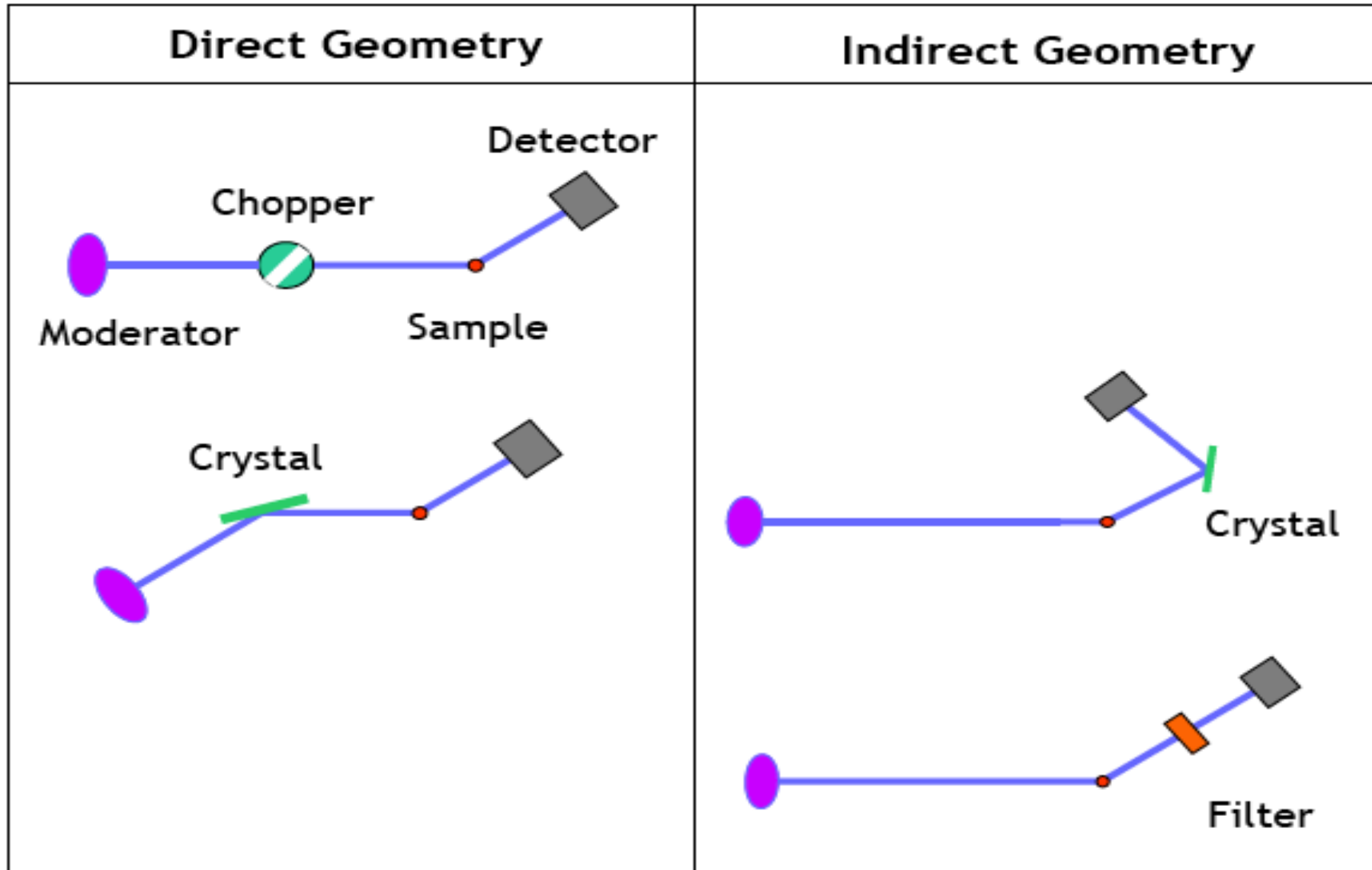


Inelastic Scattering II

Carla Andreani
Universita' degli Studi di Roma Tor Vergata-Dip. Fisica
e
Centro NAST

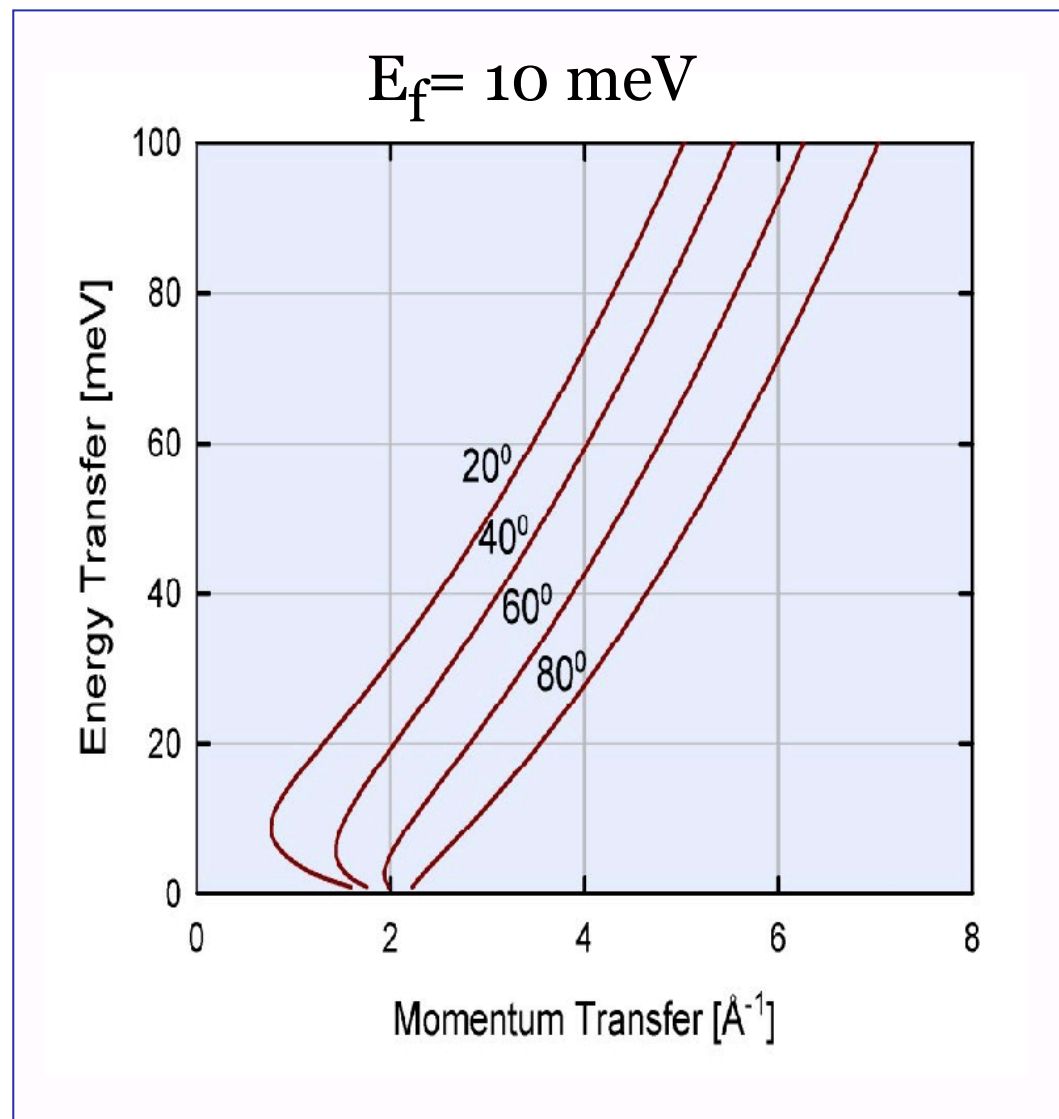
INDIRECT GEOMETRY SPECTROMETERS



TRAJECTORIES IN (Q, ω) SPACE \rightarrow INDIRECT GEOMETRY

- Similarly for indirect geometry we eliminate E_i .

$$\frac{\hbar^2 \mathbf{Q}^2}{2m} = 2E_f + \hbar\omega - 2 \cos \phi [E_f (E_f + \hbar\omega)]^{1/2}$$



RESOLUTION OF INDIRECT GEOMETRY SPECTROMETERS

Energy Resolution

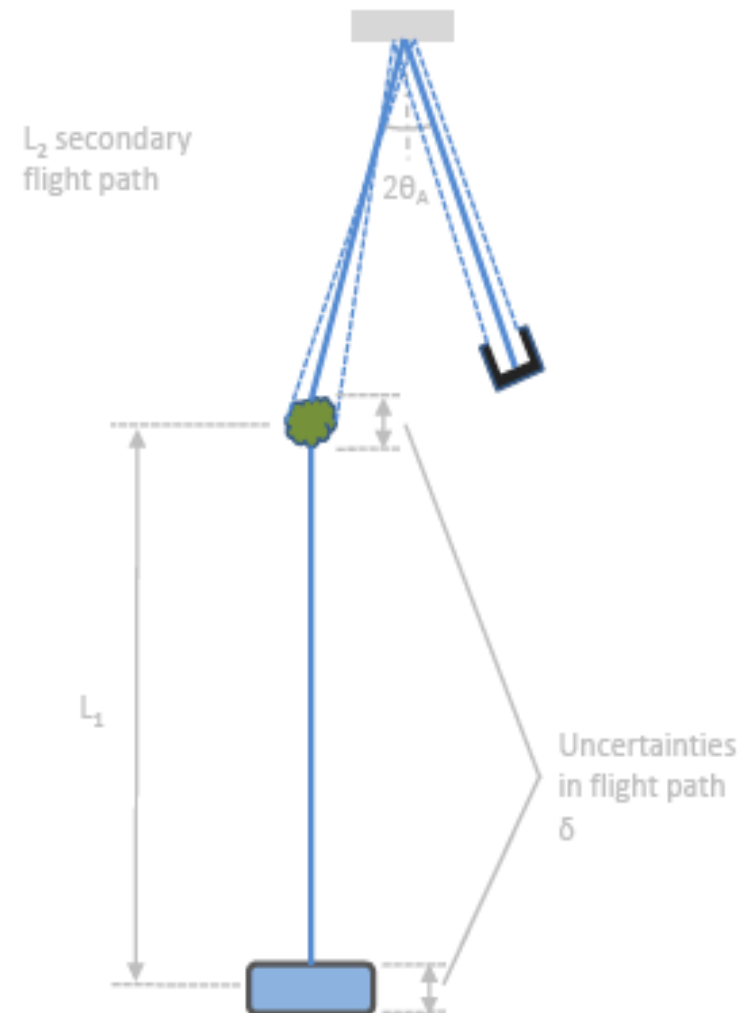
Uncertainty in the angular spread at the analyser

Timing errors due to flight path uncertainties

$$\frac{\Delta h \omega}{E_i} = 2 \left[\left(\frac{\delta}{L_1} \right)^2 + \left\{ \frac{E_f}{E_i} \cot \theta_A \Delta \theta_A \left(1 + \frac{L_2}{L_1} \left(\frac{E_i}{E_f} \right)^{3/2} \right) \right\}^2 \right]^{1/2}$$

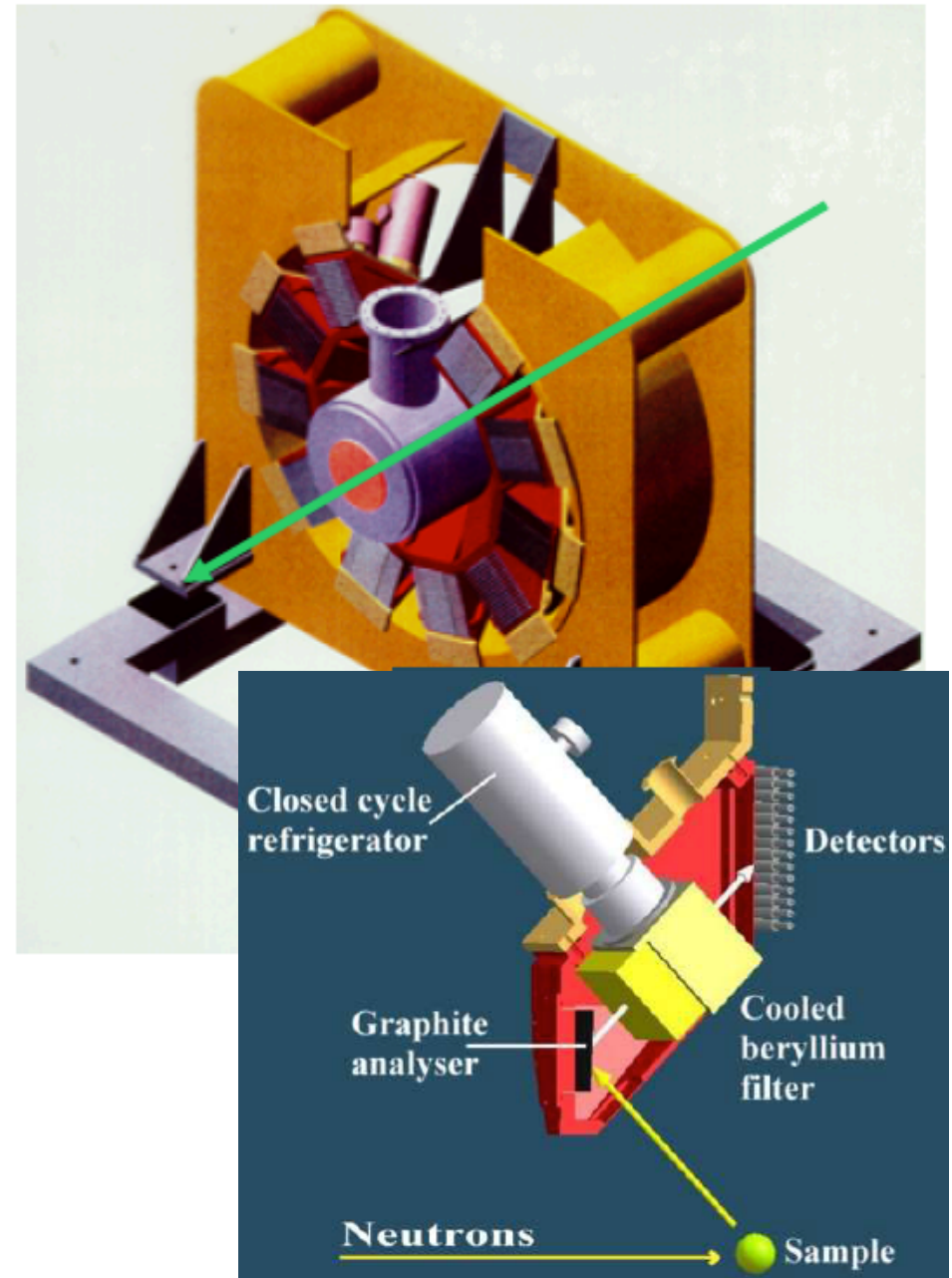
increasing θ_A or L_1 or decreasing L_2 improves resolution

fractional energy resolution becomes worse with energy transfer, the opposite of the case for direct energy instruments.

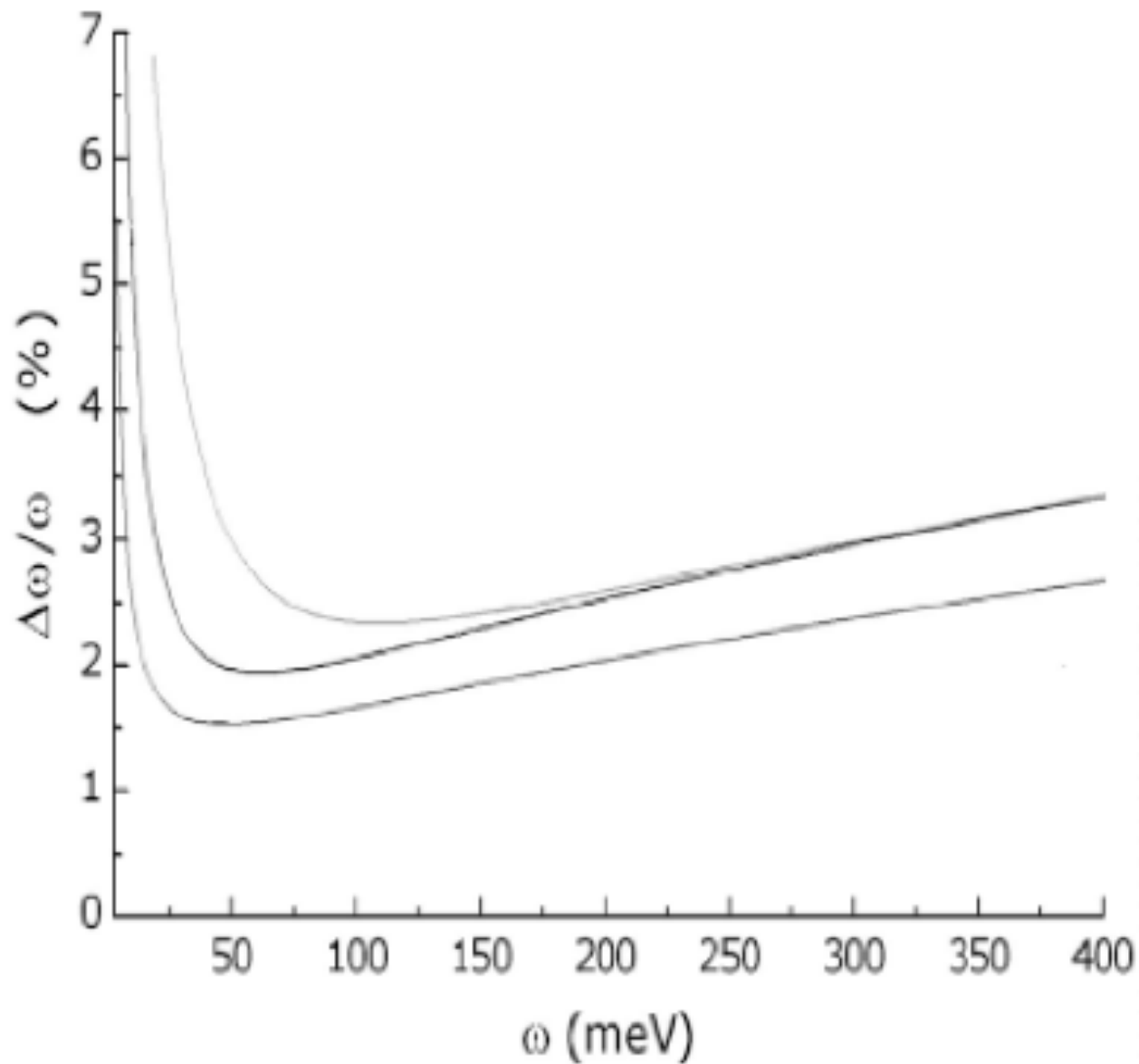


CRYSTAL ANALYZER SPECTROMETER 1 → TOSCA AT ISIS

- For Molecular spectroscopy, energy information is often much more important than Q information
- Graphite analysers, $E_f \sim 3$ meV
- Be filter removes higher order reflections. Cooled to reduce TDS
- The fact that sample, analyser and detectors are parallel reduces uncertainty in the scattered neutron flight time by time focussing.



ENERGY RESOLUTION FOR TOSCA



TOSCA → EXAMPLE OF EXPERIMENT

TOSCA Data

Gives similar information to Raman and Infra-Red

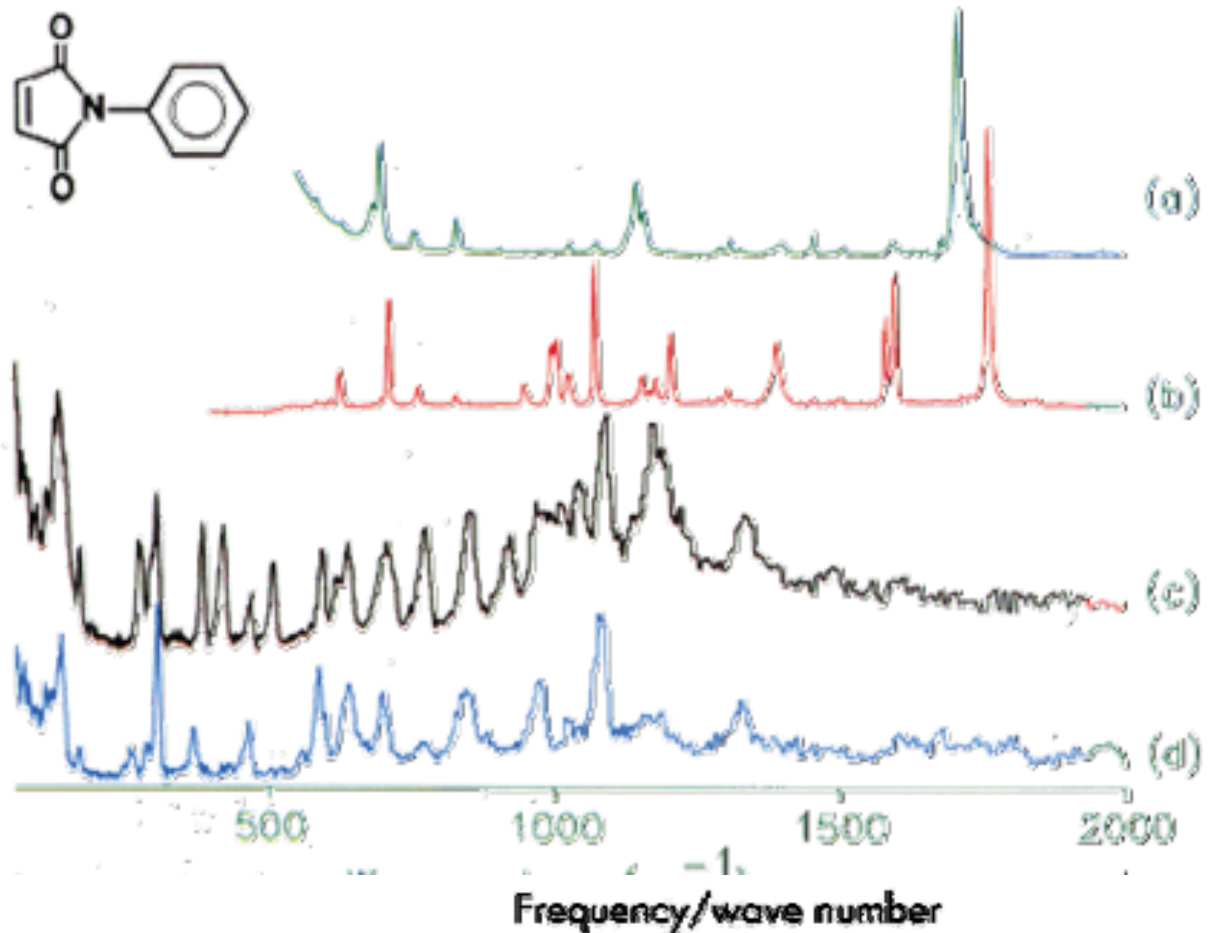
No selection rules

Intensity is proportional to mean square displacement

Cross section is element dependent..

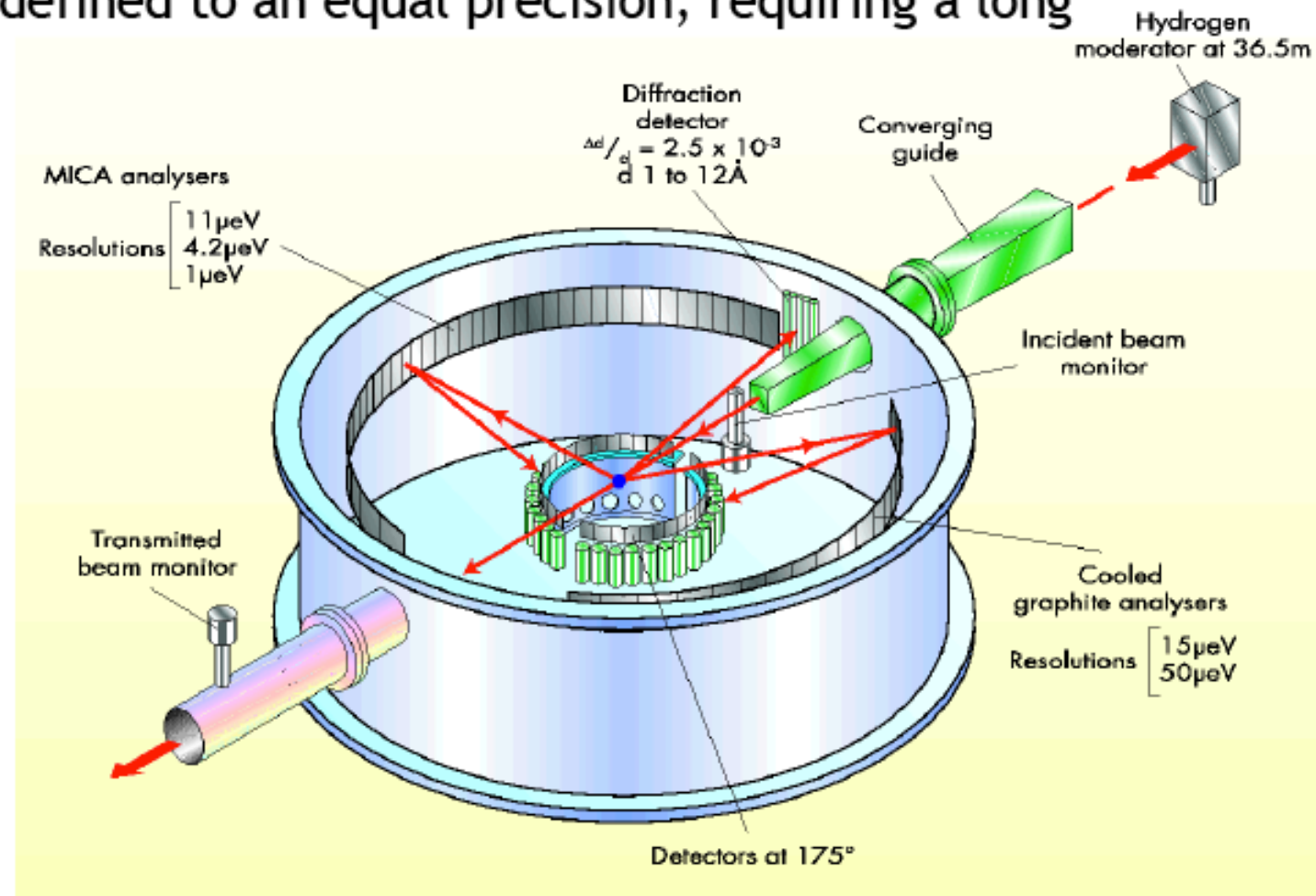
and isotope dependent

- a) Infrared
- b) (b) Raman
- c) (c) INS spectra of N-phenylmaleimide
- d) (d) INS spectrum of N-(perdeuterophenyl)maleimide



CRYSTAL ANALYZER SPECTROMETER 2 → IRIS @ ISIS

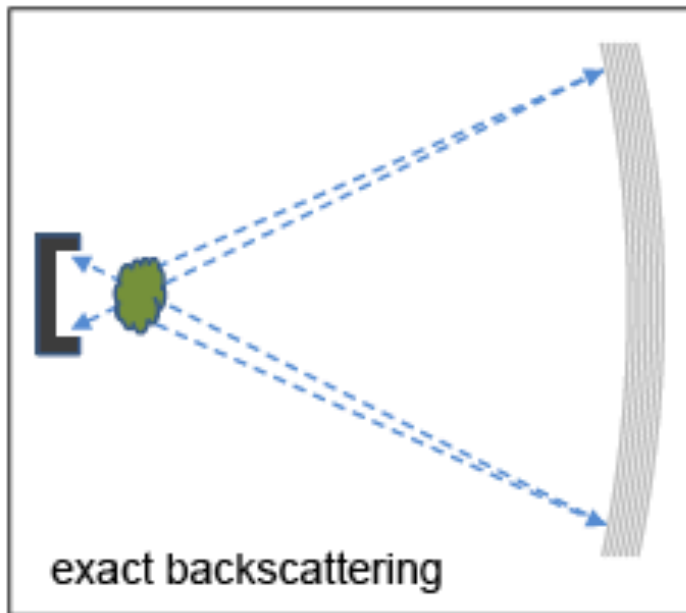
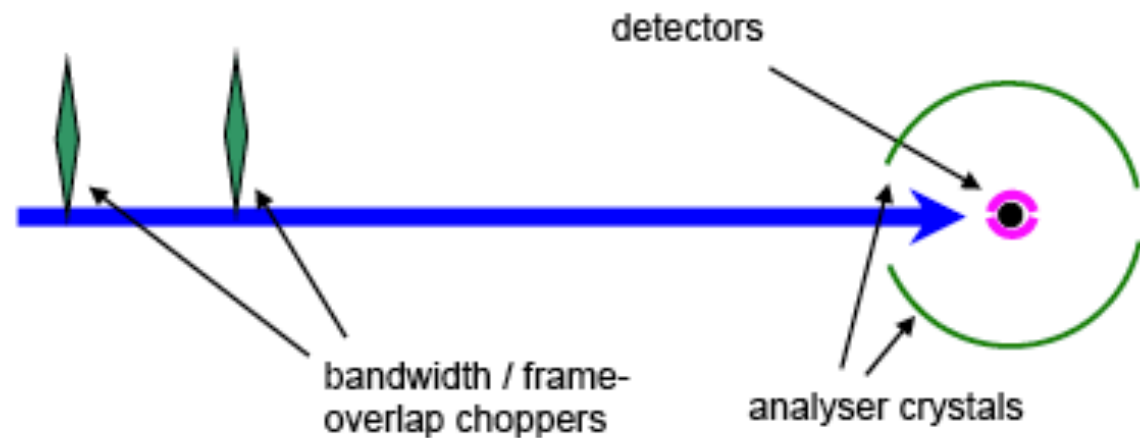
- For a crystal analyser spectrometer, the resolution contains a $\cot\theta_A$ term which can be reduced to almost zero by using a backscattering geometry.
- E_i should be defined to an equal precision, requiring a long flight path.



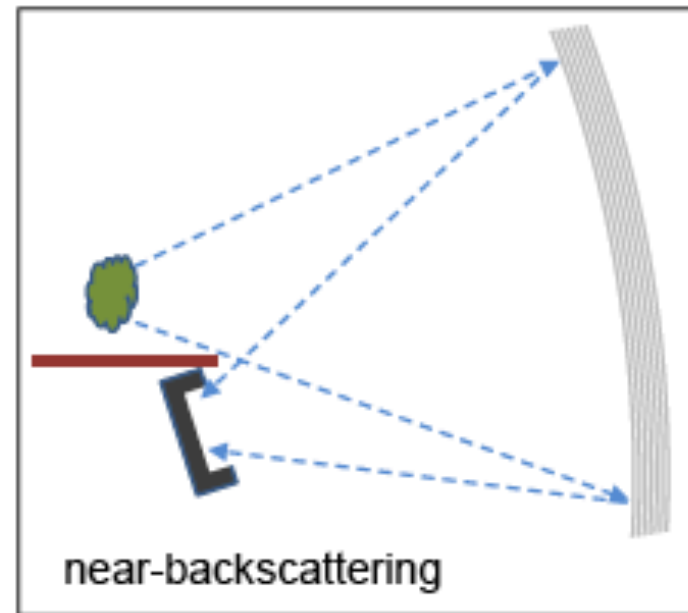
CRYSTAL ANALYZER SPECTROMETER 2

For a crystal analyser spectrometer, the $\cot \theta_A$ term can be reduced to almost zero by using backscattering

E_i should be defined to an equal precision, requiring a long flight path



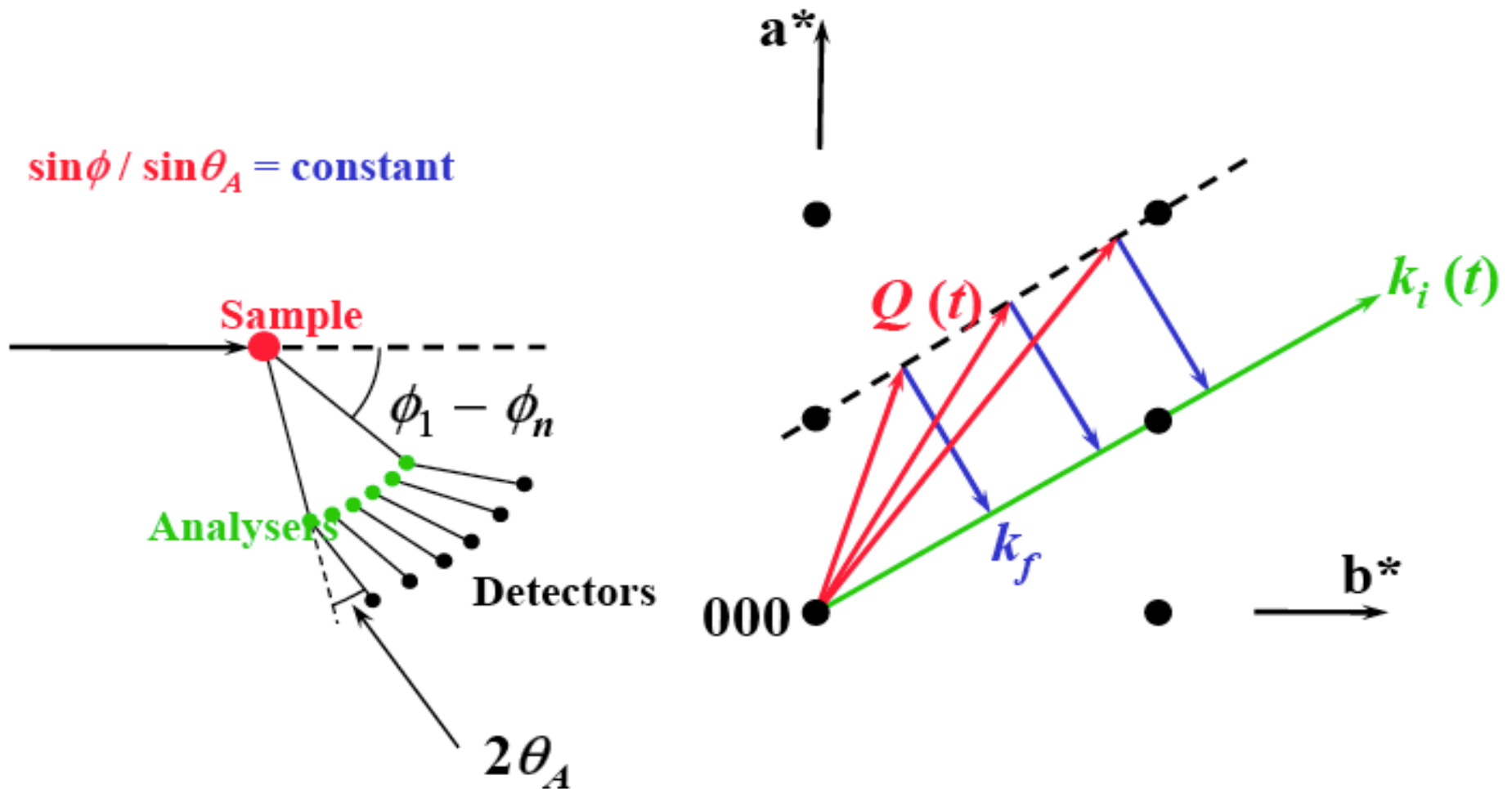
IN10, IN16



IRIS, IN13

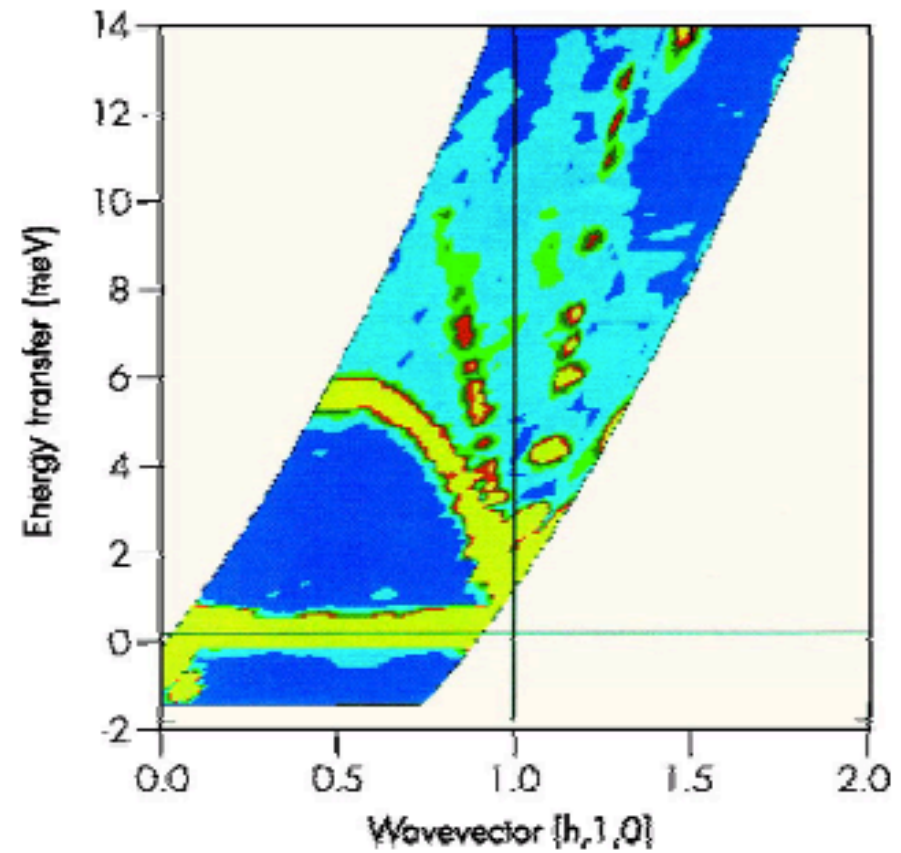
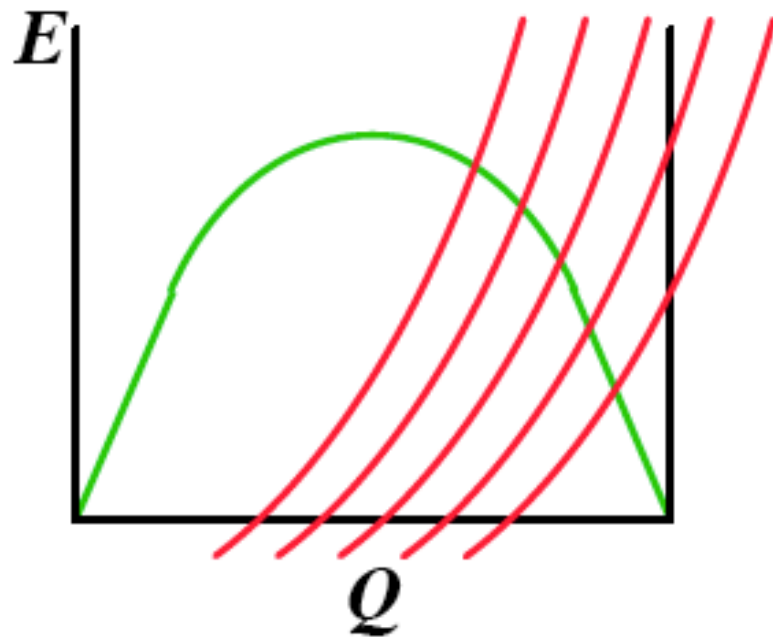
CRYSTAL ANALYZER SPECTROMETER 3 COHERENT EXCITATIONS - PRISMA

- Chopper spectrometers can be used to study coherent excitations in single crystals, **but** they are not able to directly perform scans along high symmetry directions.

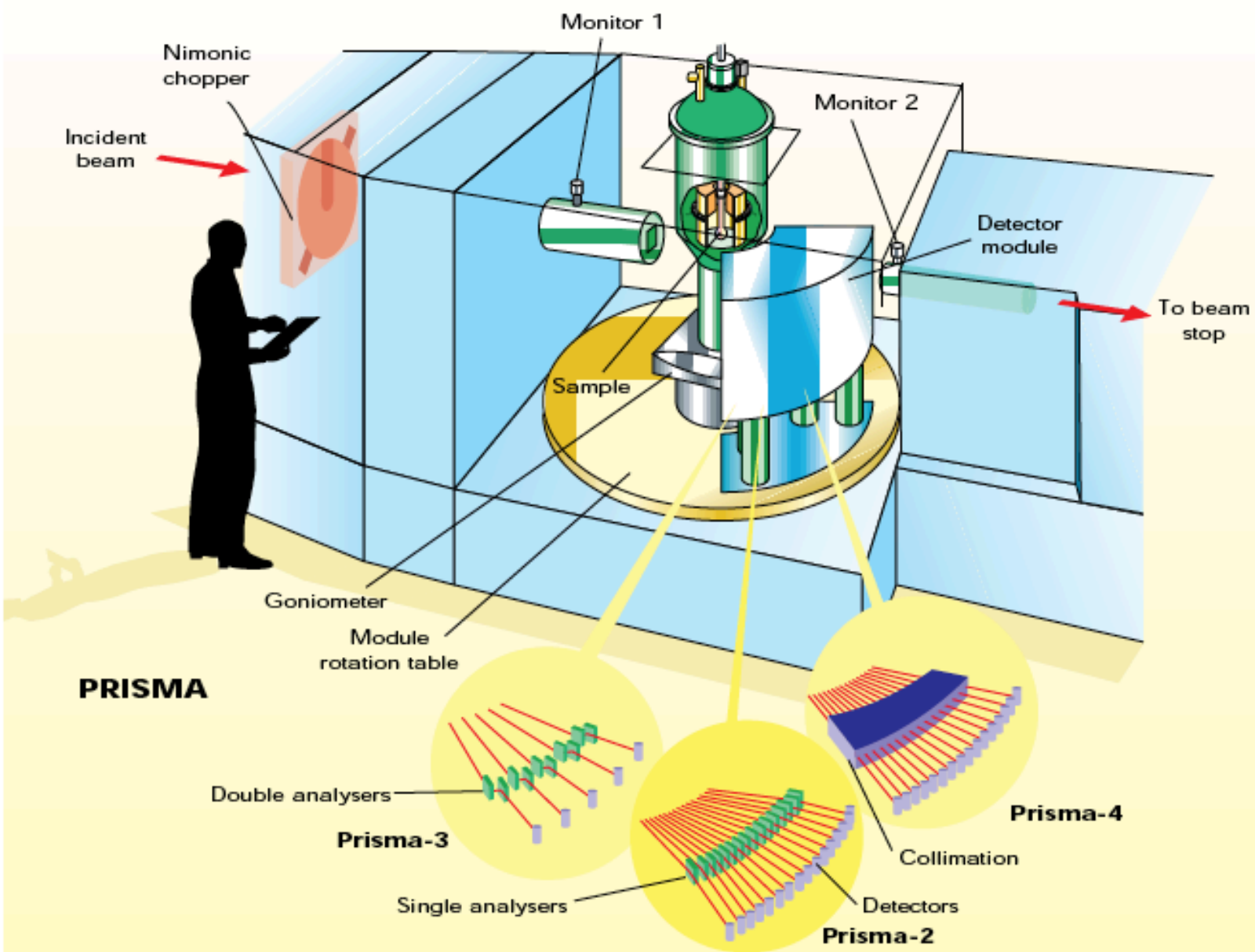


PRISMA EXPERIMENT

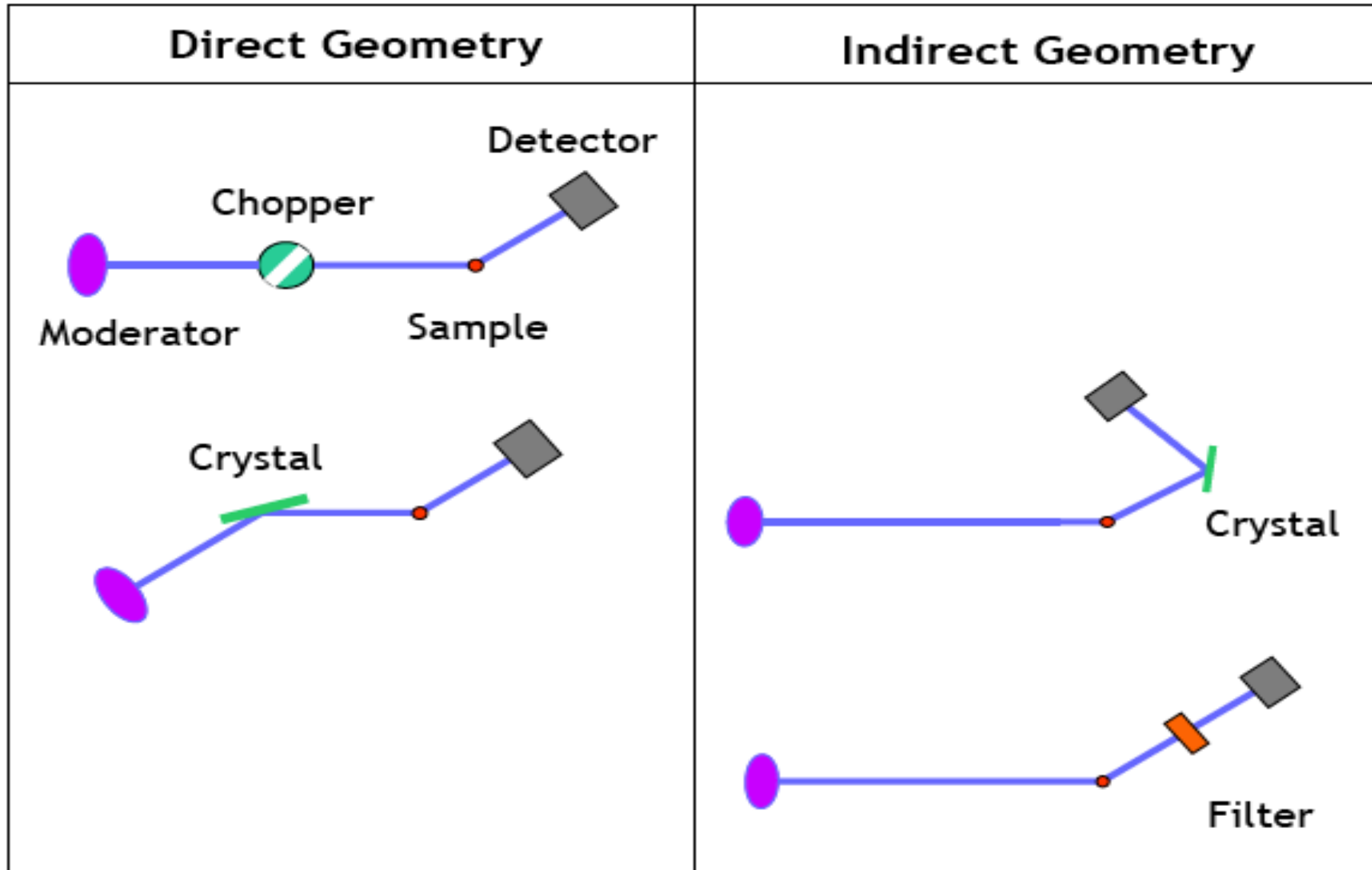
Phonons in ice 1h (Li, Bennington)



PRISMA

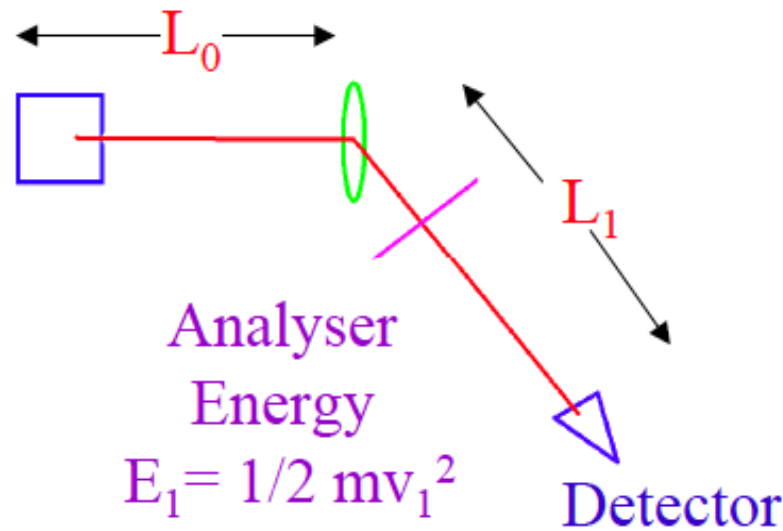


INDIRECT GEOMETRY SPECTROMETERS



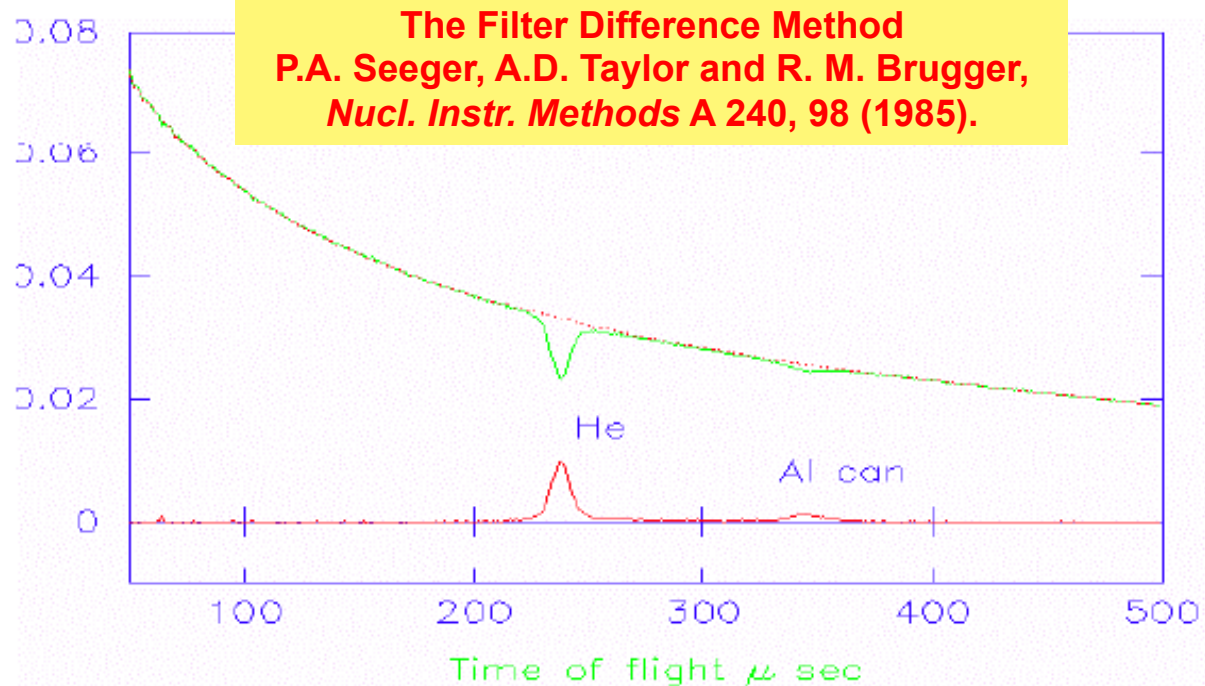
RFS mode

$$t = \frac{L_1}{v_1} + \frac{L_0}{v_0}$$



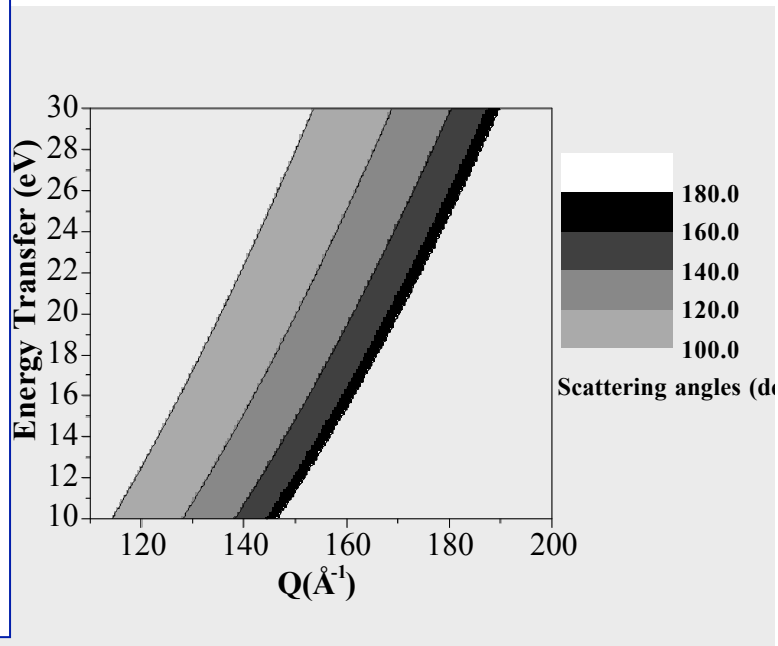
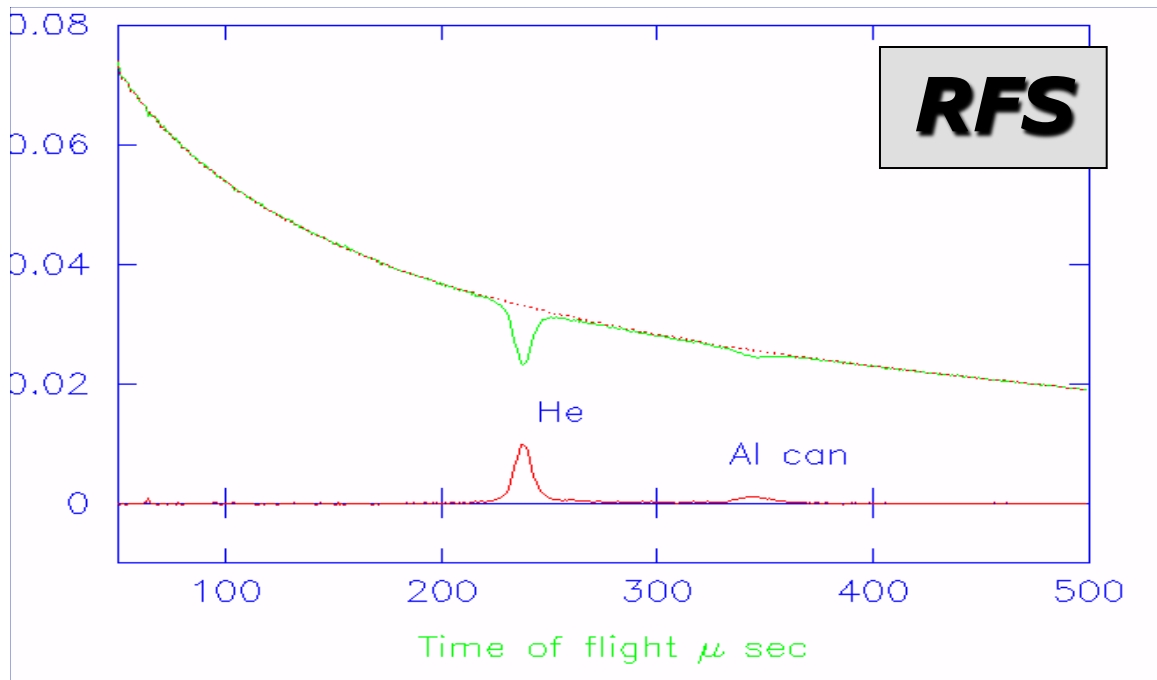
$$\hbar\omega = \frac{1}{2} m (v_0^2 - v_1^2)$$

$$q^2 = m^2 (v_0^2 + v_1^2 - 2v_0 v_1 \cos\theta)$$

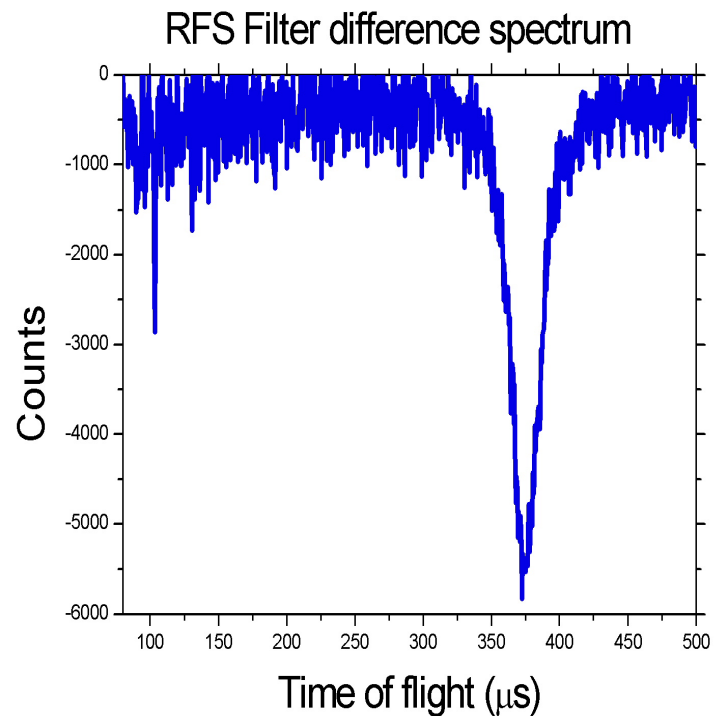
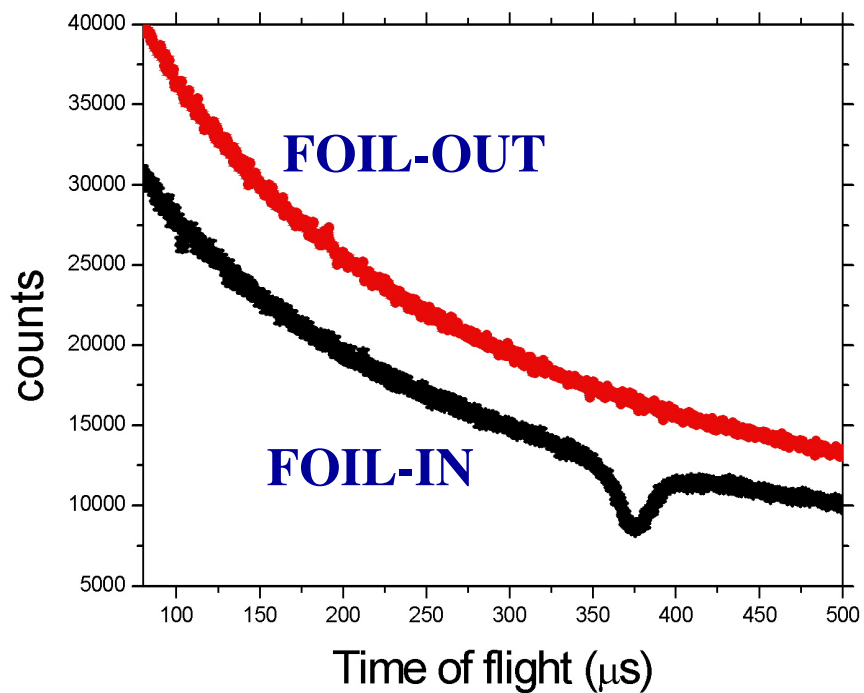


Incident neutrons of all available energies are generated at $t=0$

Scattered neutrons are energy selected by the filters and recorded by scintillators

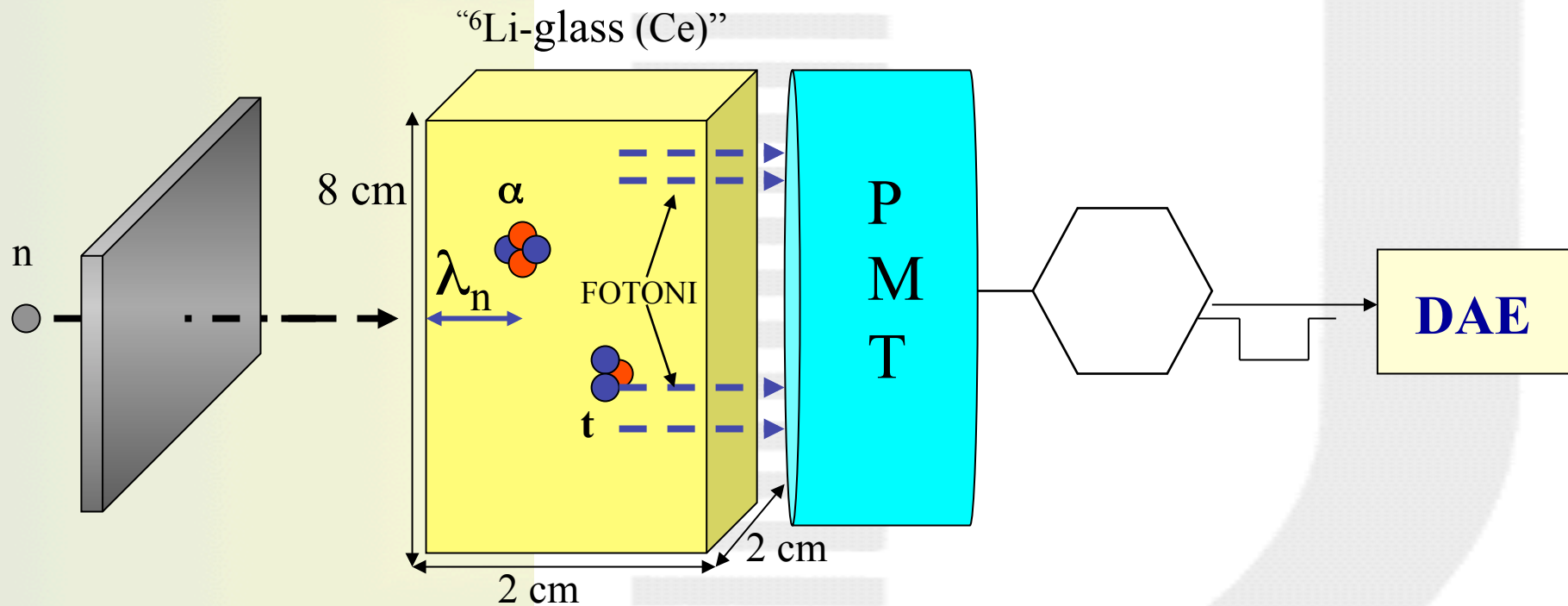


RFS Foil-out and Foil-in measurements (^{197}Au foil)



RFS mode

“Filter Difference” (FD)



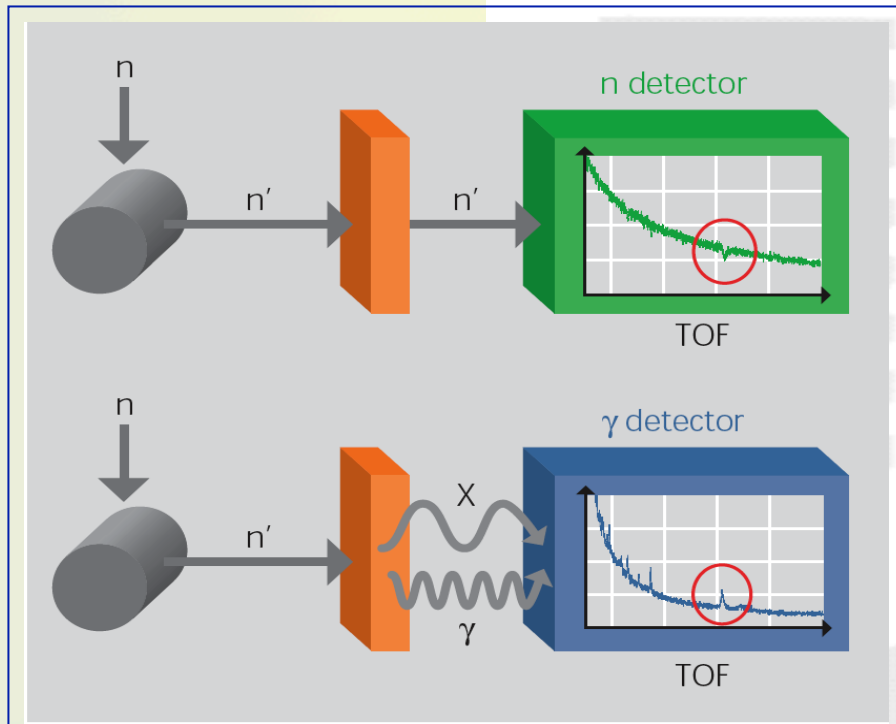
$$\eta_{\text{Li-glass}} \approx 1/v - \text{low efficiency for } E > 20 \text{ eV}$$

RFS & RDS ON VESUVIO

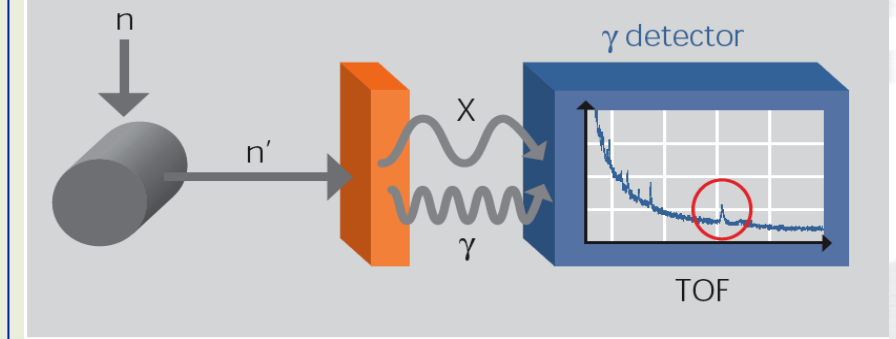


Principles of:
Resonance Filter Spectrometer (RFS)
Resonance Detector Spectrometer (RDS)

RFS



RDS



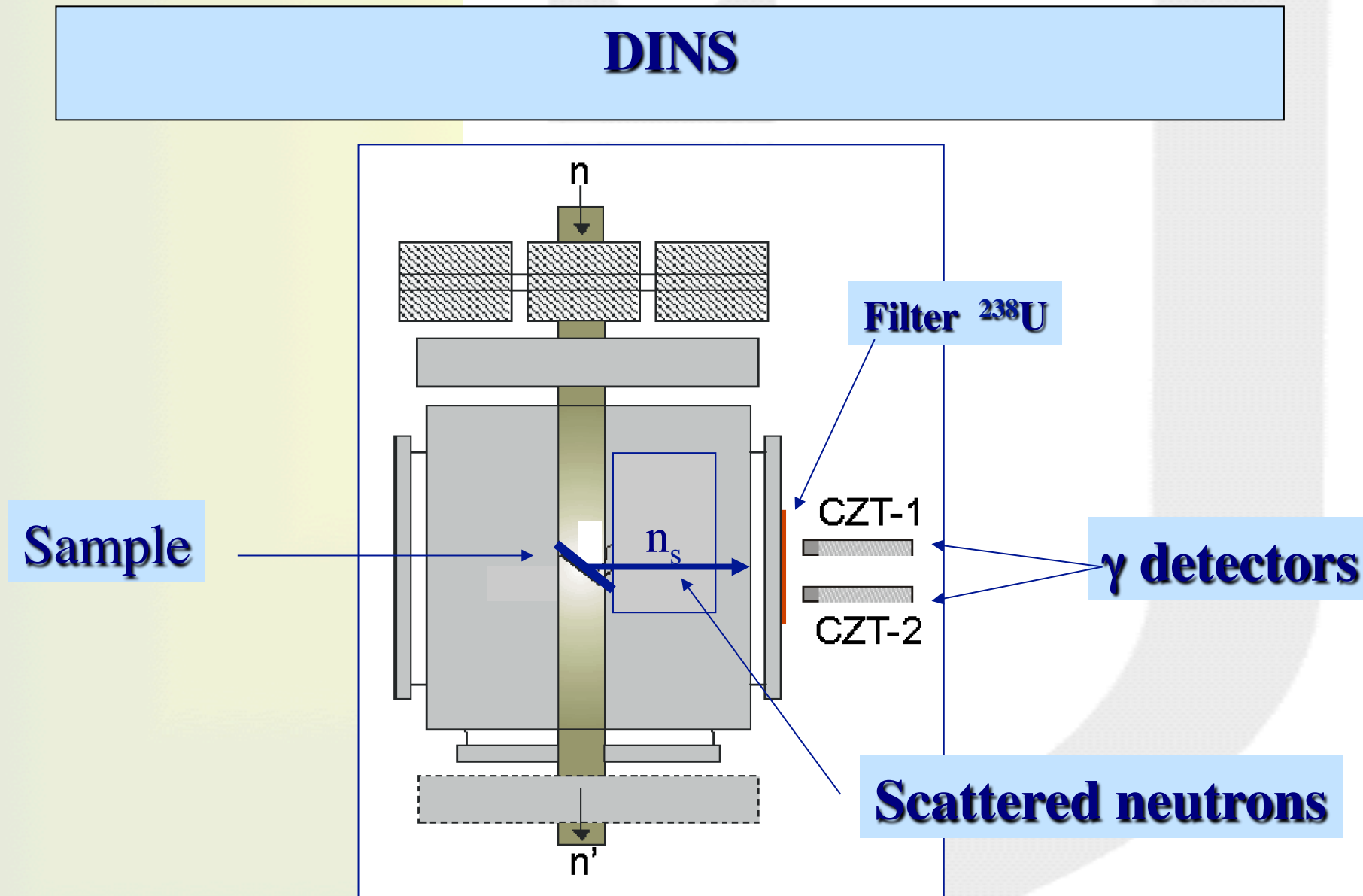
^6Li -glass
Neutron Detectors

γ
detectors

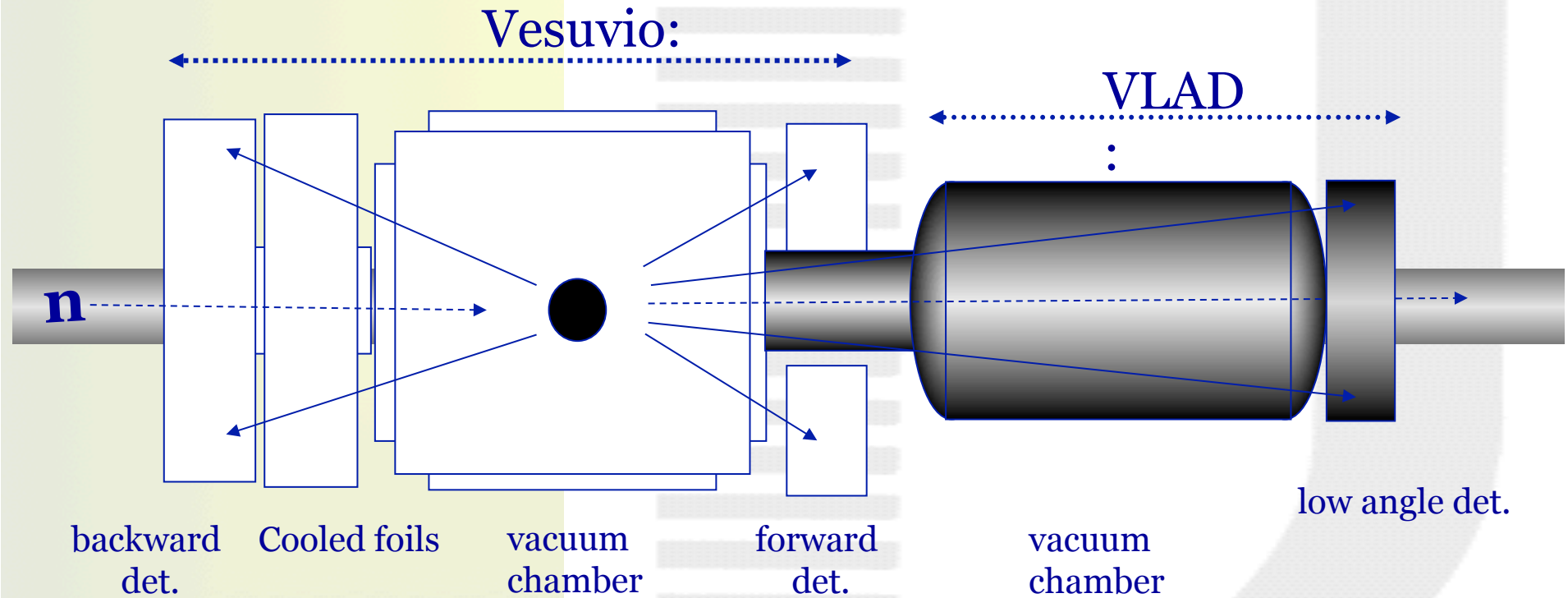


YAP scintillator

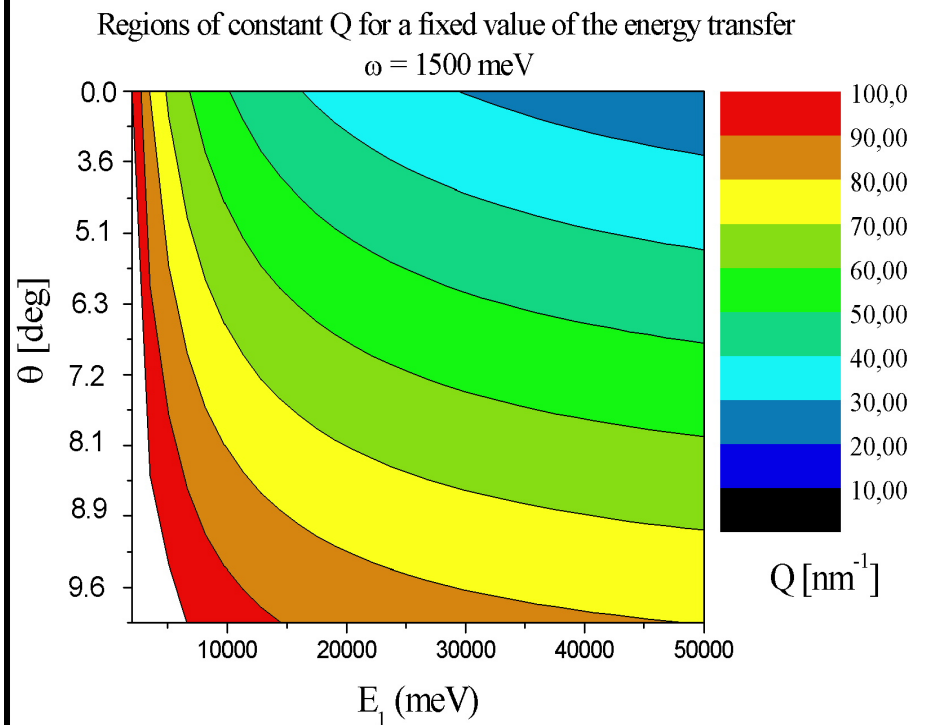
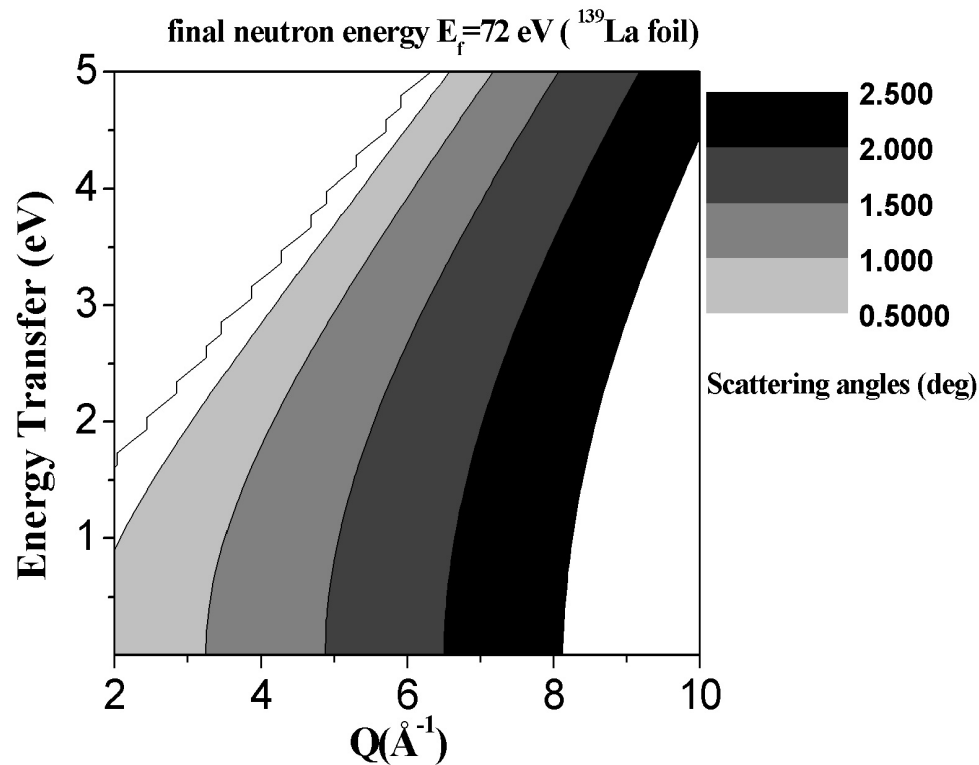
RDS mode



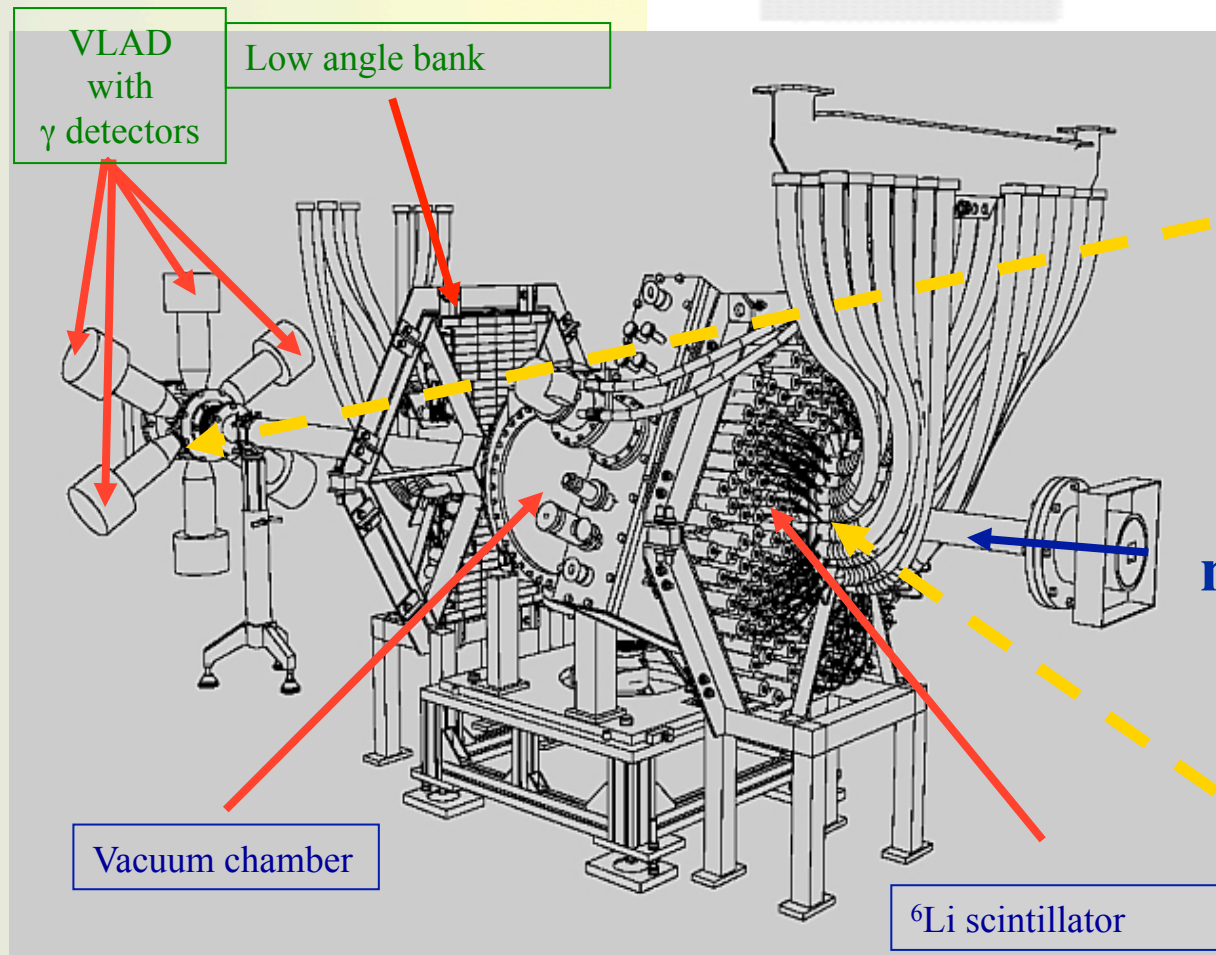
VLAD



Kinematic conditions for measurements of high energy excitation

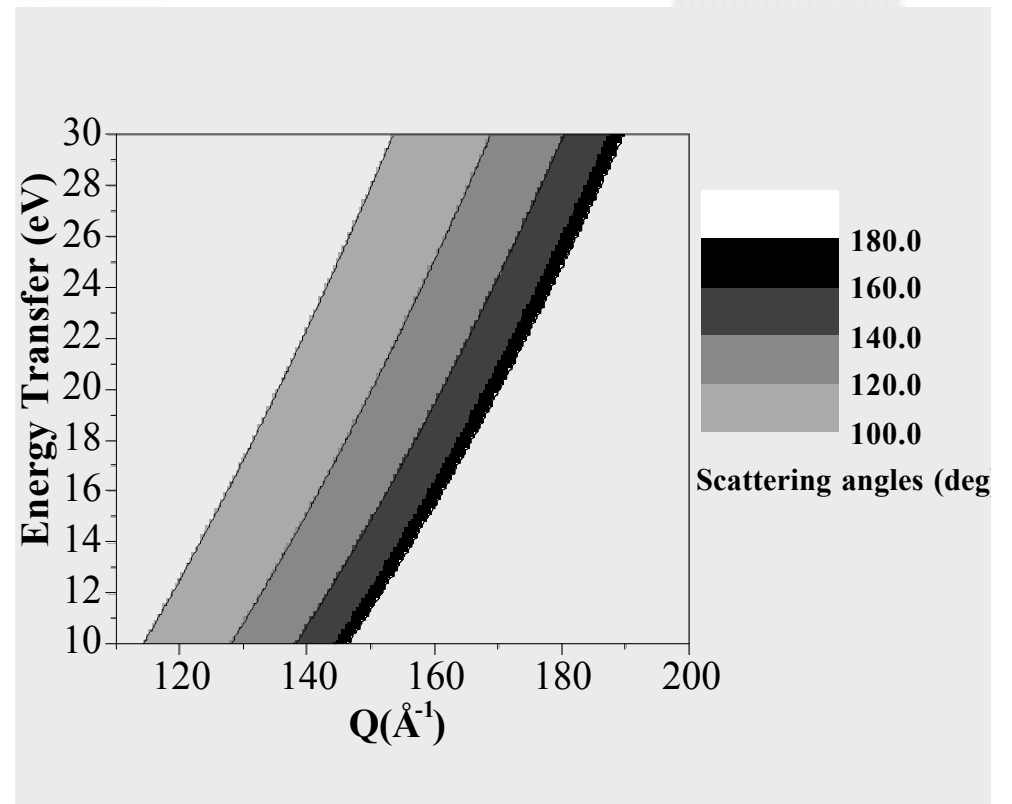
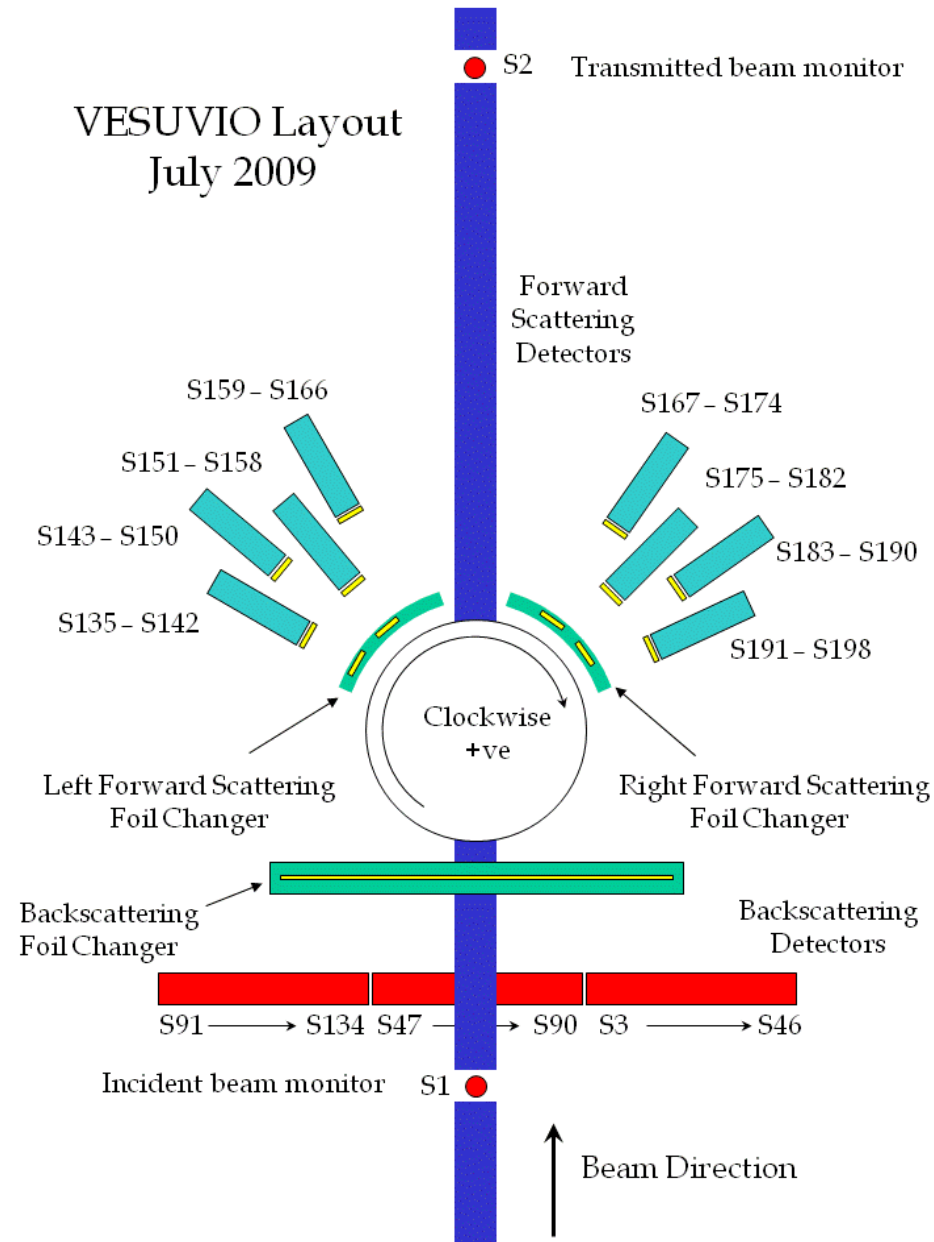


VESUVIO (DINS) & e.VERDI (HINS) RFS and RDS spectrometer



- RDS at very small scattering angles ($1^\circ < 2\theta < 5^\circ$)
- RDS at scattering angles ($20^\circ < 2\theta < 80^\circ$)
- RFS at backscattering angles

VESUVIO: Kinematic conditions



RESOLUTION COMPONENTS

Geometrical \mapsto Gaussian

Energy \mapsto Gaussian&Lorentzian

- **single difference (SD)**

example U foil

U resonances:

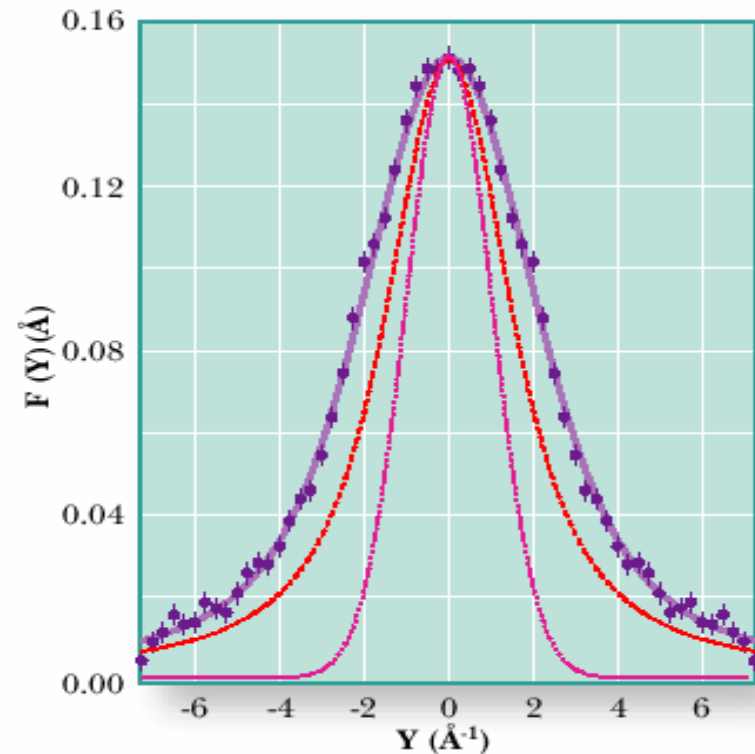
\mapsto 6.7eV, 20.7eV, 37eV..

FWHM (at 6.7 eV) \mapsto 0.04 eV

Doppler broadening

at RT \mapsto 0.11 eV

at 70 K \mapsto 0.06 eV



Scattering function $F(y)$ for the ^3He bcc solid sample. Data (full circles); best fit (purple line); resolution function SD (red line)

From Pb sample: VESUVIO resolution determined by fitting Lorentzian \otimes Gaussian convolution to the data and subtracting the Gaussian component, due to intrinsic width of the Pb sample.

HINS on VLAD $\Delta\hbar\omega/\hbar\omega$ resolution

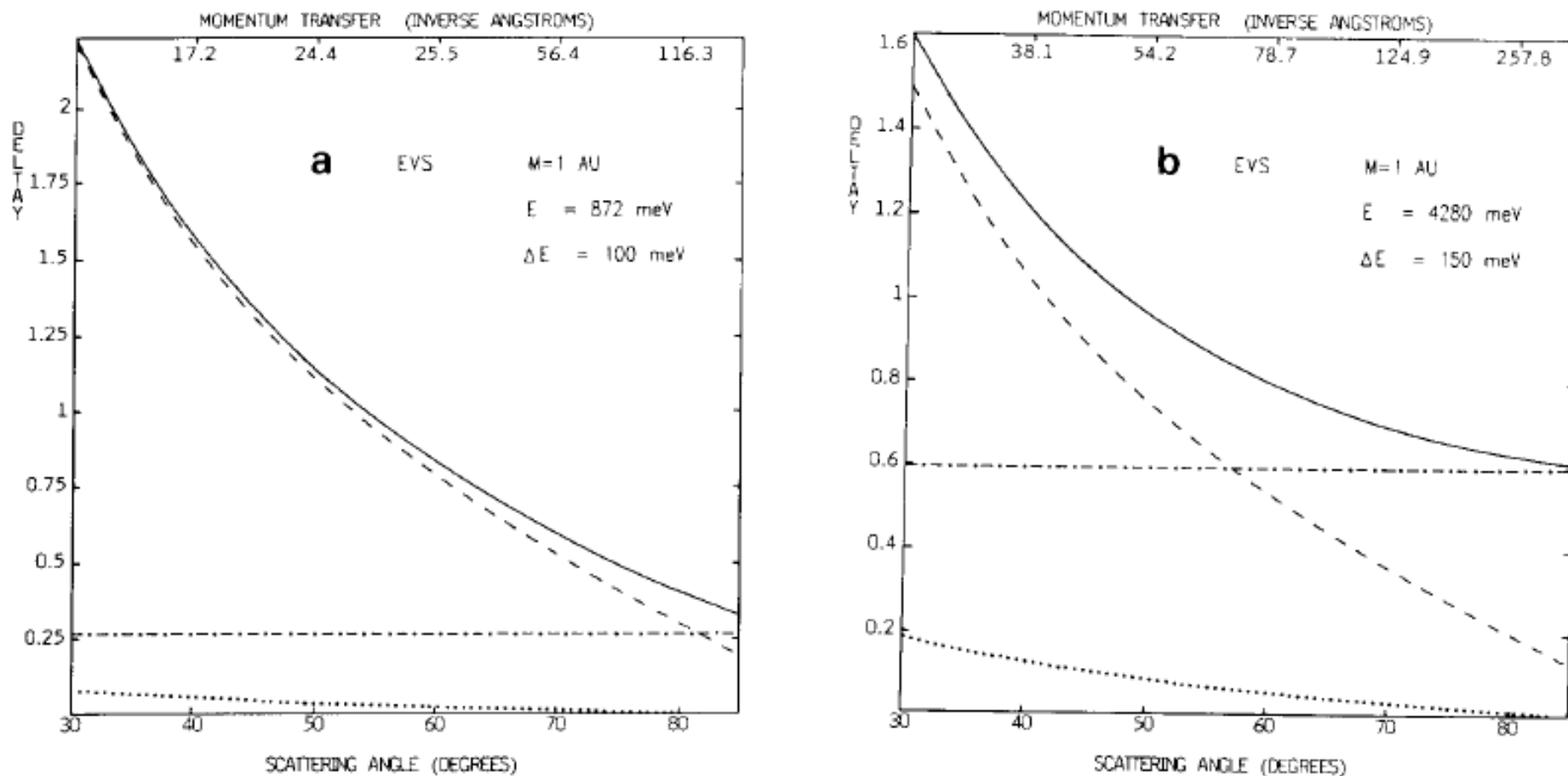


Fig. 2. The total resolution (—) and the energy (---), angular (-·-·-) and timing (·····) contributions to the resolution in hydrogen ($M=1$ amu) shown as a function of scattering angle and momentum transfer for eVS at (a) $E_1 = 872$ meV and (b) $E_1 = 4280$ meV.

HINS on VLAD $\Delta\hbar\omega / \hbar\omega$ resolution

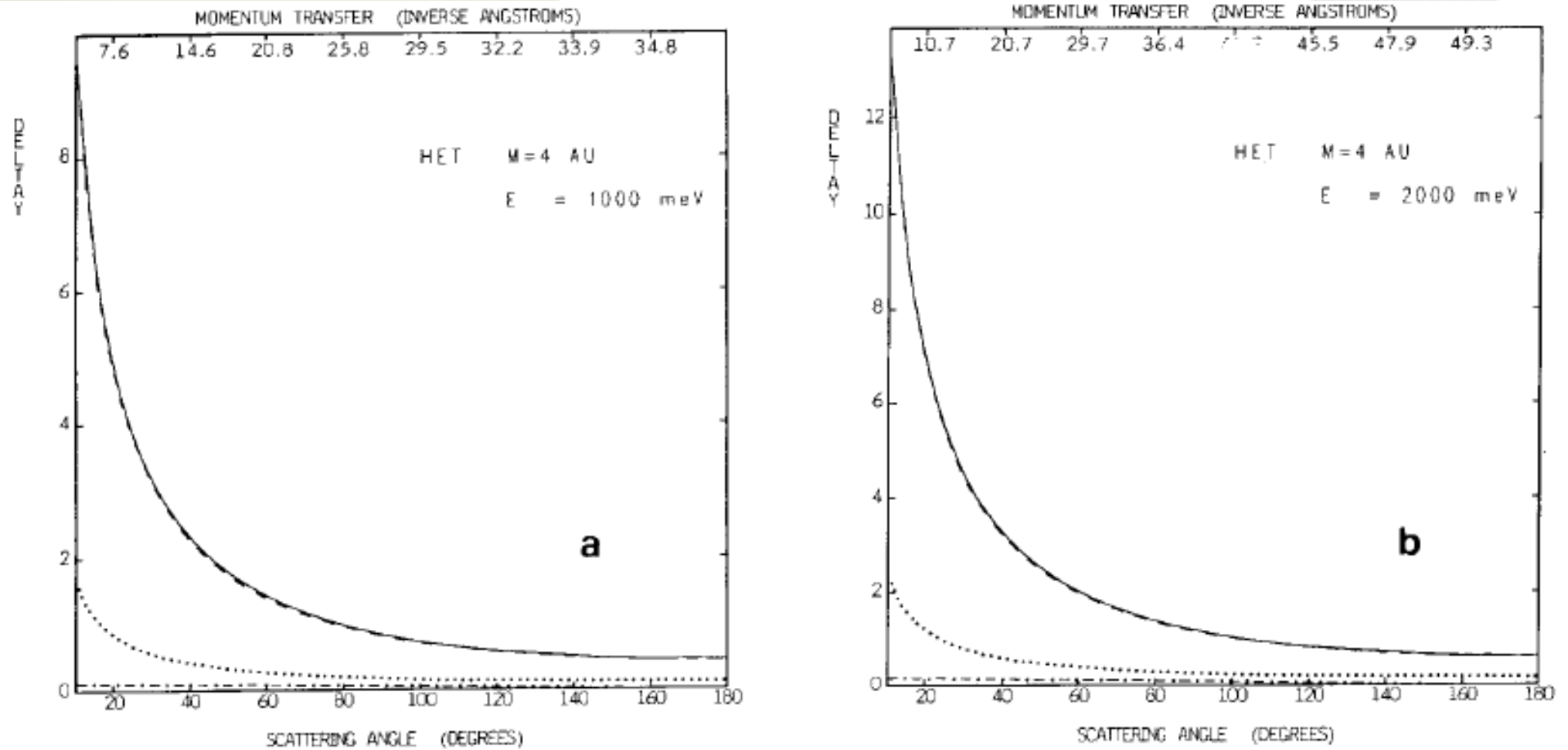
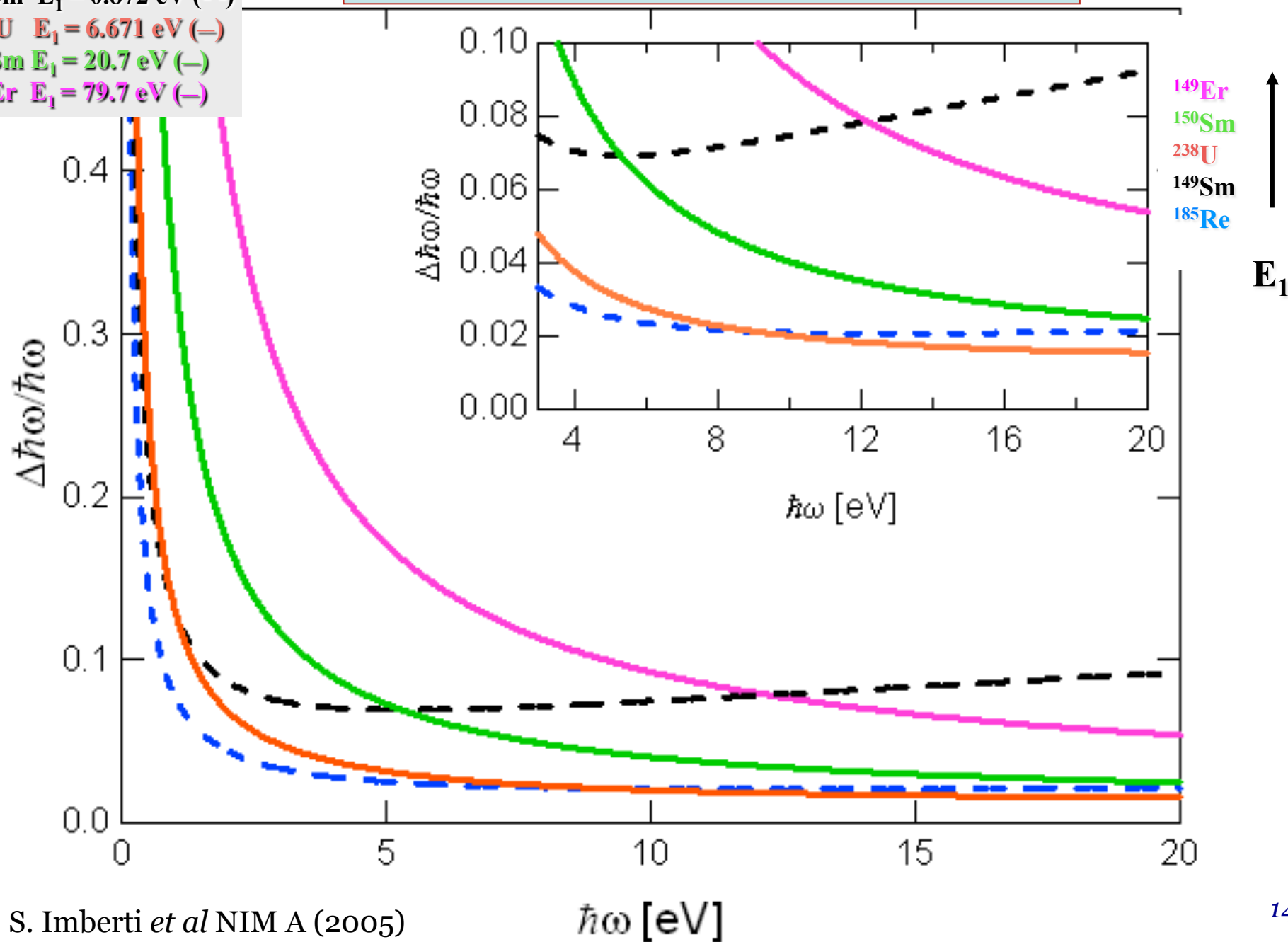


Fig. 3. The HET results for helium ($M = 4$ amu).

HINS on VLAD $\Delta\hbar\omega/\hbar\omega$ resolution

^{185}Re $E_1 = 2.16$ eV (- -)
 ^{149}Sm $E_1 = 0.872$ eV (- -)
 ^{238}U $E_1 = 6.671$ eV (-)
 ^{150}Sm $E_1 = 20.7$ eV (-)
 ^{149}Er $E_1 = 79.7$ eV (-)



Deep Inelastic Neutron Scattering

1. High q implies, because of Heisenberg indetermination principle, **scattered neutron explores regions of the sample of small dimension**, thus suited probe to study single particle properties, (no coherent effects from collective dynamics inside the system) (*Incoherent Approximation*); the distance over which the neutron phase change appreciably is much lower than the typical interparticle distance d :

$$\frac{2\pi}{q} \ll \bar{d}$$

2. High ω implies, because of the time-energy indetermination principle, that the **scattering process occurs in a very short time** (*Impulse Approximation, IA*).

Deep Inelastic Neutron Scattering, DINS

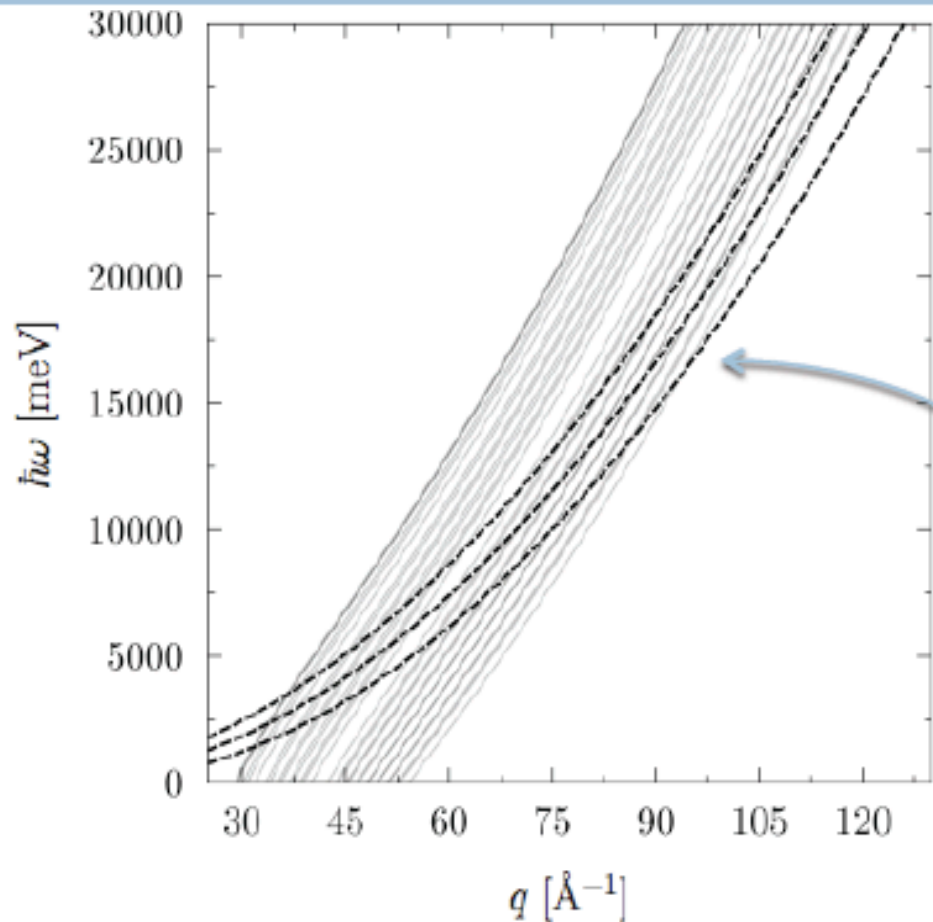
For a monoatomic system:

$$\frac{d^2\sigma}{d\Omega dE_f} = \frac{1}{\hbar} \frac{k_f}{k_i} \left[\frac{\sigma_c}{4\pi} S(\mathbf{q}, \omega) + \frac{\sigma_i}{4\pi} S_i(\mathbf{q}, \omega) \right]$$

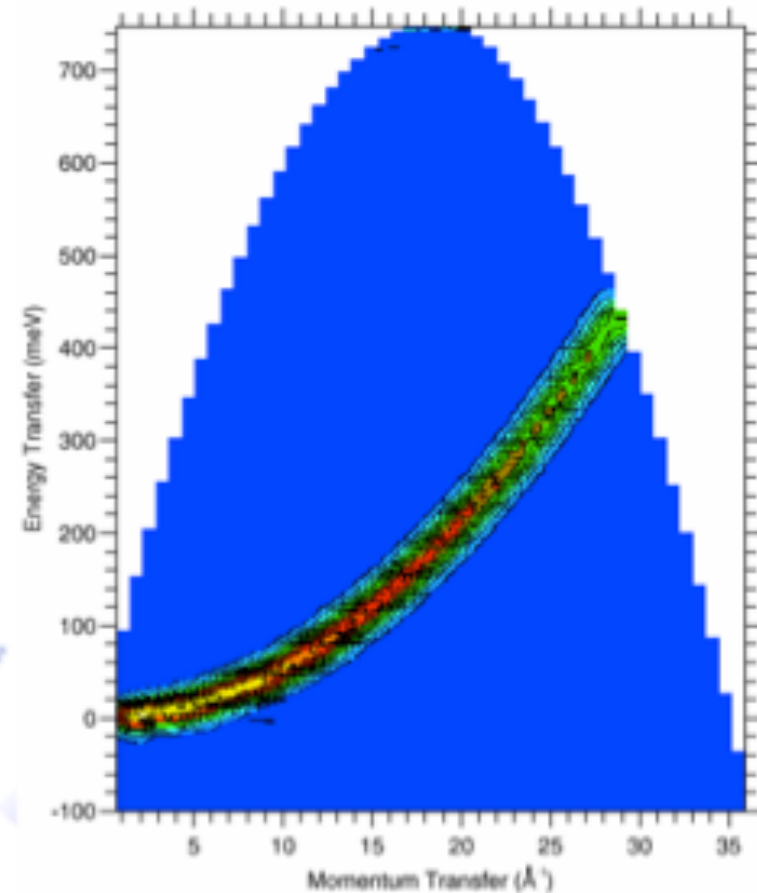
Impulse Approximation: high q and ω

$$S_{IA}(\mathbf{q}, \omega) = \int \underline{n(\mathbf{p})} \delta \left(\hbar\omega - \frac{\hbar^2 q^2}{2M} - \frac{\hbar\mathbf{q} \cdot \mathbf{p}}{M} \right) d\mathbf{p}$$

Nuclear quantum effects on proton momentum distribution



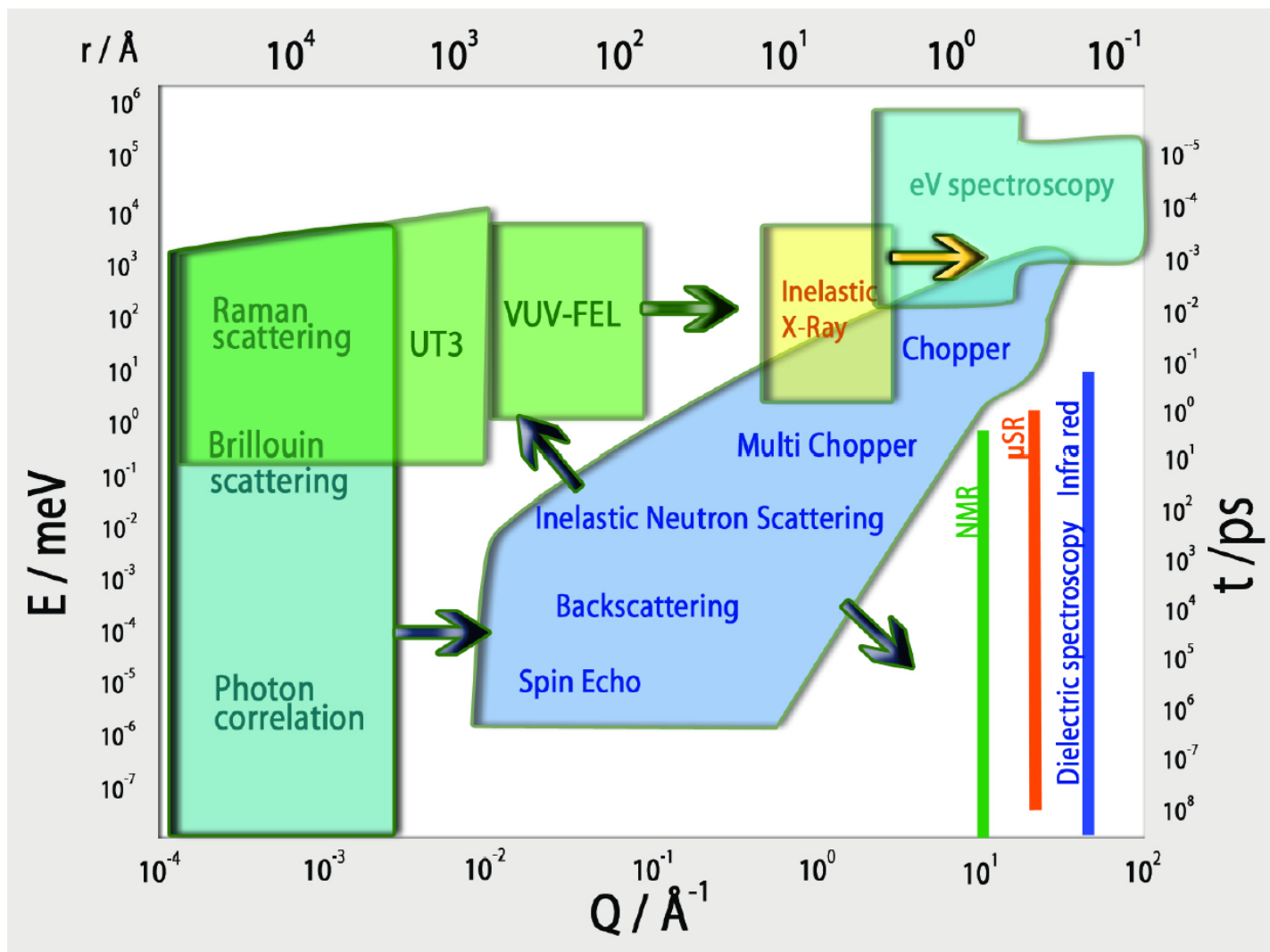
^4He (Stirling et al)



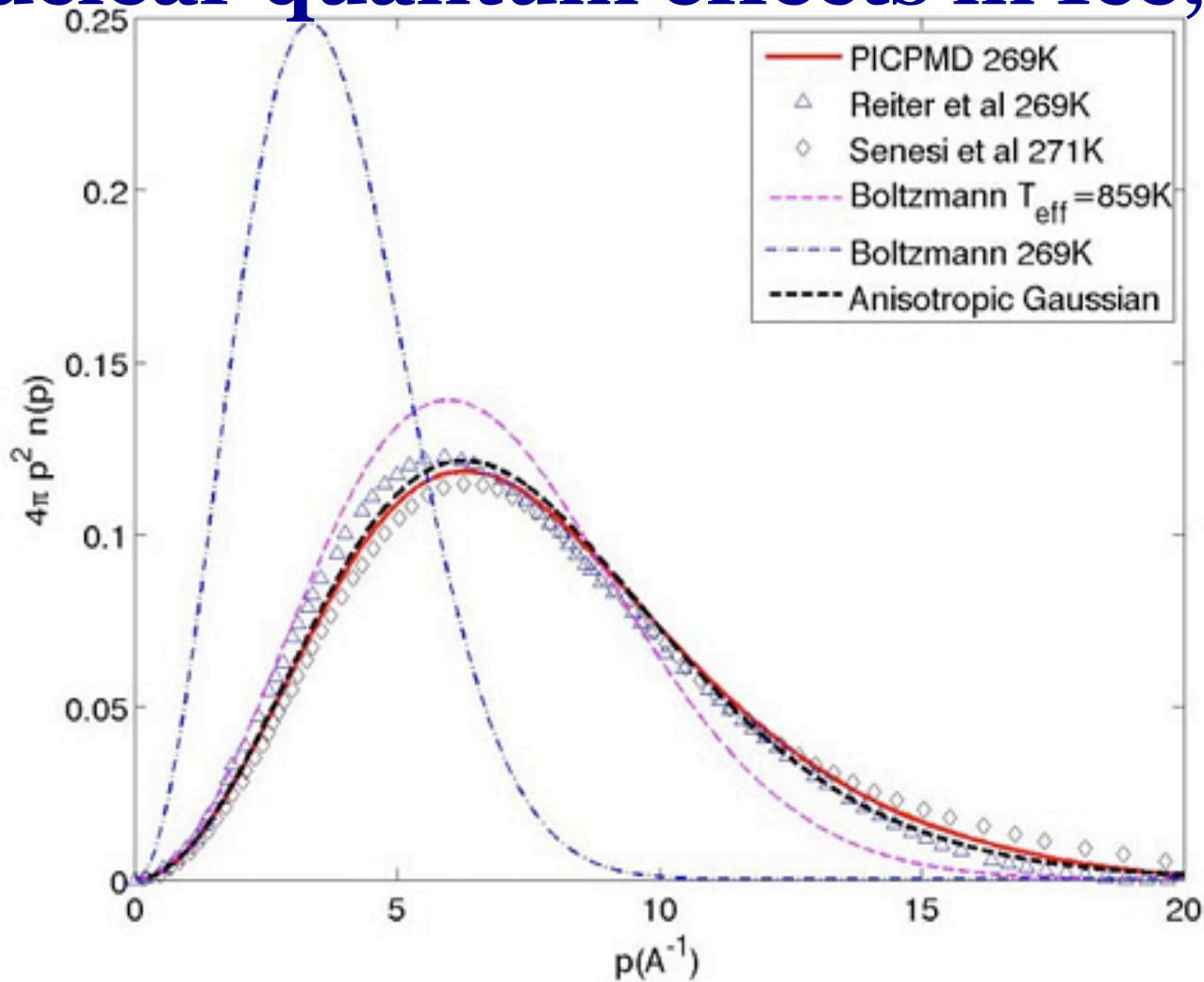
Vesuvio Beamline at the ISIS neutron source -UK

DINS

Figure 2. Schematic plot illustrating the kinematical $(q, \hbar\omega)$ range accessed by different spectroscopic techniques; on the top right of the figure the t and r ranges for DINS (10^{-5} ps $< t < 10^{-3}$ ps, $0.1 \text{ \AA} < r < 0.2 \text{ \AA}$) and HINS (10^{-6} ps $< t < 10^{-2}$ ps, $0.2 \text{ \AA} < r < 2 \text{ \AA}$) spectroscopy is shown.



Nuclear quantum effects in Ice, T=296K

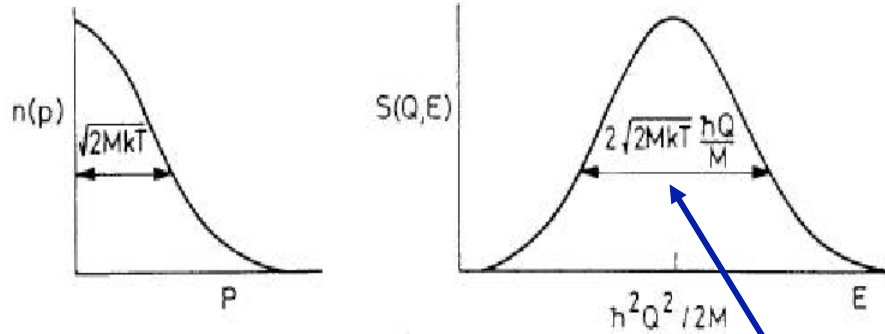


The spherically averaged momentum distribution of the protons (shown here for ice at T=296 K) is quite different from the classical Maxwell-Boltzmann distribution at the same temperature. The path integral ab-initio molecular dynamics (PICPMD) result agrees well with two experiments (Reiter and Senesi). The Maxwell-Boltzmann distribution that fits better the PICPMD data has T=859 K. An even better fit is obtained with an anisotropic (multivariate) Gaussian distribution.

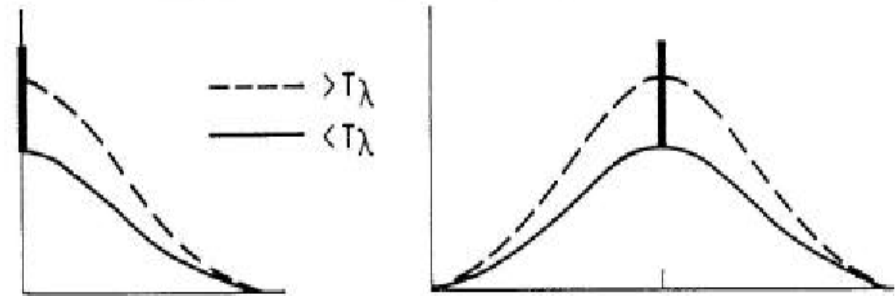
ulture (2011)

Courtesy of Roberto Car (Princeton University)

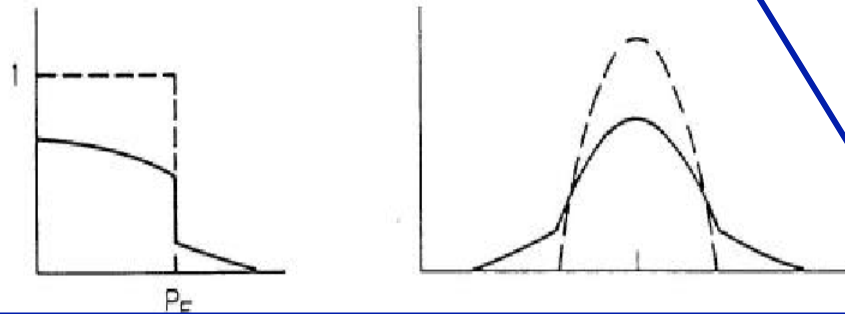
CLASSICAL IDEAL GAS



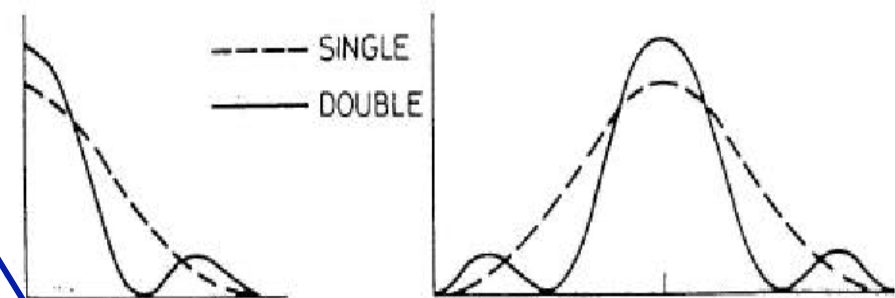
BOSE CONDENSATE (⁴He)



FERMI LIQUID (³He)



PARTICLE IN POTENTIAL WELL



$$n(p) \propto \exp\left[-p^2 / (4Mk_B T)\right] \Rightarrow \sigma^2 = 2Mk_B T \quad \text{Classical system}$$

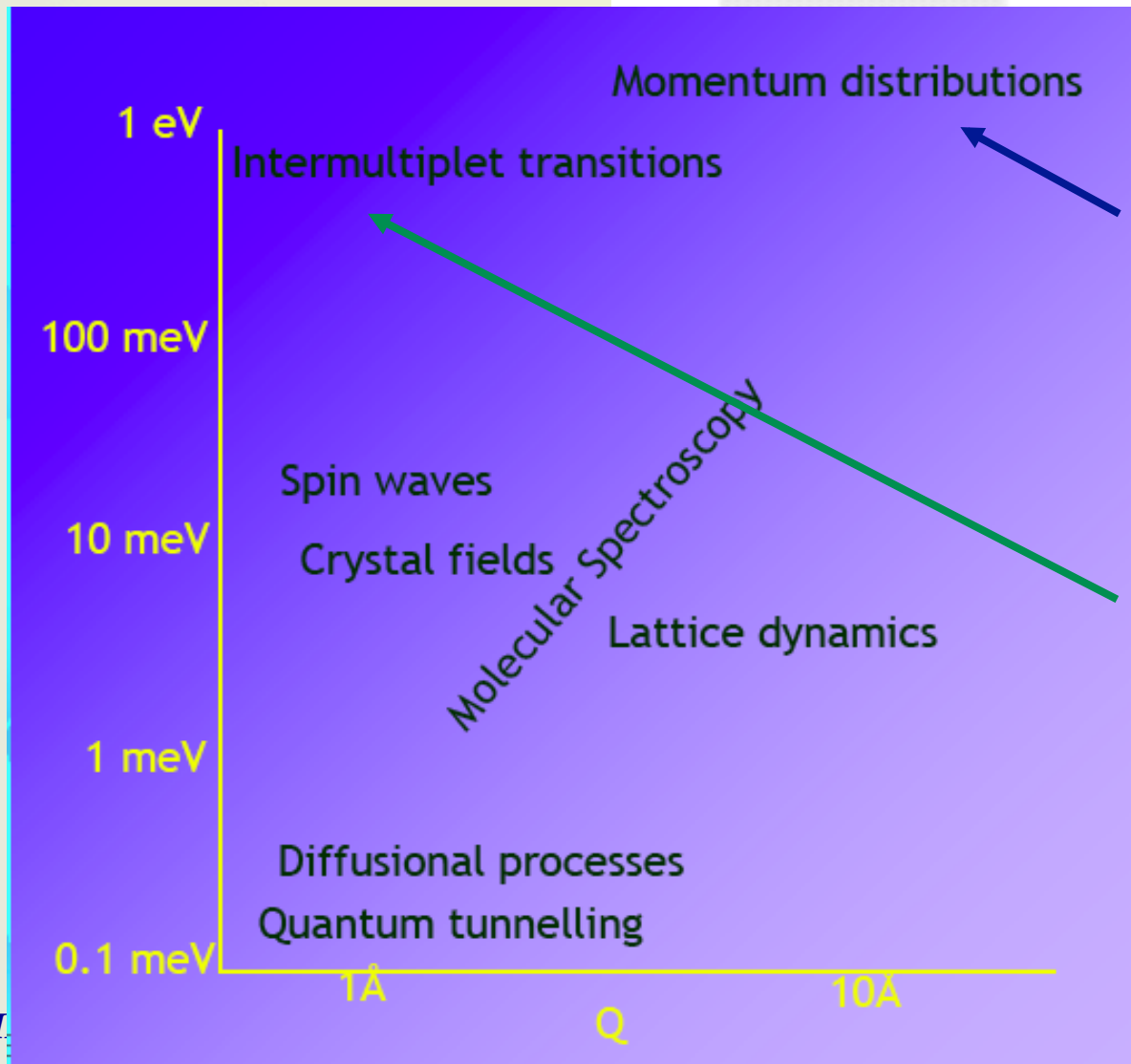
Peak width of $S(Q, \omega)$ provides a direct measure of $\langle E_k \rangle$

$$\langle E_k \rangle = \frac{1}{2M} \int n(p) p^2 dp$$

$$n(\vec{p}) = \left| \int \psi(\vec{r}) \exp(i\vec{p} \cdot \vec{r}) d\vec{r} \right|^2$$

Momentum Distribution is "Diffraction Pattern" of Wave function

eV SPECTROSCOPY



(DINS):

- $\langle E_k \rangle$

- $n(p)$

•(HINS):

- **high energy excitations**

0.1 meV
 1 meV
 10 meV
 100 meV
 1 eV



EXAMPLES

Light and heavy particles quantum dynamics

Nuclear Quantum Effects

35

$$\Delta x \Delta p \geq \hbar / 2$$

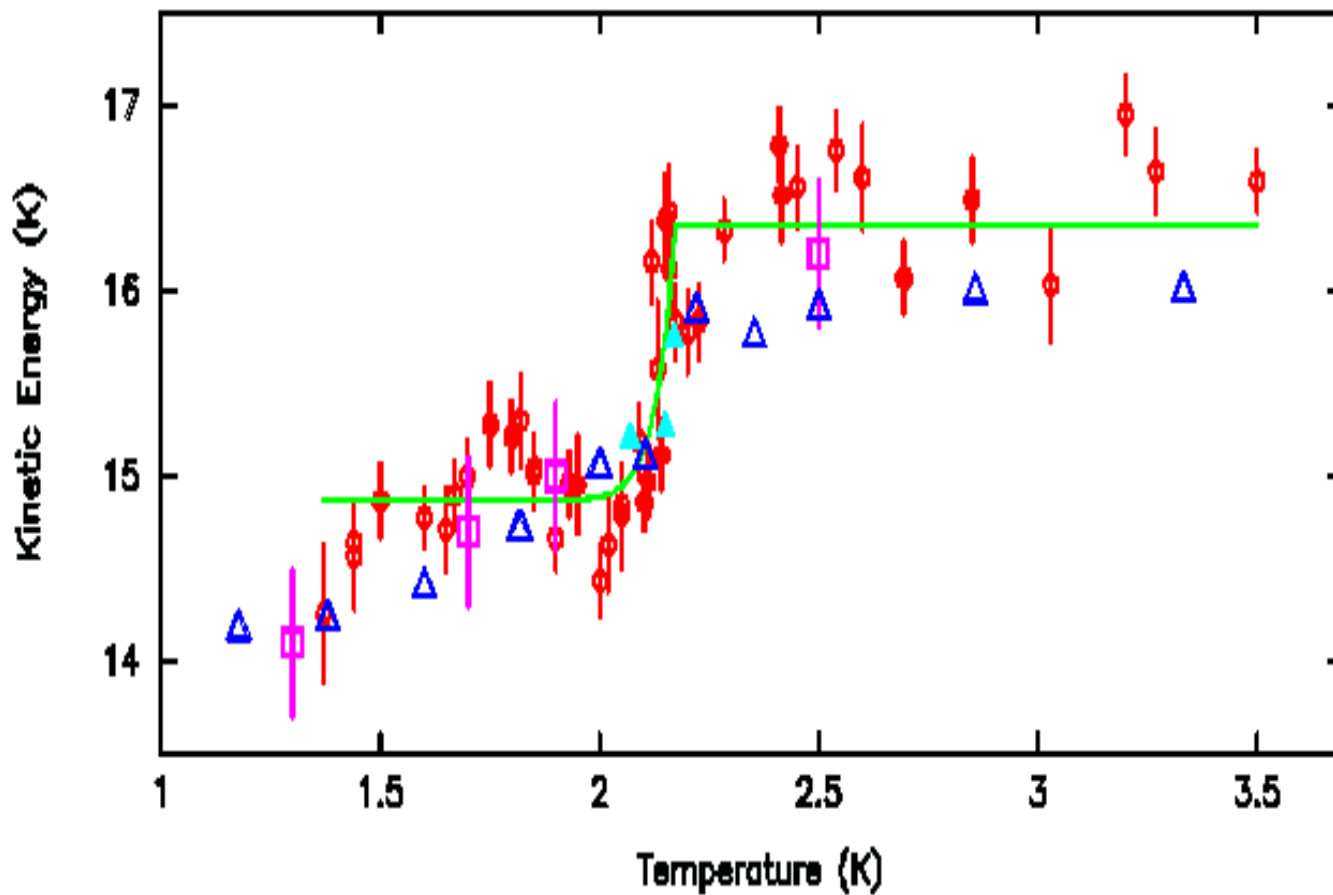
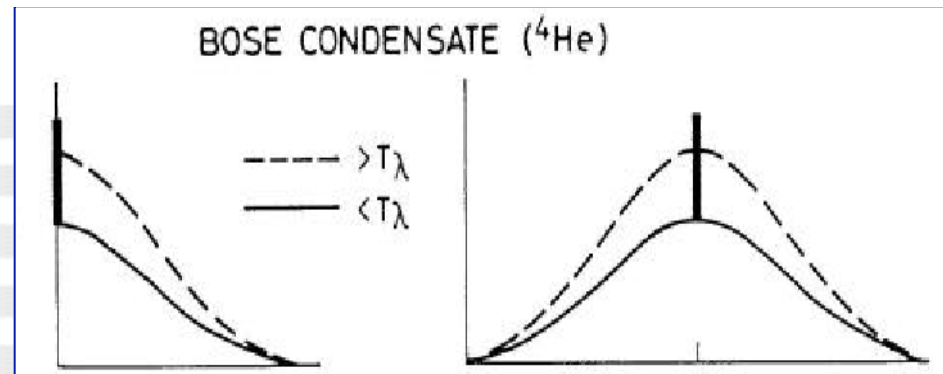
$\langle E_K \rangle$ & $n(p) \rightarrow$ PES

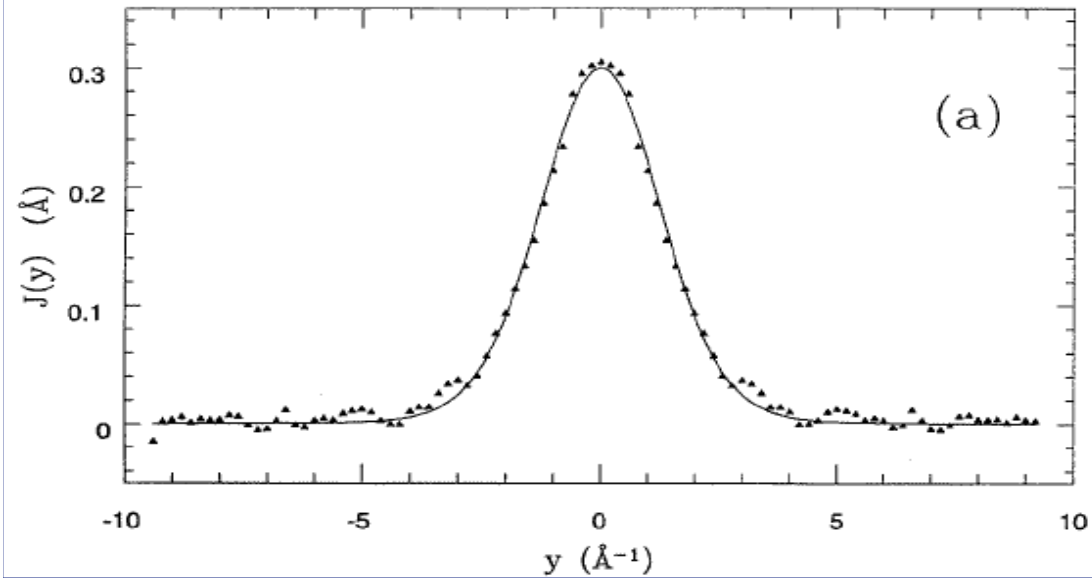
$$n(\vec{p}) = \left| \int \psi(\vec{r}) \exp(i\vec{p} \cdot \vec{r}) d\vec{r} \right|^2 \quad \langle E_k \rangle = \frac{1}{2M} \int n(\mathbf{p}) p^2 d\mathbf{p}$$

localization \rightarrow **excess of $\langle E_K \rangle$**

DINS on VESUVIO

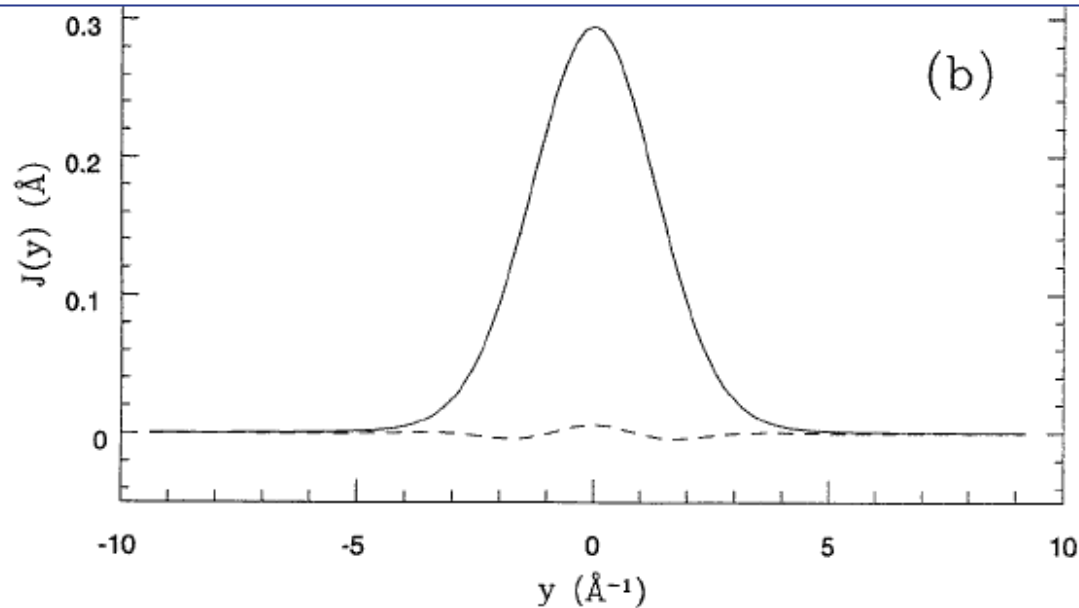
Mean Kinetic Energy of ^4He





^4He at 2.5 K

Figure 3. (a) The response function $J(y)$ for ^4He at 2.5 K from the cooled U filter: triangles are the experimental data; the solid line is the result of the fit. (b) The solid line is the Gaussian component of the fit; the dashed line is the non-Gaussian component, with both including a resolution contribution.

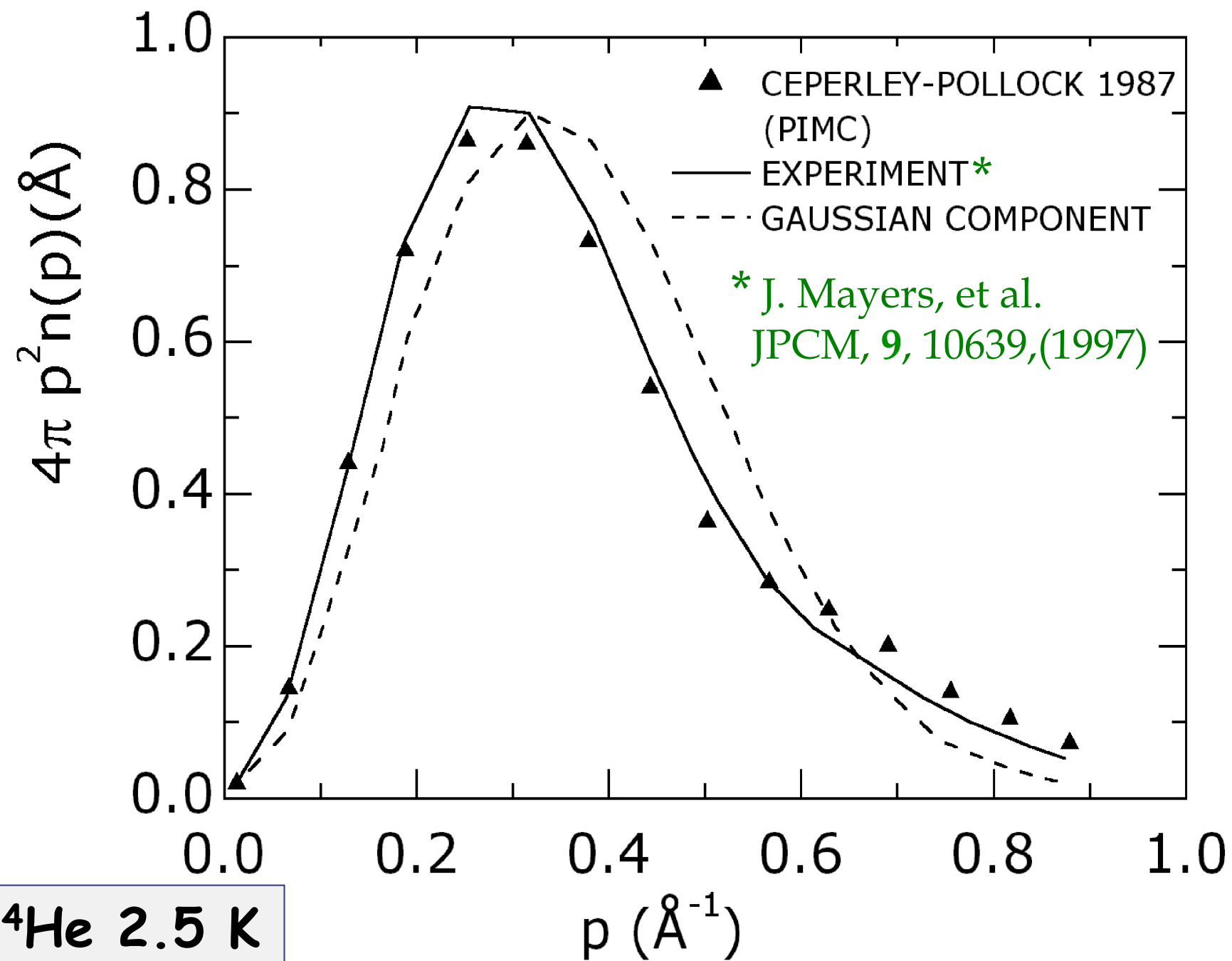


$$\delta = 0.63 \pm 0.06$$

* J. Mayers, et al.

on and Centre for Scientific Culture (2014)
JPCM, 9, 10639, (1997)

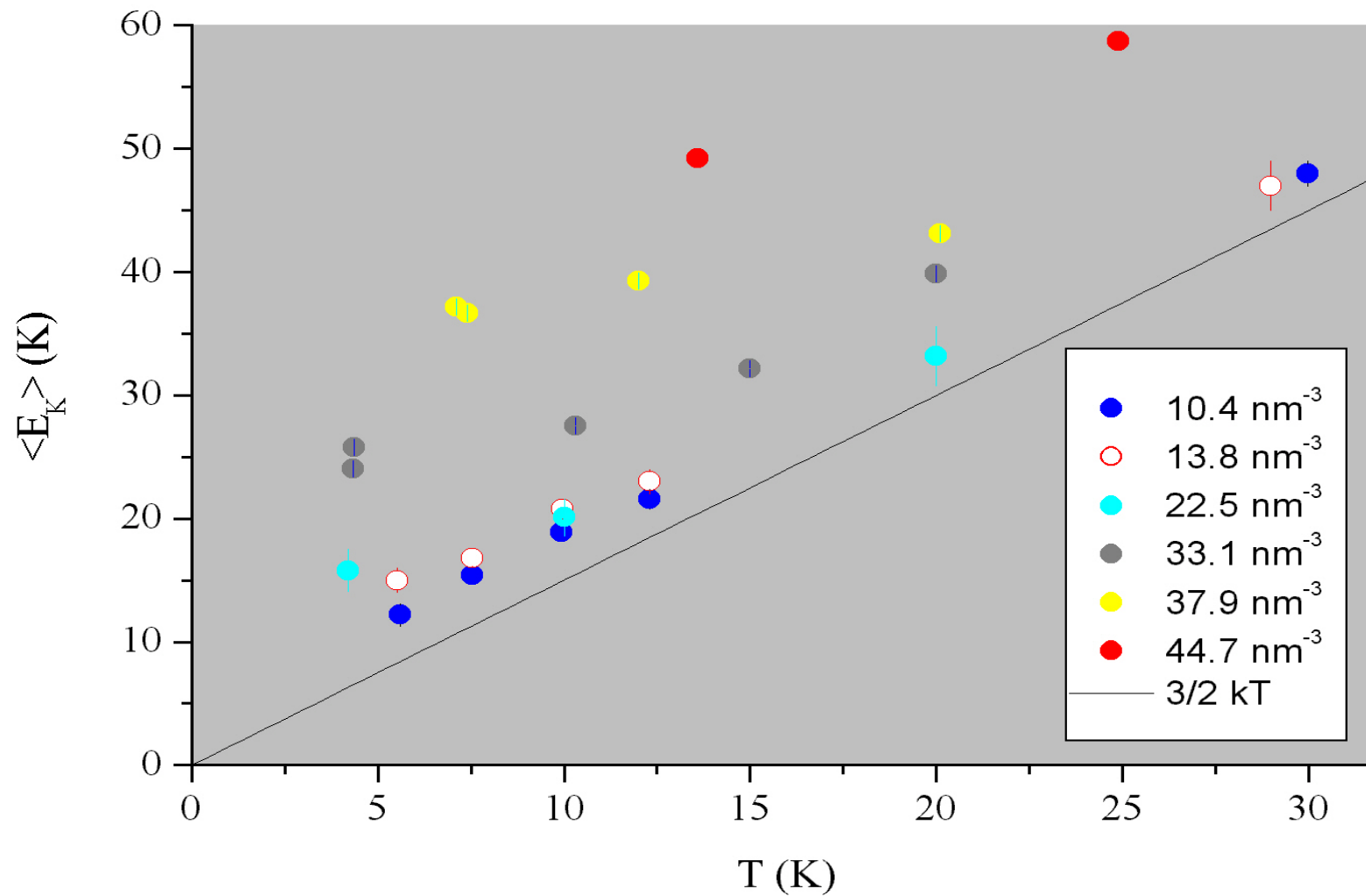
SPERICALLY AVERAGED $n(p)$



^4He 2.5 K

▲ CEPERLEY-POLLOCK 1987 (PIMC)
— EXPERIMENT*
- - - GAUSSIAN COMPONENT
* J. Mayers, et al. JPCM, 9, 10639, (1997)

Liquid ^4He $\langle E_K \rangle > 3/2 kT$!



DINS in $p\text{-H}_2$

N(p) Gaussian
 $\langle E_K \rangle_{\text{TRAS}}$ vs T

■ $\rho = 22.41 \text{ nm}^{-3}$

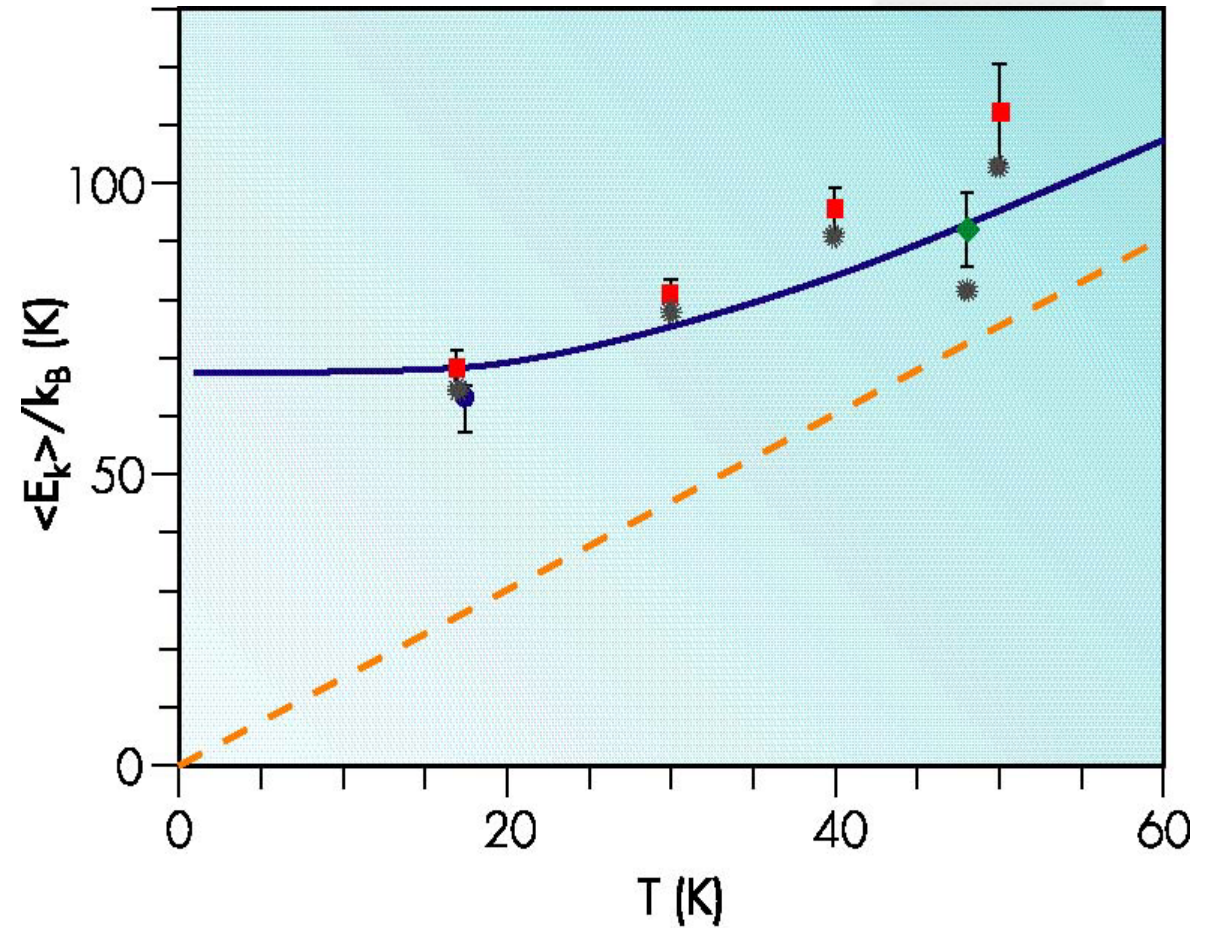
◆ $\rho = 10.45 \text{ nm}^{-3}$

● Langel et al.

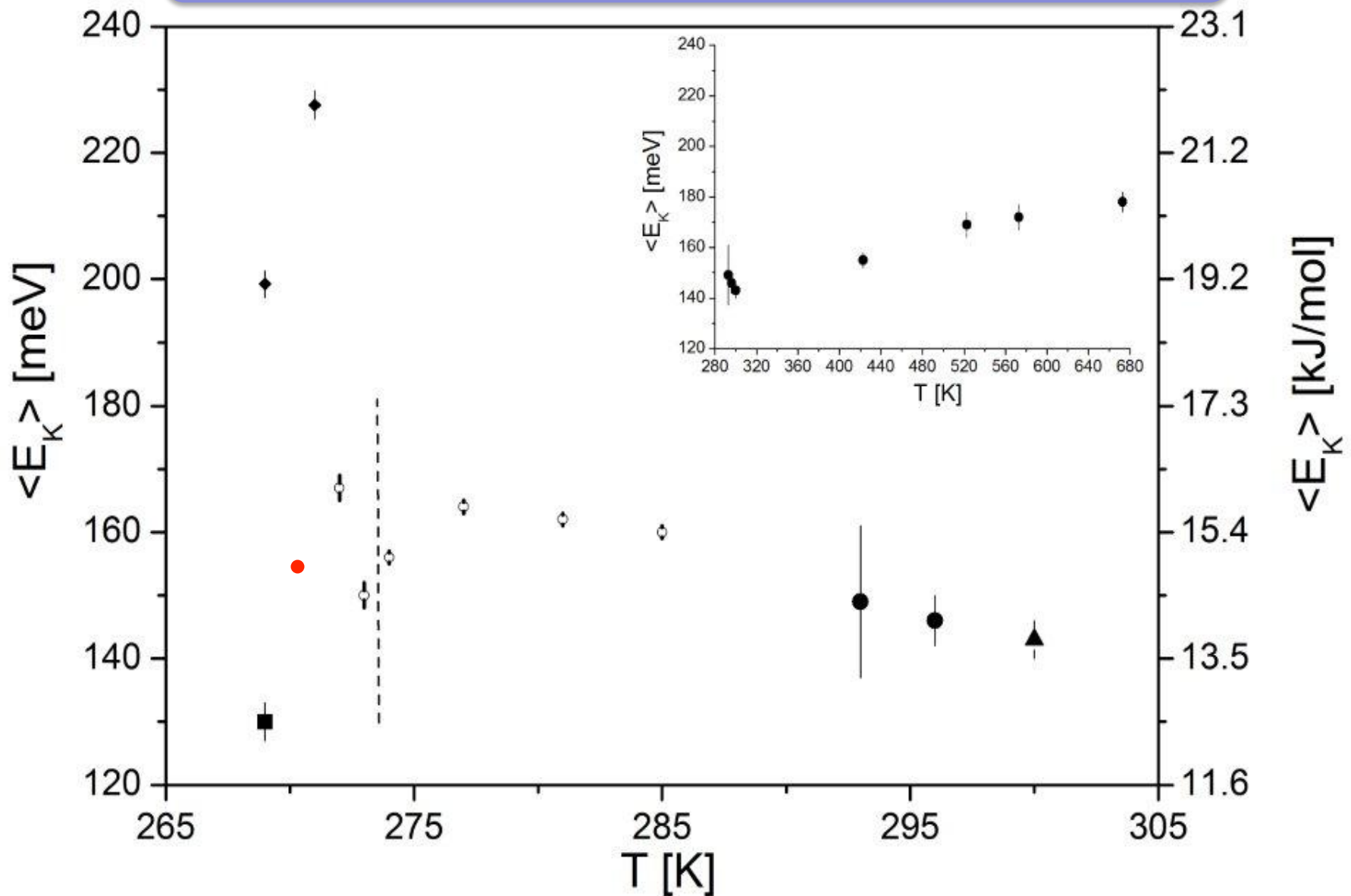
* PIMC

-- classic model

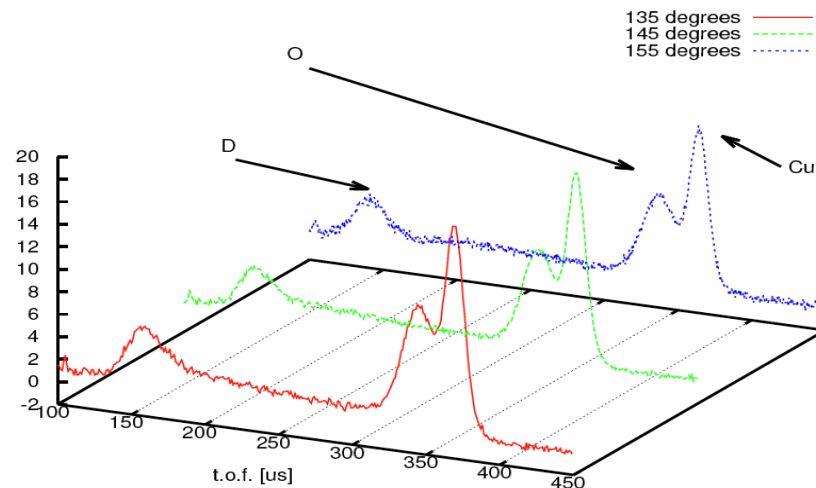
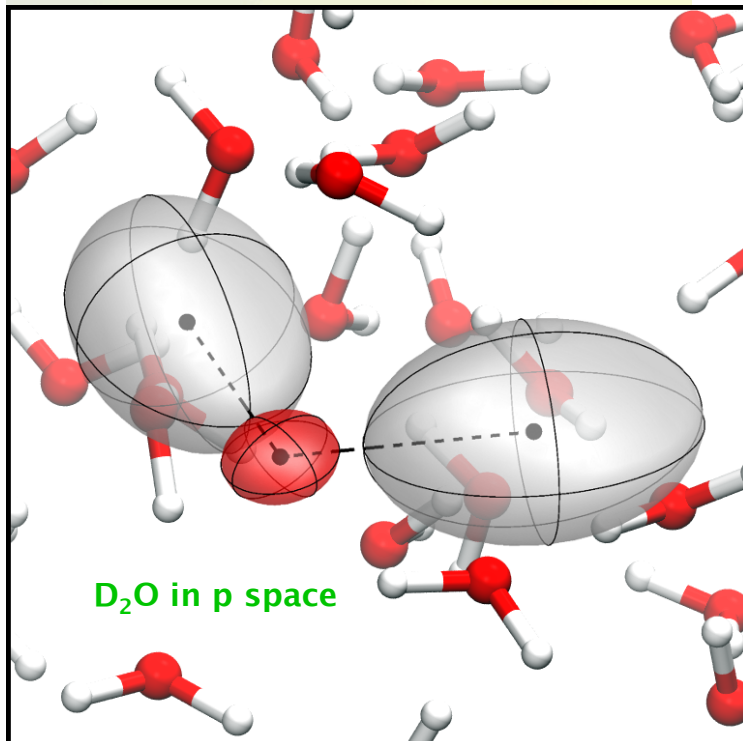
— harmonic model



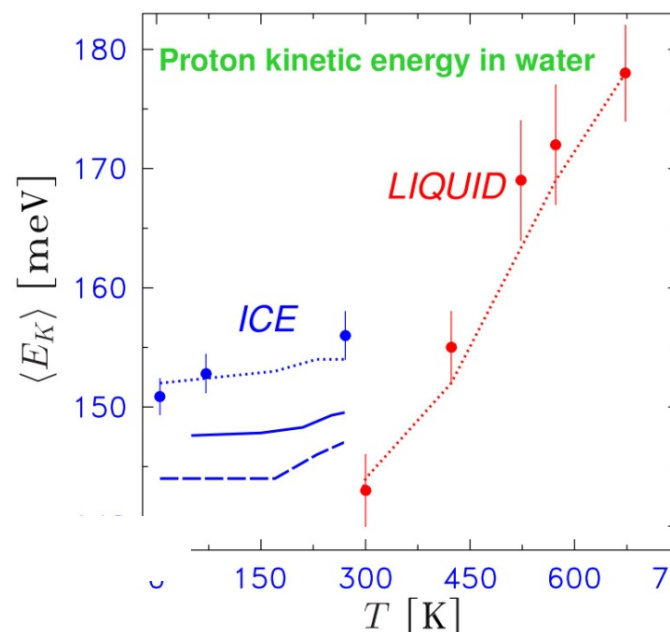
Water: Stable and Metastable Phases



Results from H, D and O in D₂O



G. Romanelli, *et. al.* J.P.C.L. 4 (19), 3251 (2013).

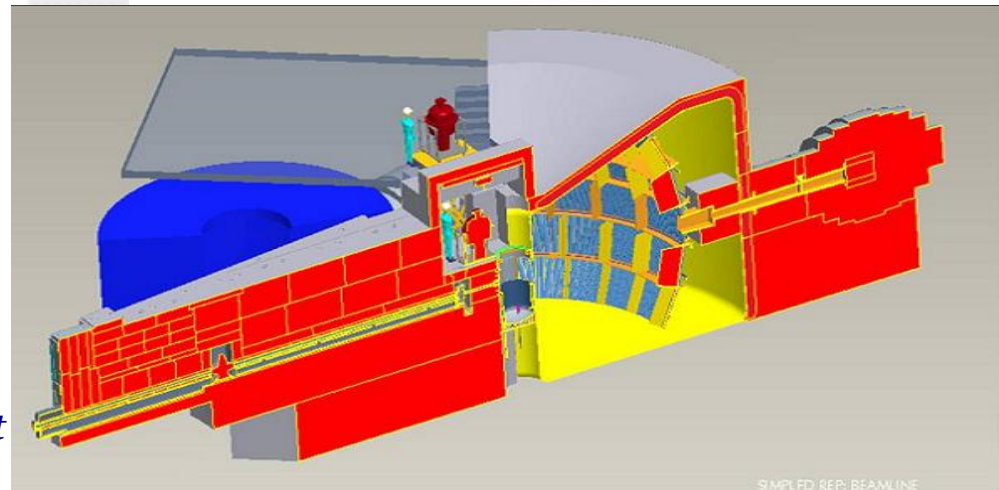
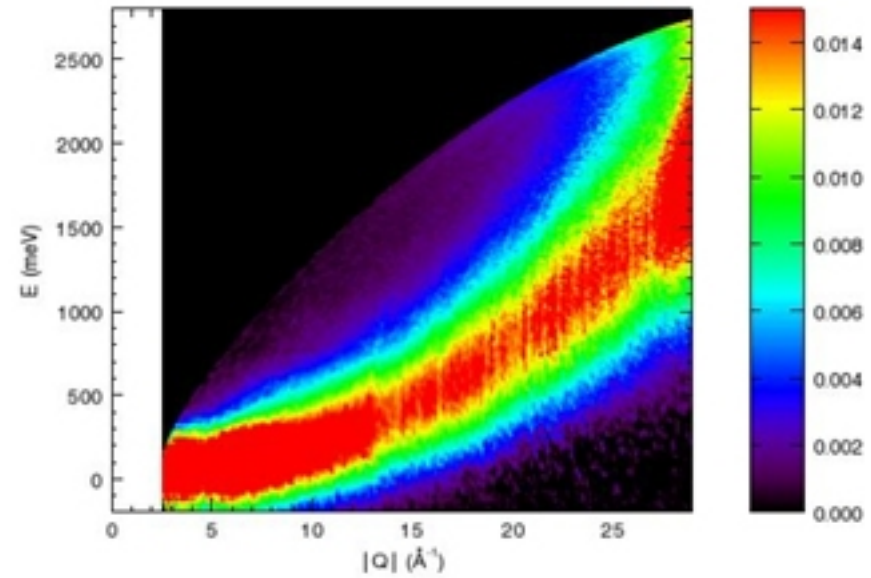
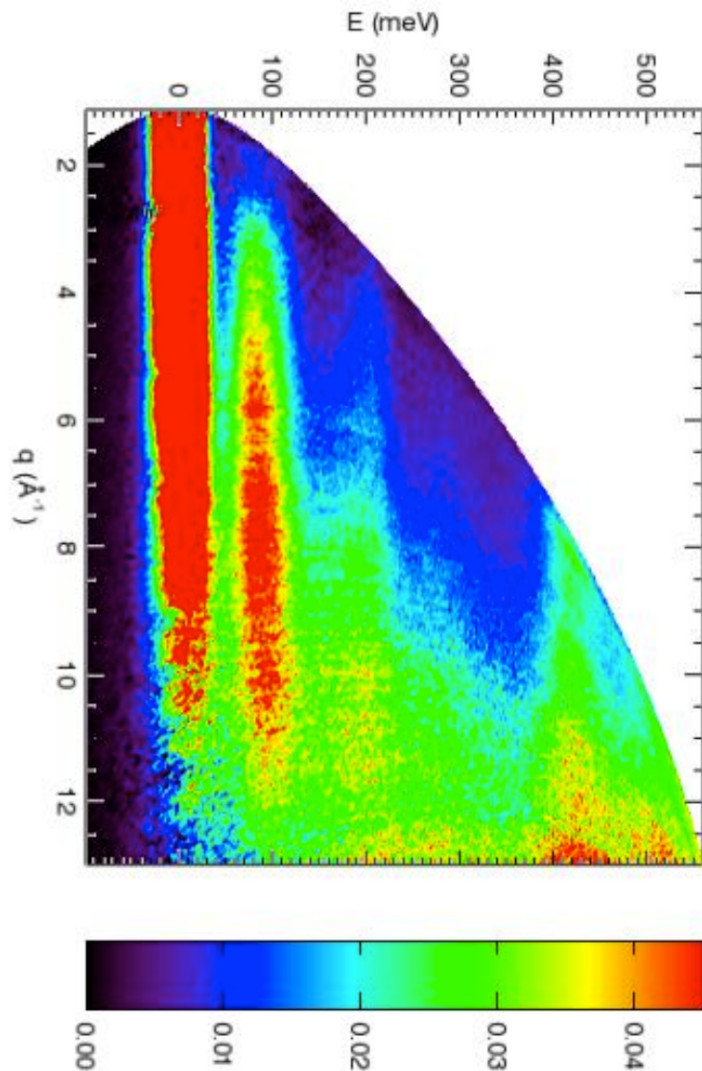


XII R. Senesi, *et al* CP 427, 111 (2013)

Scientific Culture (2014)

Simultaneous INS and DINS measurements possible at SNS- SEQUOIA beamline

43



at

re (2014)

Inelastic Neutron scattering: a probe for H vibrations and Zero Point Kinetic Energy components...

$$\lim_{q \rightarrow 0} \frac{S_{inc}(q, E)}{q^2} 2ME \frac{e^{[2W(q)]}}{[n(E) + 1]} = g_{exp}(E)_H$$

$$S(\vec{q}, \omega) = \hbar \int d\vec{p} n(\vec{p}) \delta \left(\hbar\omega - \frac{\hbar^2 q^2}{2M} - \frac{\vec{p} \cdot \hbar \vec{q}}{M} \right)$$

↓
For the OH
Stretching
range

Low wave vector +
high energy transfers

$$\langle E_K \rangle = \frac{\langle p^2 \rangle}{2M}$$

$$\langle E_K \rangle_{OH} = \frac{3}{4} \int_{355}^{480} g_{exp}(E)_{OH} E dE$$

↓
Harmonic+decoupling
assumption

High wave vector + high energy transfers
= Deep Inelastic Neutron Scattering

$$\langle E_K \rangle \approx \langle E_K \rangle_{OHstr} + \langle E_K \rangle_{bend} + \langle E_K \rangle_{libr}$$

$\langle E_K \rangle$ components and H-bond

$$\langle E_K \rangle_{OH} = \frac{3}{4} \int_{355}^{480} g_{exp}(E)_{OH} E dE$$

$$\langle E_K \rangle = \frac{\langle p^2 \rangle}{2M}$$

$$\frac{\langle E_K \rangle_{OH}}{\langle E_K \rangle}$$

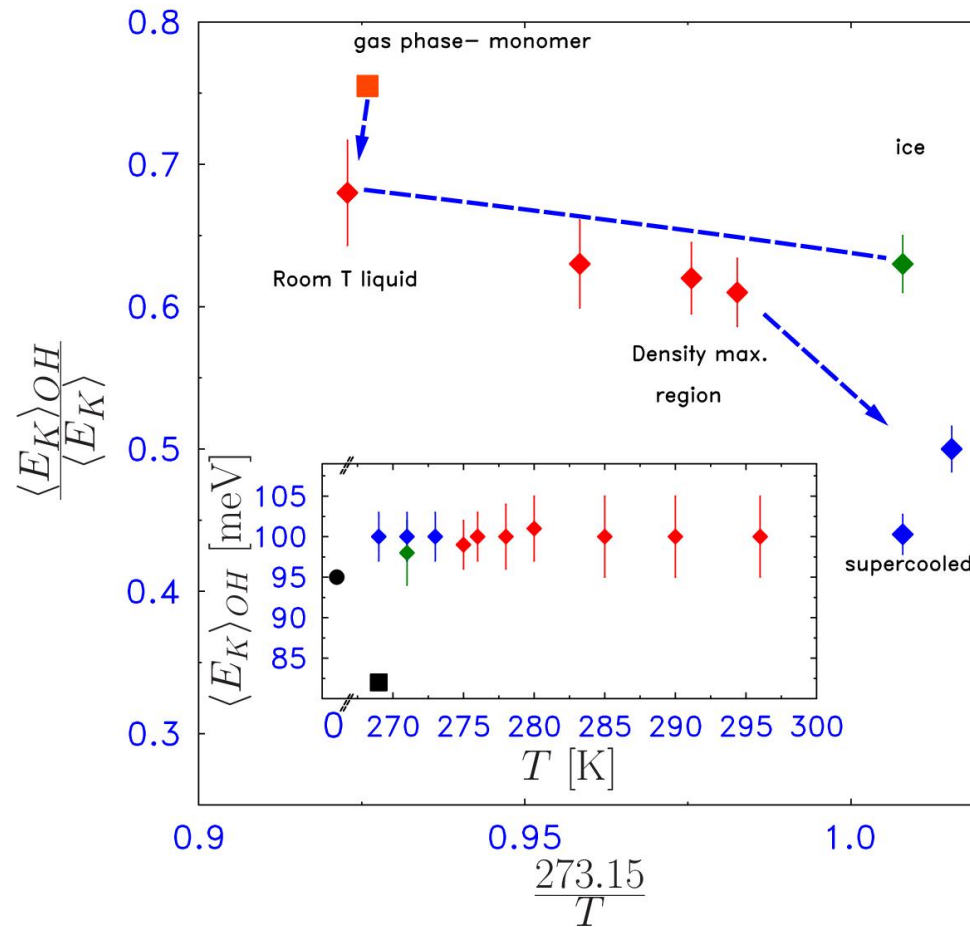
For various phases
and temperatures

1- Evaluate the stretching kinetic energy from INS data (example from SEQUOIA)

2- Use total kinetic energy at same temperatures from previous DINS data (ISIS-VESUVIO)

3- Ratio of stretching to total kinetic energy is sensitive to intermolecular interactions, including H-bonding?

Mean kinetic energy components and H-bond



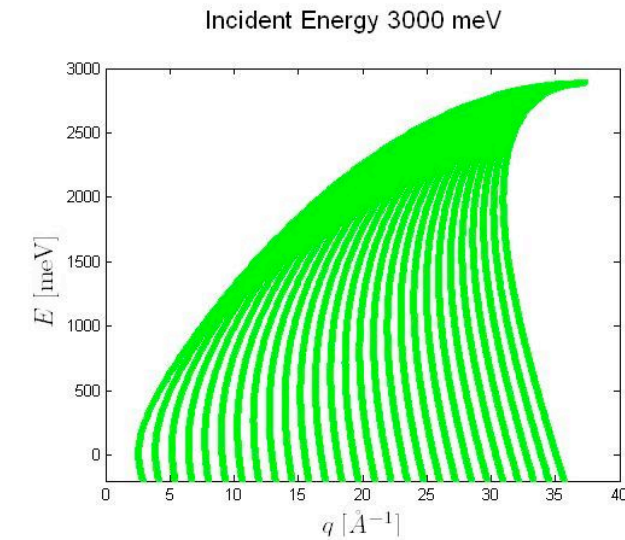
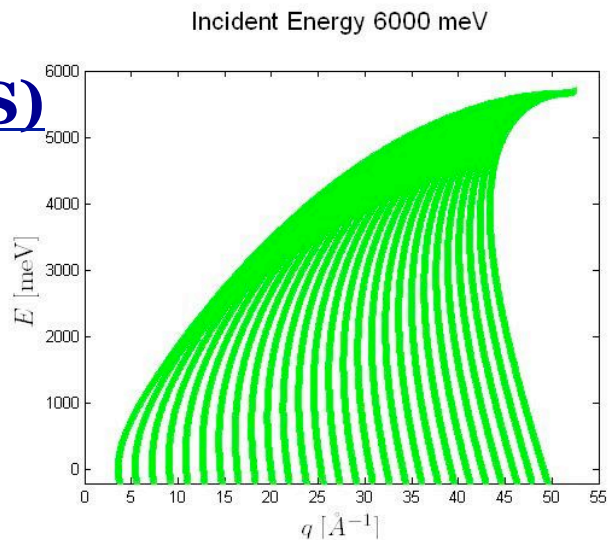
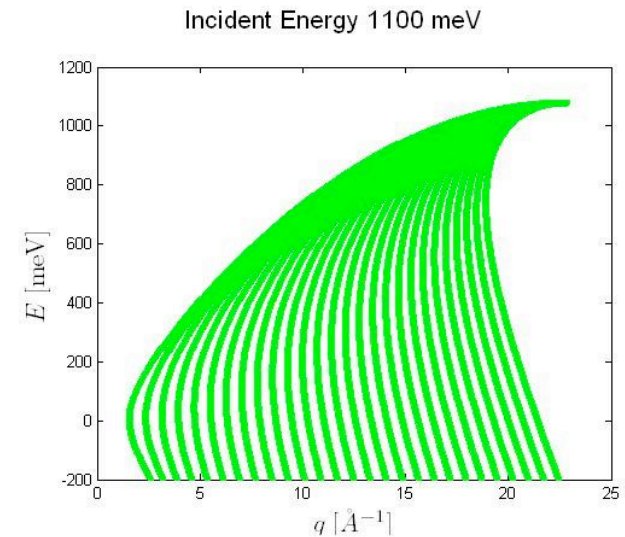
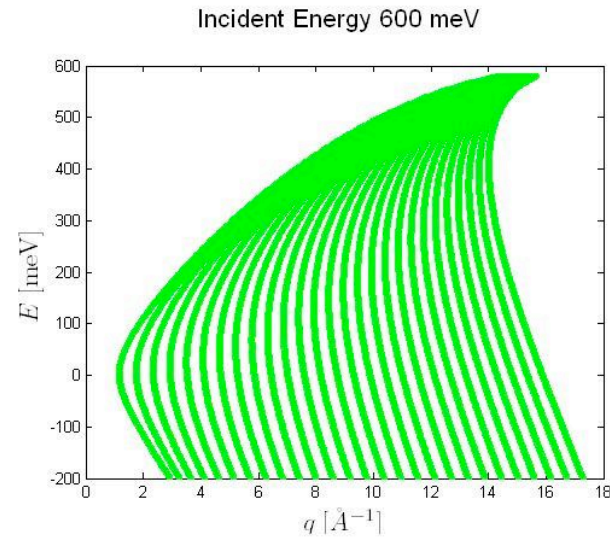
$$\langle E_K \rangle_{OH} = \frac{3}{4} \int_{355}^{480} g_{exp}(E)_{OH} E dE$$

$$\langle E_K \rangle = \frac{\langle p^2 \rangle}{2M}$$

Supercooled water*

DINS + **INS**

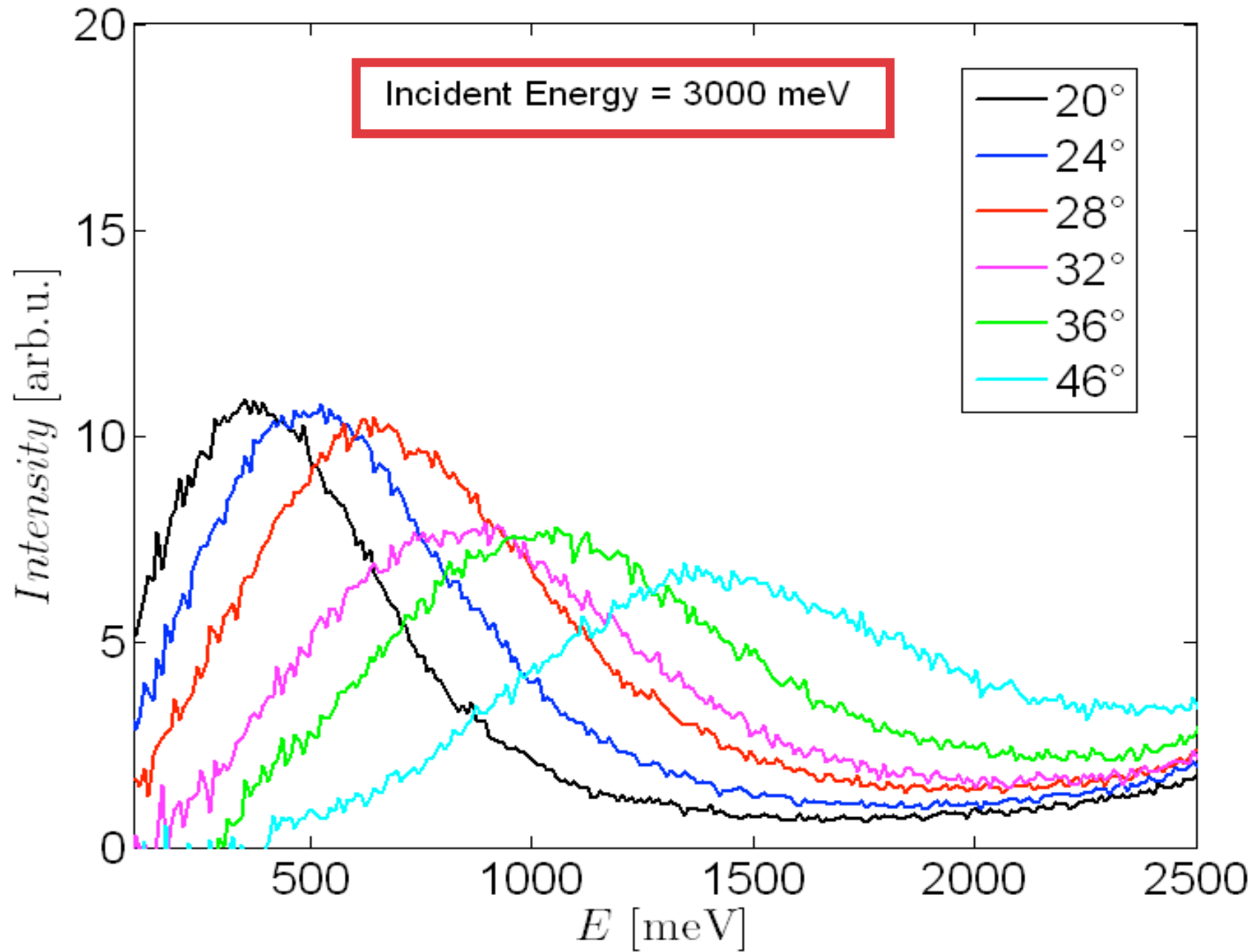
	T [K]
INS	263, 265, 268, 269, 270, 271, 274, 277, 281, 283
DINS	265, 268, 270



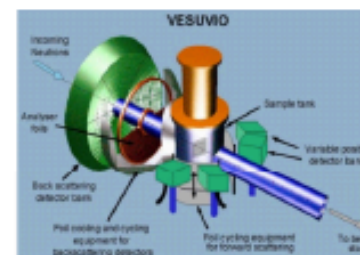
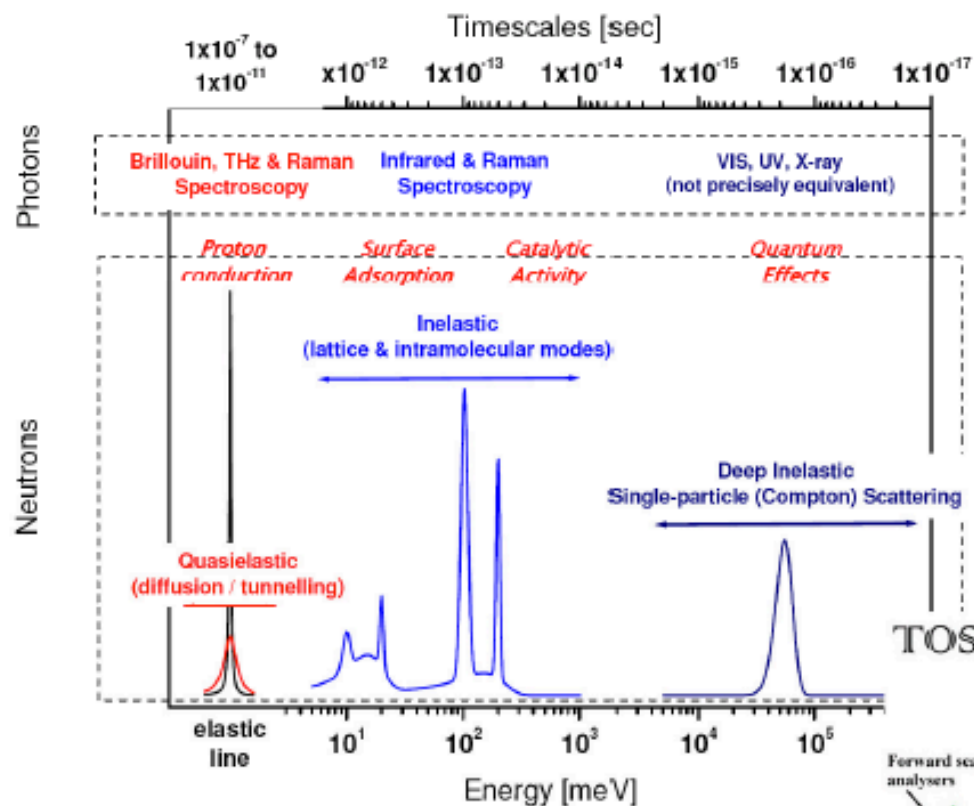
DINS and INS
On SEQUOIA (SNS)

Al CELL
THICK = 0.25 mm

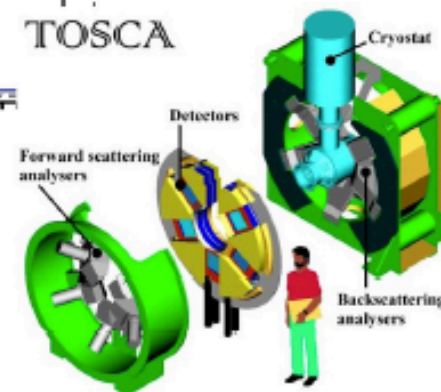
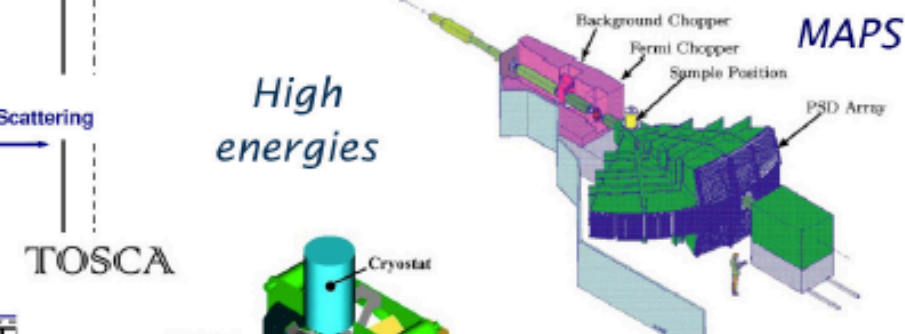
Hydrogen Water recoil at $T = 296$ K



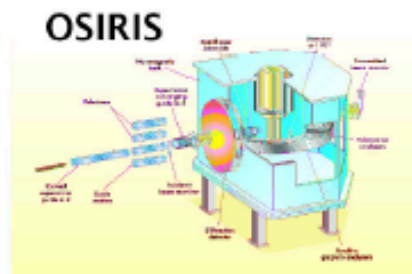
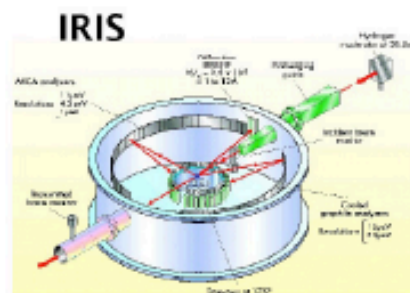
Neutron Spectroscopy at ISIS



Simultaneous high-resolution diffraction



Widest spectral range in the World



Intermediate energies (infrared, Raman)

Low energy (Brillouin, THz)

The background of the slide is a close-up, slightly blurred image of blue water with gentle ripples. The lighting is soft, creating a calm and serene atmosphere. The text is centered and rendered in a white, serif font.

THANK YOU

QUESTIONS?