



Neutron imaging – Data and image analysis

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Measurement





Measurement



Back projection



Back projection



Back projection



Back projection



Definition:

Tomography is the process of reconstructing a three dimensional distribution of the attenuation coefficients in the volume from many two dimensional projections of the sample, taken at different angles.

Assumptions:



$$N = N_0 \exp\left\{-\int_{\substack{beam\\path}} \mu(x, y) ds\right\}$$

$$p_{\theta}(t) = -\ln \frac{N_{\theta}(t)}{N_{0}(t)} = \int_{ray(\theta,t)} \mu(x, y) ds =$$
$$= \int_{0}^{+\infty+\infty} \delta(x\cos\theta + y\sin\theta - t)\mu(x, y) dxdy$$

, where $t = x \cos \theta + y \sin \theta$

 $-\infty -\infty$



$$N = N_0 \exp\left\{-\int_{\substack{beam\\path}} \mu(x, y) ds\right\}$$



$$p_{\theta}(t) = -\ln \frac{N_{\theta}(t)}{N_{0}(t)} = \int_{ray(\theta,t)} \mu(x, y) ds =$$

$$= \int_{-\infty-\infty}^{+\infty} \delta(x\cos\theta + y\sin\theta - t)\mu(x, y)dxdy$$

, where $t = x \cos \theta + y \sin \theta$

$$P_{\theta}(\omega) = \int_{-\infty}^{+\infty} p_{\theta}(t) e^{-2\pi i \omega t} dt$$

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$$S(u,v) = \int_{-\infty-\infty}^{+\infty+\infty} \mu(x,y) e^{-2\pi i(ux+vy)} dxdy$$





$$\mu(x, y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} S(u, v) e^{-2\pi i (ux + vy)} du dv$$



2.3 Filtered Back-projection

In reality, the number of rays and the number of projections is limited. The function S(u,v) is known only at a few points on radial lines.



Measured values in frequency domain



•With increasing radial distance, the density of measured values decreases, and interpolation incertainty increases.

•A simple reconstruction can be performed by simply summing up the two-dimensional Fourier Transforms of the single lines.

•Because of the linearity of the Fourier Transform, this can be done either in spatial or in frequency domain.

•But as the density of measured values decreases towards high frequencies, the high frequencies do not get enough weight, and the reconstructed image appears smoothed or smeared.



Each of k projections over 180° must deliver the information for a "cake slice" of width $2\pi |\omega|/k$.

But as it delivers only a single line, it is weighted with a ramp filter of height $2\pi |\omega|/k$, so that the new wedge has the same "mass" as the cake slice.



(a)

(b)

(c)

required, real and filtered representation of the data in frequency domain

The filter $|\omega|$ can be obtained mathematically exact by transformation to polar coordinates in frequency space. Then we obtain

$$\Sigma(x, y) = \int_{0}^{\pi} \int_{0}^{+\infty} S(\omega, \theta) e^{2\pi i \omega (x \cos \theta + y \sin \theta)} |\omega| d\omega d\theta$$
$$= \int_{0}^{\pi} \int_{0}^{+\infty} S(\omega, \theta) e^{2\pi i \omega t} |\omega| d\omega d\theta \qquad \text{with} \quad t = x \cos \theta + y \sin \theta$$

If we now substitute the one-dimensional Fourier Transform $P_{\theta}(\omega)$ of the projection at angle θ for the two-dimensional Fourier Transform $S(\omega,\theta)$, we obtain

$$\Sigma(x, y) = \int_{0}^{\pi} \left[\int_{0}^{+\infty} P_{\theta}(\omega) e^{2\pi i \omega t} |\omega| d\omega \right] d\theta \quad \text{with} \quad t = x \cos \theta + y \sin \theta$$
$$= \int_{0}^{\pi} Q_{\theta}(x \cos \theta + y \sin \theta) d\theta \quad \text{with} \quad Q_{\theta}(t) = \int_{0}^{+\infty} P_{\theta}(\omega) e^{2\pi i \omega t} |\omega| d\omega$$

This Equation describes a filter operation with the filter $|\omega|$.

The simple multiplication of $P_{\theta}(\omega)$ by ω in the frequency domain can be replaced by a convolution of $p_{\theta}(t)$ with the Fourier Transform of $|\omega|$ in the spatial domain:



w (frequency) or t (distance)

The ideal filter $|\omega|$ in spatial and frequency domain

- The value of each raysum is shared out equally among all the voxels through which the ray passed. This is called backprojection.
- As such an image is formed, albeit a poor one. An unwanted starburst pattern is obtained.





- The attenuation profile is convolved with a filter function prior to back-projection.
- The action of the filter is such that the starburst pattern disappears and an accurate representation of the original object is obtained.





The filtered back-projection is being "smeared" along the original ray path across the reconstruction field.



Back-projection of filtered data

Difficulties with the ideal filter |ω| occur with noisy data, as noise consists mainly of high frequencies, which are much enhanced by this filter.
The ideal filter is therefore often replaced by special filter functions that decrease again towards high frequencies.

•For neutrons, the inherent beam unsharpness (see below) often attenuates most high frequencies towards the Nyquist limit.



The ideal filter $|\omega|$ and some alternative functions



2.4 Number of projections

The number of projections should be in the same order as the number of rays in one projection.

For M projections with N rays over 180°, the angular increment δ between two consecutive projections is given in Fourier space as

$$\delta = \frac{\pi}{M}$$

For distance T between two neighboring rays, the highest measured spatial frequency ω_{max} in the projection is given by Nyquist's Theorem as

$$\mathcal{D}_{\max} = \frac{1}{2T}$$

This is the radius of a disk in the frequency domain that contains all measured values.

The distance d between two consecutive values on the circle is:



For N measured values for each projection in spatial domain, there are also N measured values for each measured line in the frequency domain, so that the distance ε between two consecutive measured values on a radial line (or diameter) in frequency domain is given as

$$\varepsilon = \frac{2\omega_{\max}}{N} = \frac{1}{TN}$$

For the worst azimuthal resolution in frequency domain to match the radial resolution, we must demand:

$$\frac{1}{2T}\frac{\pi}{M} \approx \frac{1}{TN}$$

<u>Number of projections</u> = $\frac{M}{\approx} = \frac{\pi}{m}$ Number of rays \mathcal{N}

For practical neutron radiography, most detector systems cannot - at least for sub-millimeter resolution - measure down to the nominal Nyquist resolution given by their pixel size.

The greatest limiting factor is almost always the geometry of the neutron beam and its deviation from the ideal parallel ray model.



Beam optimisation



- D Collimator aperture
- *L* Distance Collimator-Object
- l Distance Object-Detector





Beam optimisation



$$d = \frac{l}{L/D}$$
 Example:
$$l = 10 \text{ cm}$$
$$L/D = 500$$
$$=> d = 0.02 \text{ cm}$$

For sample width of 20 cm $N = 1000 \Rightarrow M \sim 1500$ projections

For sample width of 2 cm $N = 100 \Rightarrow M \sim 150$ projections

Neutron tomography principle



Phantom



A) 8 views in frequency domain

C) reconstruction from image A

B) 180 views in frequency domain

D) reconstruction from image B

Neutron tomography – sinogram preparation



Selecting one z_0 slice from the tomography volume (t,z, θ) and stacking all its projections as a function of the rotating angle theta we will obtain the sinogram, which is used later for the tomography reconstruction.



Attenuation Contrast

Combustion chamber







<u>Time scale</u>

- 1 old sediments
- 3 fresh sediments





Sample No. 1





Volume sample: 3235 mm³ Volume impregnation: 0 mm³



Sample No. 6

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Volume sample: 3596 mm³ Volume impregnation: 639 mm³



Sample No. 10





Volume sample: 2989 mm³ Volume impregnation: 283 mm³



Particle size distribution in battery, Typ AAA



Anode

$$Zn \rightarrow Zn^{2+} + 2e^{-}$$

 $Zn^{2+} + 2OH^{-} \rightarrow Zn(OH)_{2}$

Kathode

$MnO_2 + H_2O + e^- \rightarrow MnO(OH) + OH^ MnO(OH) + H_2O + e^- \rightarrow Mn(OH)_2 + OH^-.$

Boolean transformation



Mn-Ring Artifacts: Steel case, Powder

Zn-Powder and Steel case

Morphological transformation



Noise reduction - Opening process

SiC





Boolean transformation – intermediate result



Morphological transformation



EDT: Euclid Distance Transformation

									2	2	2	2	2	3	2	2	2	2	2	3
					1	1	1		2	1	1	1	2	3	2	1	1	1	2	3
					1		1		2	1		1	2	3	2	1		1	2	3
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2,24	1,41	1	1,41	2,24
2	1		1	2
2	1		1	2
2,24	1,41	1	1,41	2,24



Image: Soille. Morphological Image Analysis. Springer.

Segmentation of powder particles



Segmentation of powder particles



Result – chemical components of the battery



Initial point 3D-Data set Final result Multi-component data set

Battery – different discharge levels





Digital Mummy Project

A Collaborative Effort between Stanford Universitγ NASA/Stanford Biocomputational Center Silicon Graphics Inc. and The Rosicrucian Egyptian Museum



Thank you !