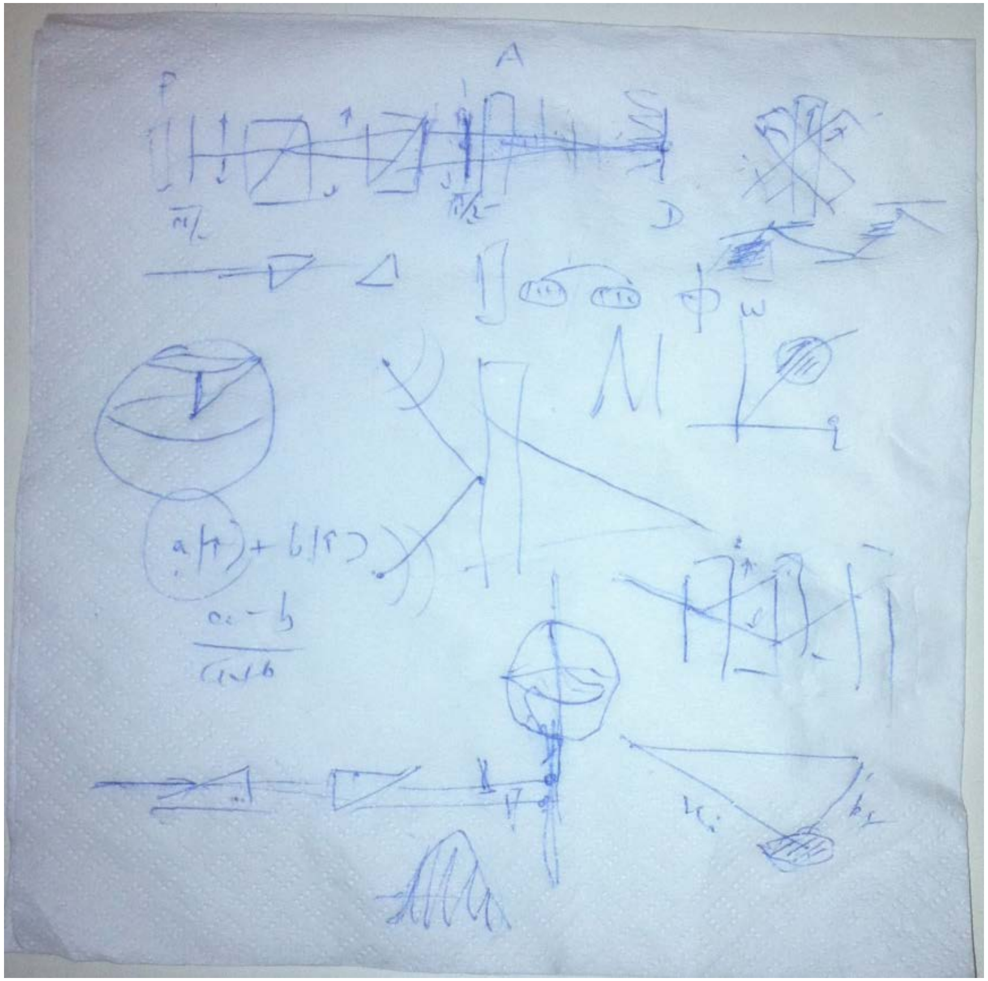
Know the limits of the model.

Check the predictions of the model.

If it is wrong do not use it.

If it is right and you don't like it: don't use it.

Thank you for your attention!



BACKUP Slides

Wave Packet Spread

A wavepacket $\Psi(x,t) = \int A(k) e^{i(kx-\omega t)} dk$

with envelope function $A(k) = A e^{-(k-k_0)^2 d^2}$

gives probability to find a particle:

$$|\Psi(x,t)|^2 = \frac{1}{d\sqrt{2\pi(1+\Delta^2)}} \exp\left\{-\frac{(x-vt)^2}{2d^2(1+\Delta^2)}\right\} \quad \text{with} \quad \Delta = \frac{t\hbar}{2m_n d^2}$$

spatial uncertainly $\Delta x = d\sqrt{1+\Delta^2}$

numbers: assuming 4 Å neutrons travelling 1 m, i.e. *t*=1ms

$$\Delta x(t=0) = d = 1\AA$$
$$\Delta x(t=1ms) = 0.314m$$

$$\Delta x(t=0) = d = 1cm$$
$$\Delta x(t=1ms) = 0.01m$$