Spin Waves in 1 Dimensional Spin Chain

VIII School of Neutron Scattering F. P. Ricci - S. Margherita di Pula 2006 -

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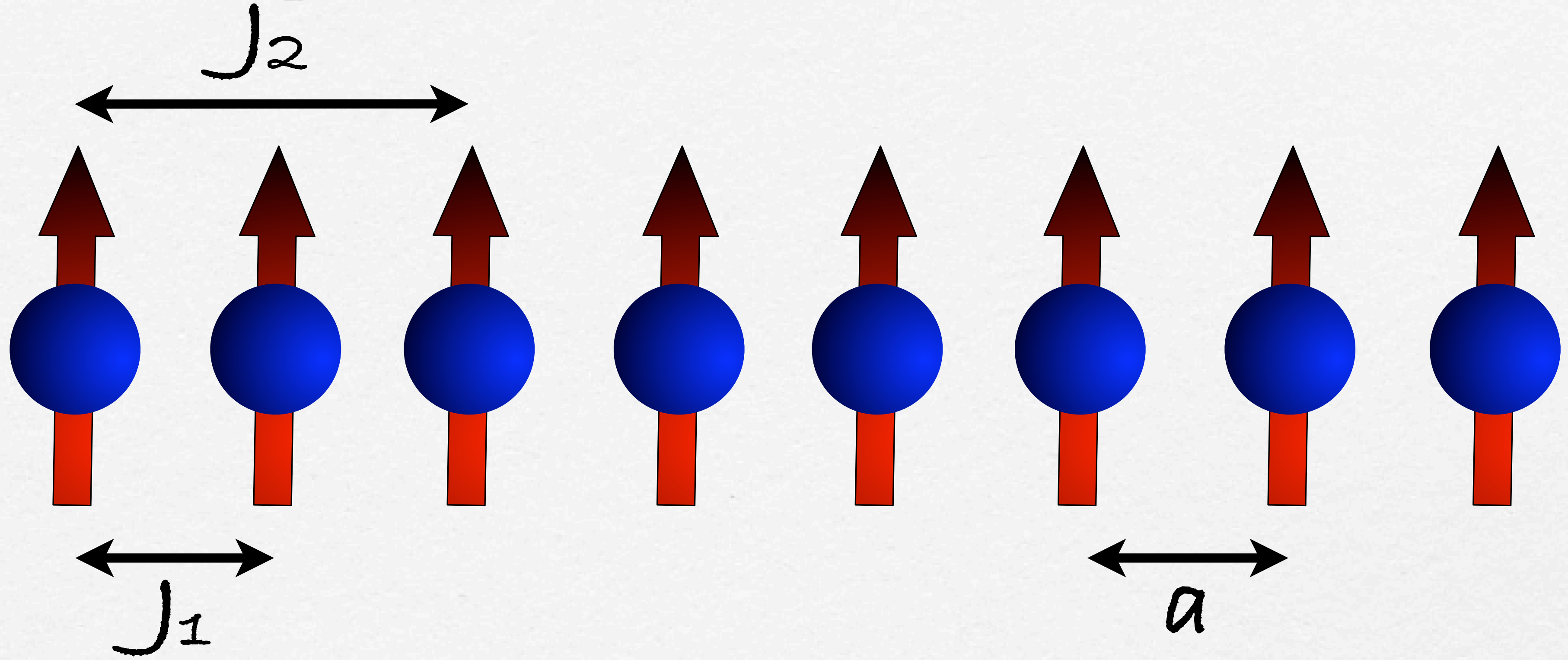
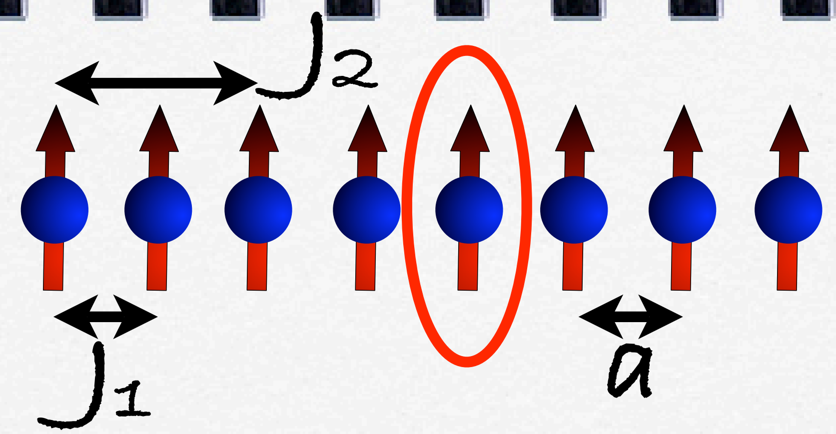
1D Spin Chain



Where:

- "a" is the distance between two adjacent spins
- "J_1" is the exchange energy term between 2 near neighbors
- "J_2" is the exchange energy between 2 next near neighbors

A bit of math



$$H = -J\sum_n \overline{S}_n \overline{S}_{n+1}$$

$$H = -\sum_{nm} J(R_{nm}) S_n S_m = -\sum_q J(q) S_q S_{-q}$$

$$J(q) = \sum_n J(R_n) exp(-iqR_n)$$

$$J(q) = J_1 e^{iqa} + J_1 e^{-iqa} + J_2 e^{i2qa} + J_2 e^{-i2qa}$$

using Eulero

$$J(q) = J_1 2cosqa + J_2 2cos2qa = 2J_1 cos2\pi h + 2J_2 cos4\pi h$$

$$= 2J_1 cos2\pi h + 2J_2[2cos^2 2\pi h - 1]$$

$$q = \frac{2\pi}{a} h$$

- Q is the q that maximize the j function (Energy minimized)
- H corresponds to the ordering vector of Q

Some more math

we **blindly** believe that:

$$E = -NS^2 J(Q)$$ Dispersion relation for spin waves

Linearizing the above expression, the following equation can be obtained

$$\hbar\omega(q) = 2S\{[J(Q) - \frac{1}{2}J(Q+q) - \frac{1}{2}J(Q-q)][J(Q) - J(q)]\}^{\frac{1}{2}}$$

We need to minimize the energy maximizing the j(q)

Differentiating the J(q) function we can find 3 solutions

Dispersion relations

The solutions of

$$J(h) = 2J_1 cos 2\pi h + 2J_2[2cos^2 2\pi h - 1]$$

are

$$H = 0 \qquad H = \frac{1}{2} \qquad cos 2\pi H = -\frac{J_1}{4J_2}$$

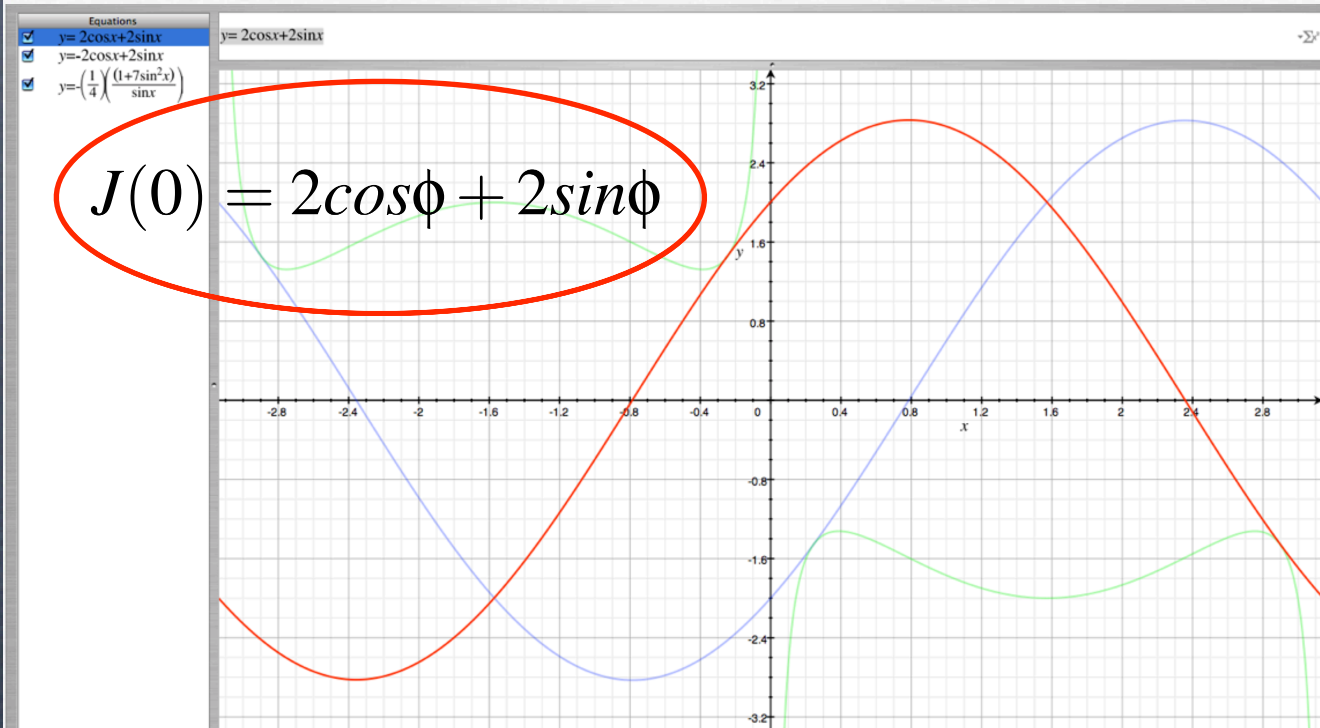
That leads to the following dispersion relations

$$\hbar\omega(h) = 2(J_1(1 - cos 2\pi h)) + J_2(1 - cos(4\pi h))$$

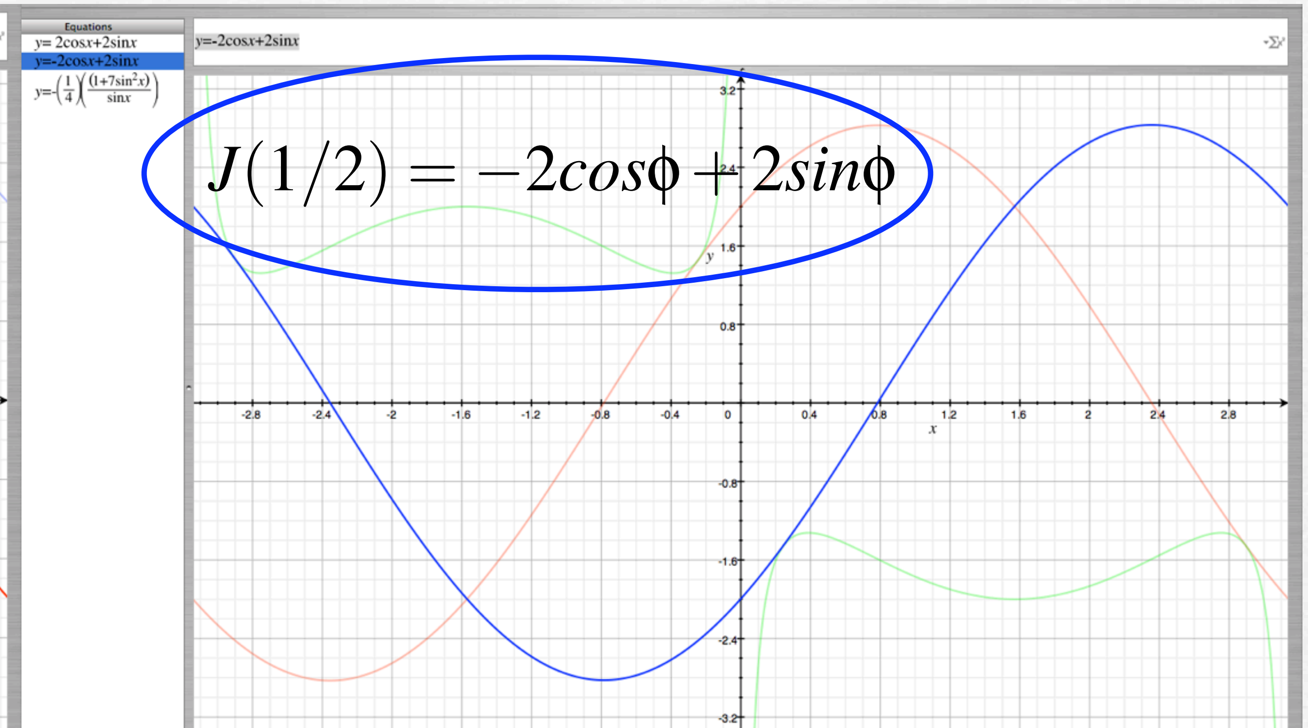
$$\hbar\omega(h) = 4S\{[-2J_1 + 2J_2 - 2J_2 cos 4\pi h]^2 - [2J_1 cos 2\pi h]^2\}^{1/2}$$

$$\hbar\omega(h) = \left\{\left(-4J_2 + \frac{J_1^2}{2J_2} cos 2\pi h - \left(\frac{J_1^2}{2J_2} - 4J_2\right) cos^2 2\pi h\right)\left(-\frac{J_1^2}{4J_2} - 2J_2 - 2J_1 cos 2\pi h - 2J_2\left(2cos^2 2\pi h - 1\right)\right\}^{1/2}$$

Solutions 1-2-3



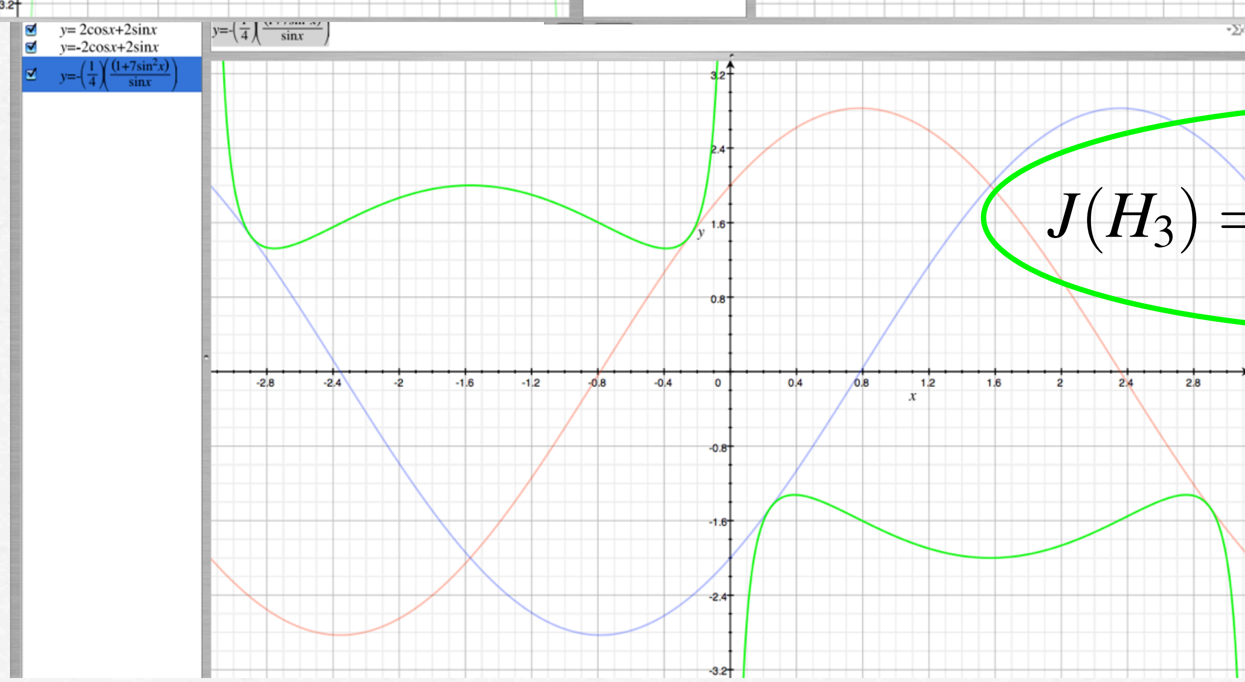
$$J(0) = 2cos\phi + 2sin\phi$$



$$J(1/2) = -2cos\phi + 2sin\phi$$

$$J_1 = cos\phi$$

$$J_2 = sin\phi$$



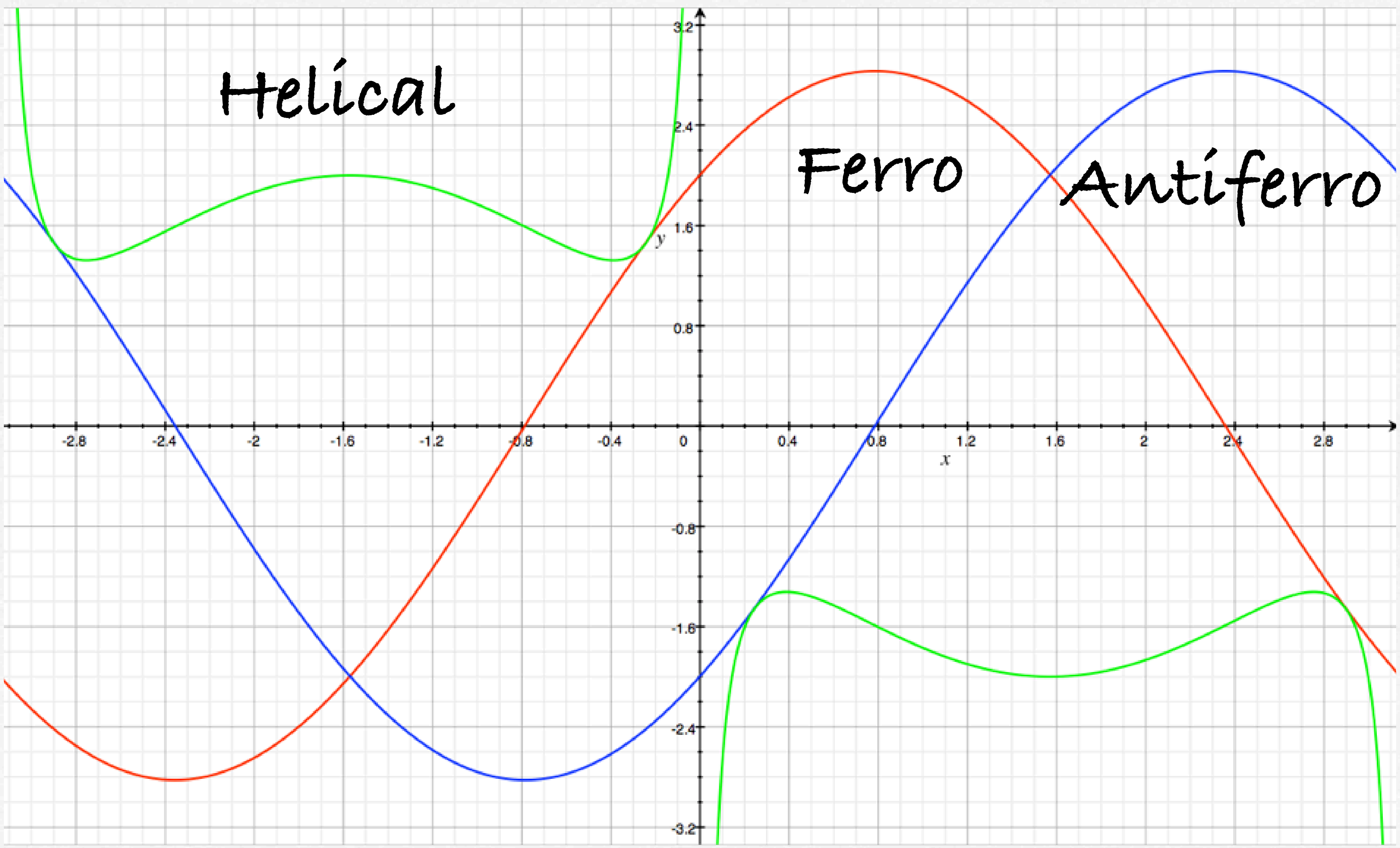
$$J(H_3) = -\left(\frac{1}{4}\right)\left(\frac{1+7sin^2\phi}{sin\phi}\right)$$

Phases

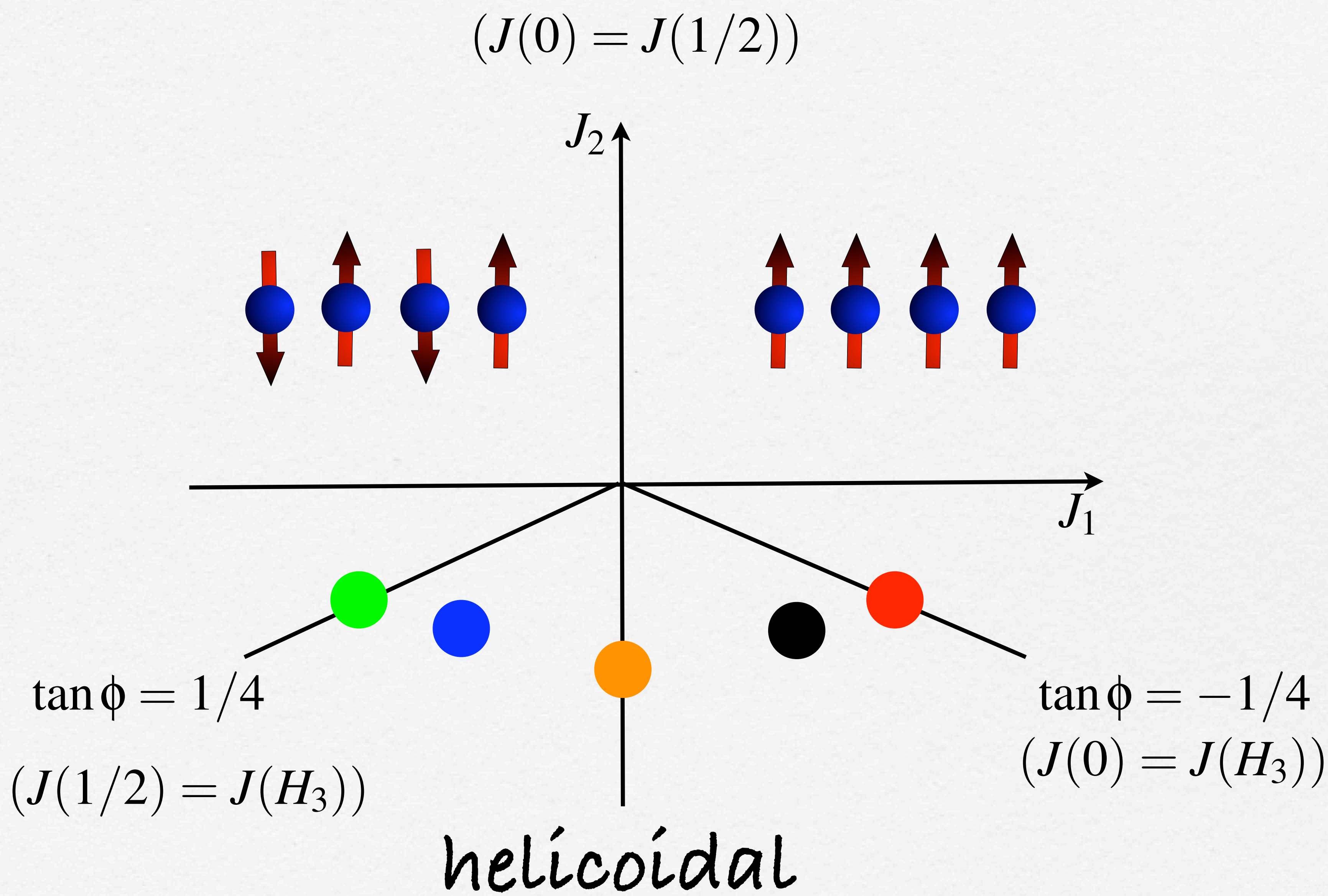
$J(0) = 2cos\phi + 2sin\phi$

$J(1/2) = -2cos\phi + 2sin\phi$

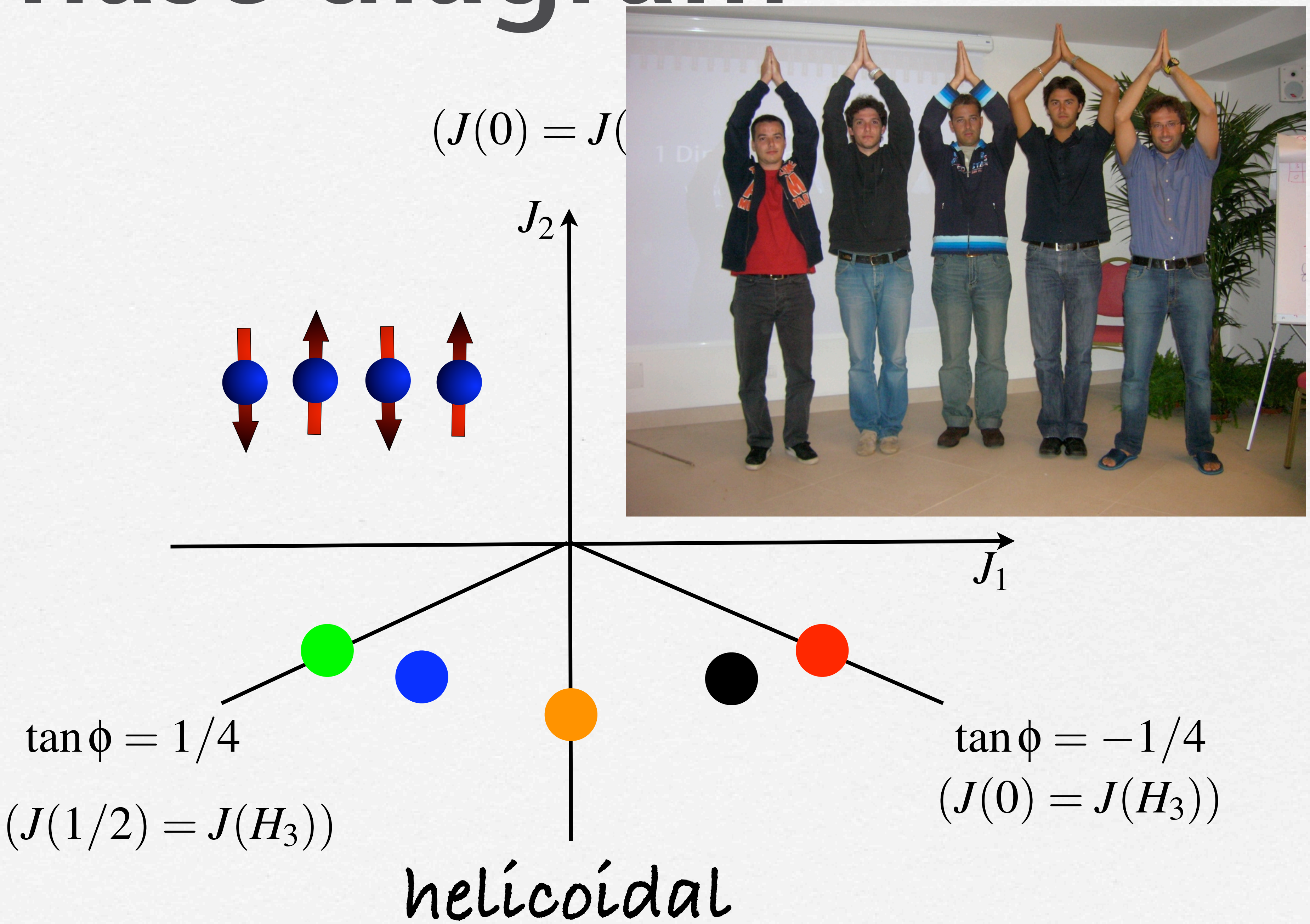
$J(H_3) = -\left(\frac{1}{4}\right)\left(\frac{1+7sin^2\phi}{sin\phi}\right)$



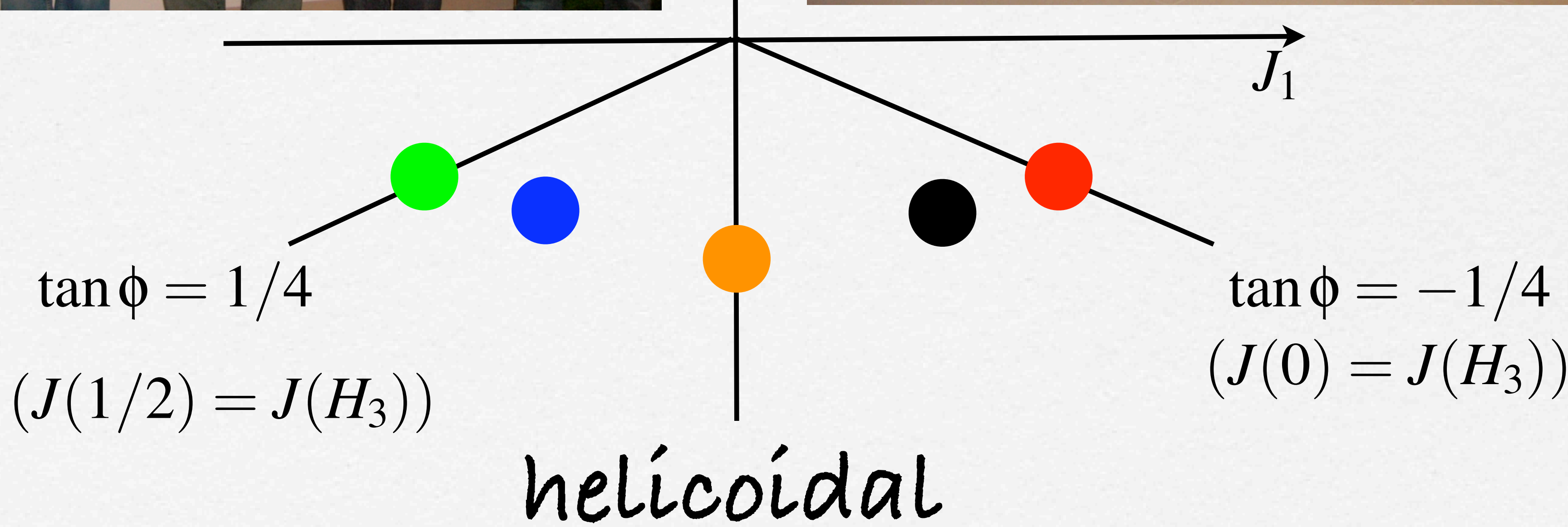
Phase diagram



Phase diagram



Phase diagram



Phase diagram

$) = J($

J_2

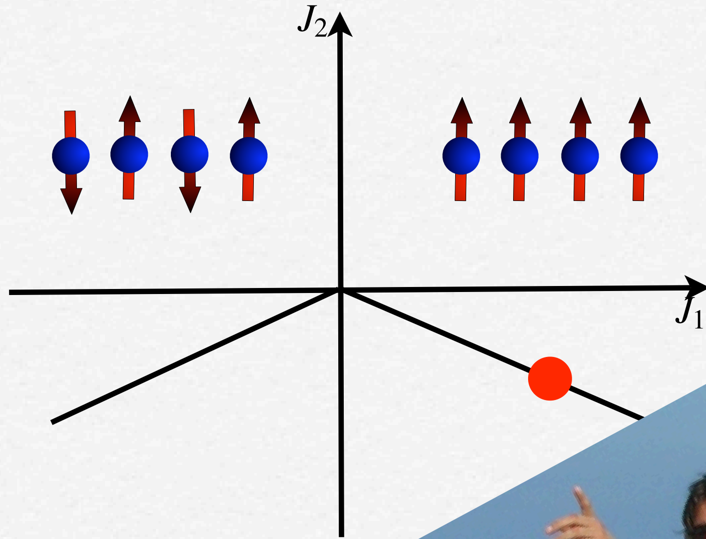
J_1

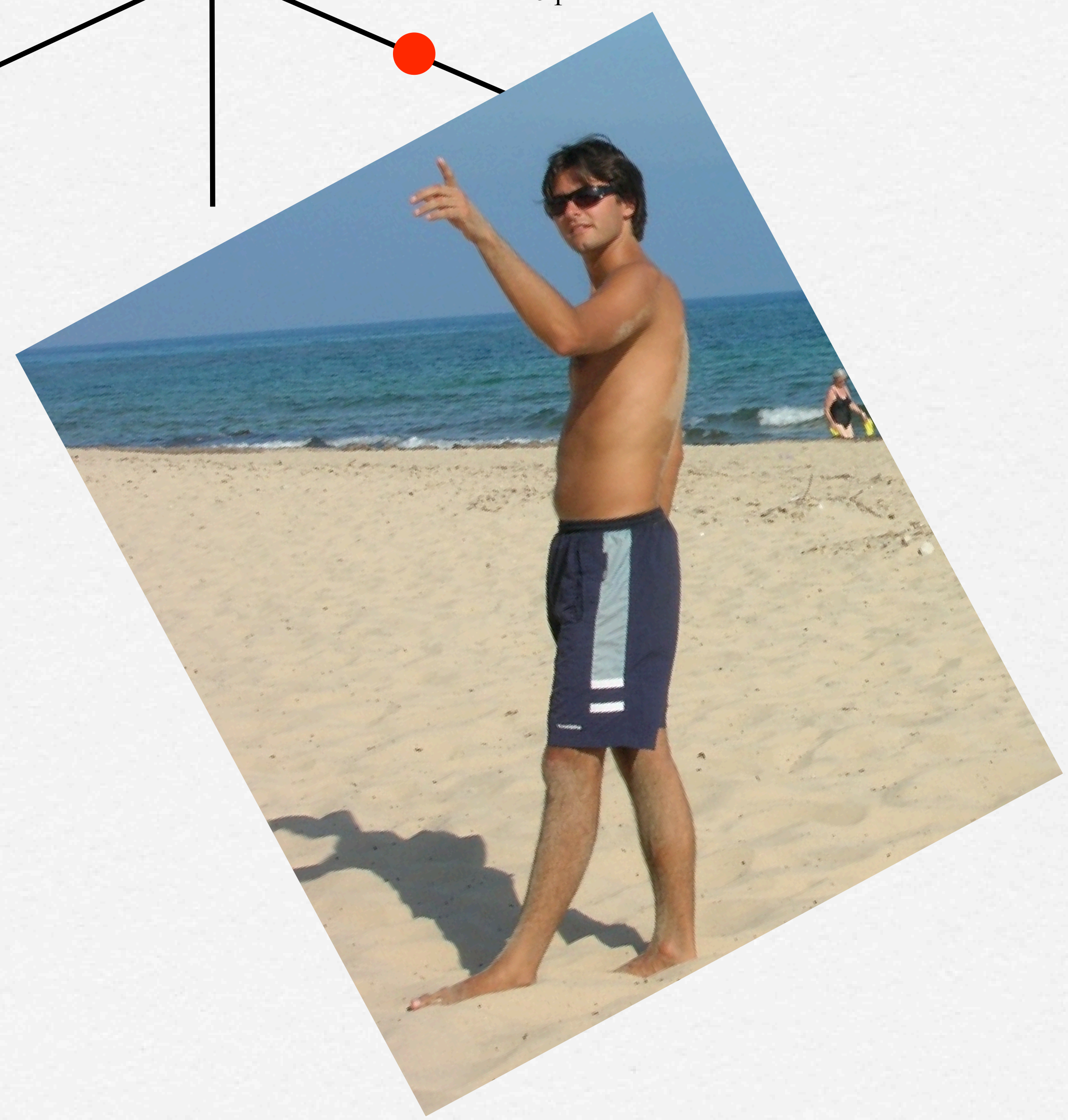
$\tan\phi = 1/4$

$(J(1/2) = J$

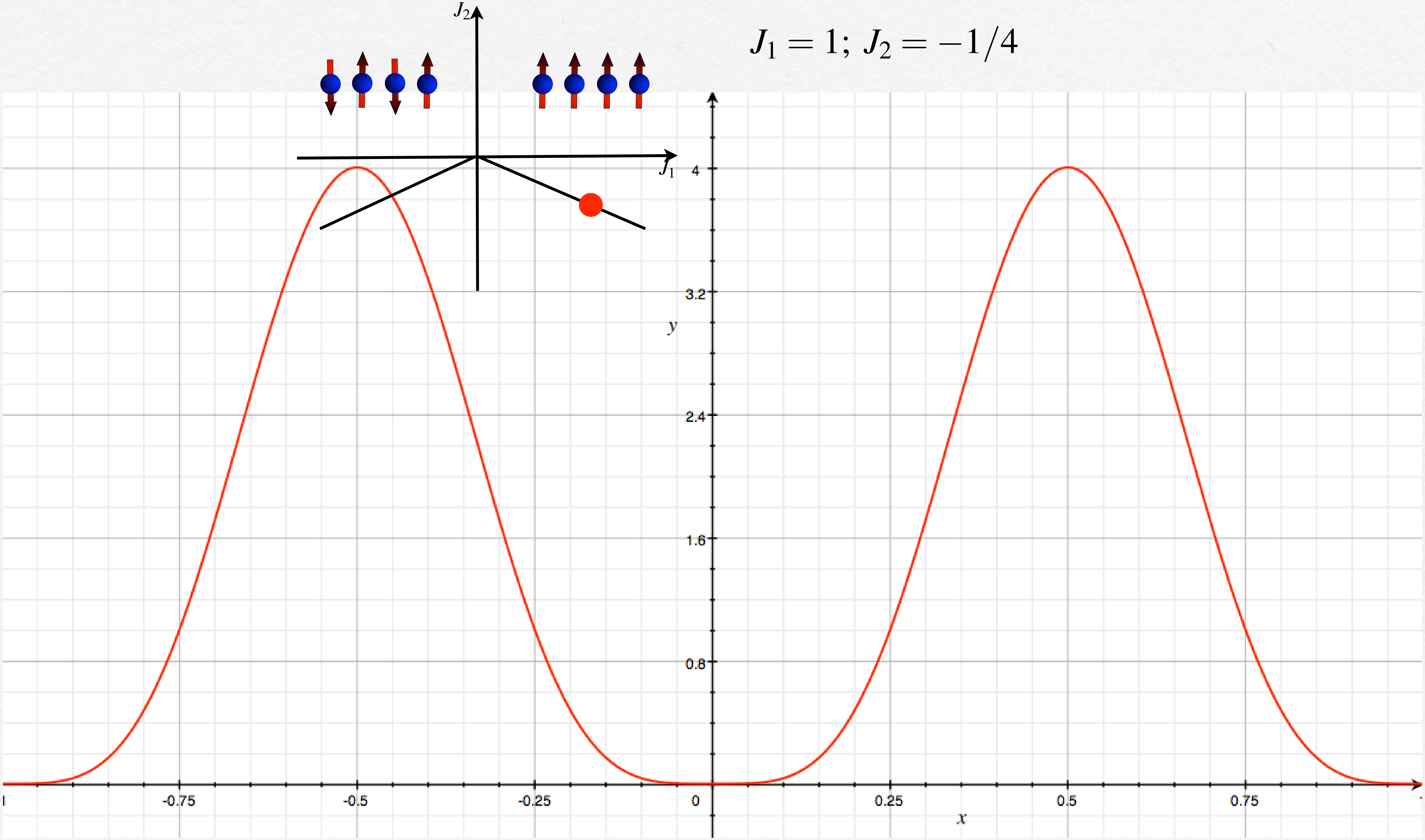
$\tan\phi = -1/4$

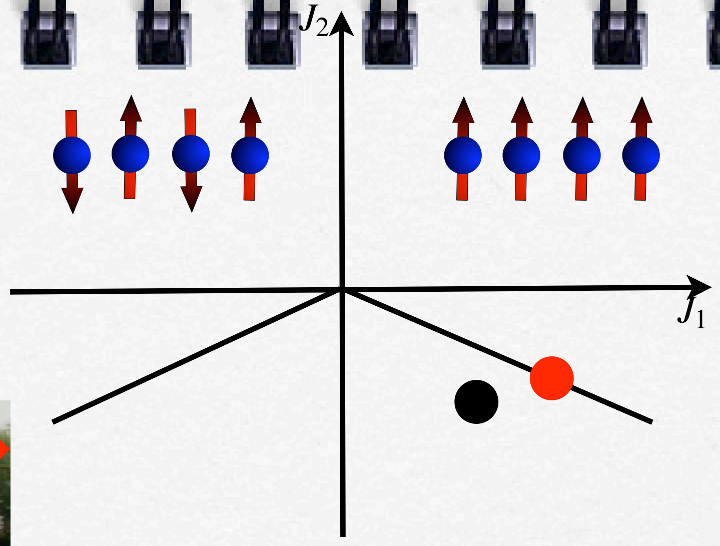
$(J(0) = J(H_3))$


J_2
J_1



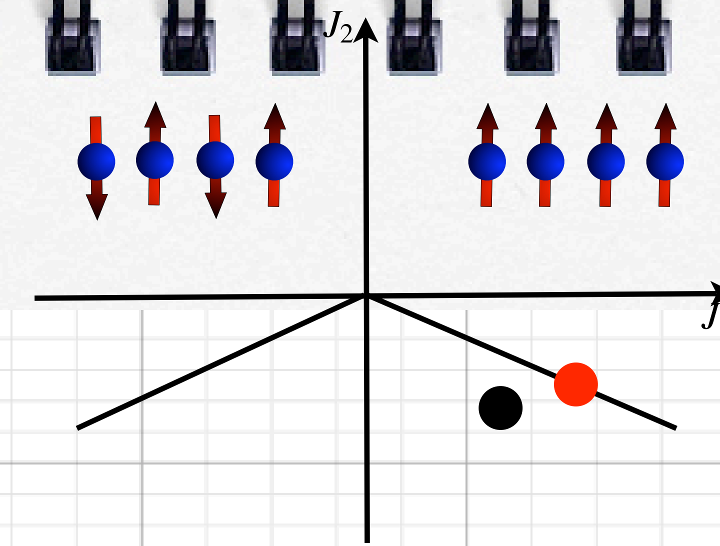
$J_1 = 1;\ J_2 = -1/4$

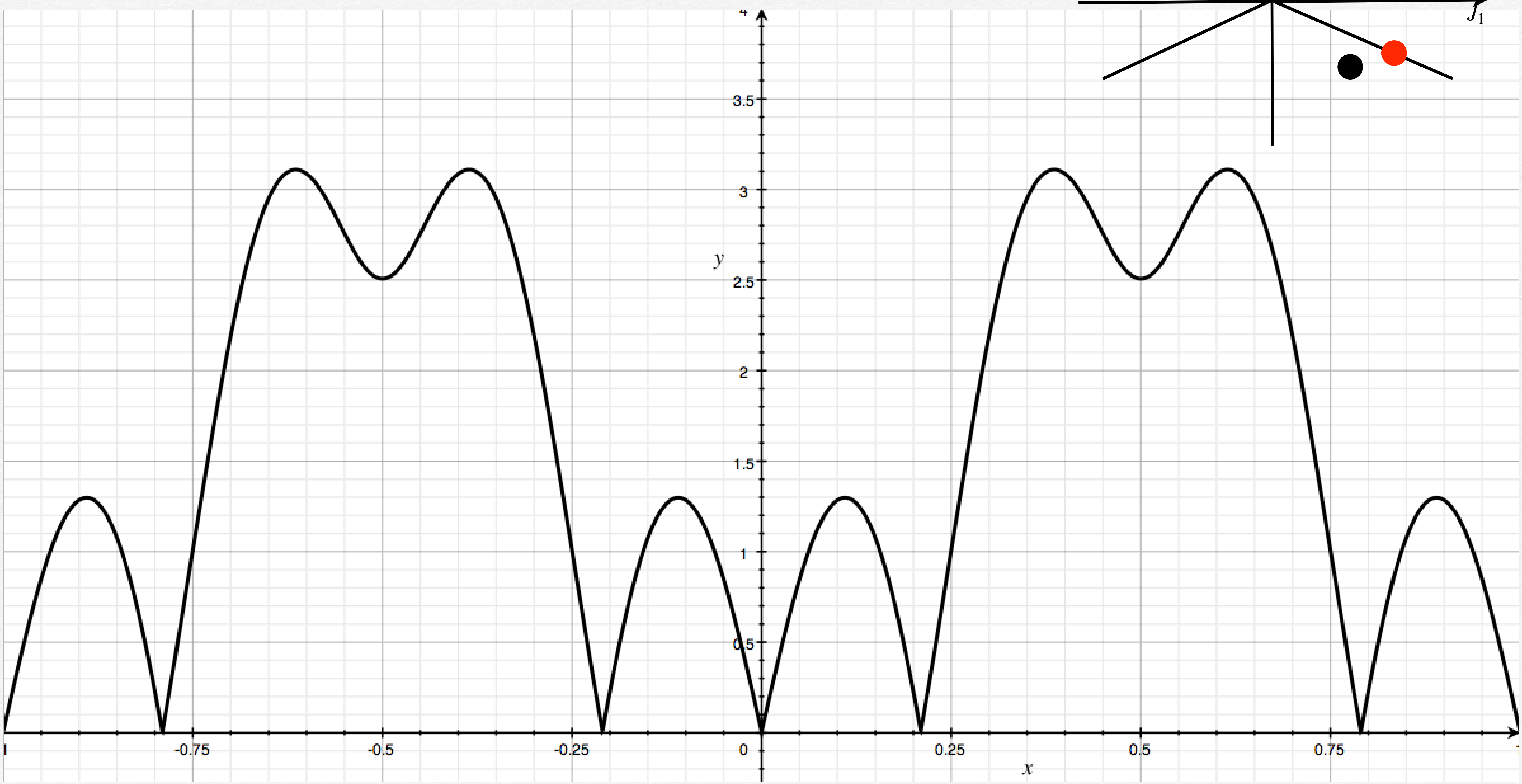


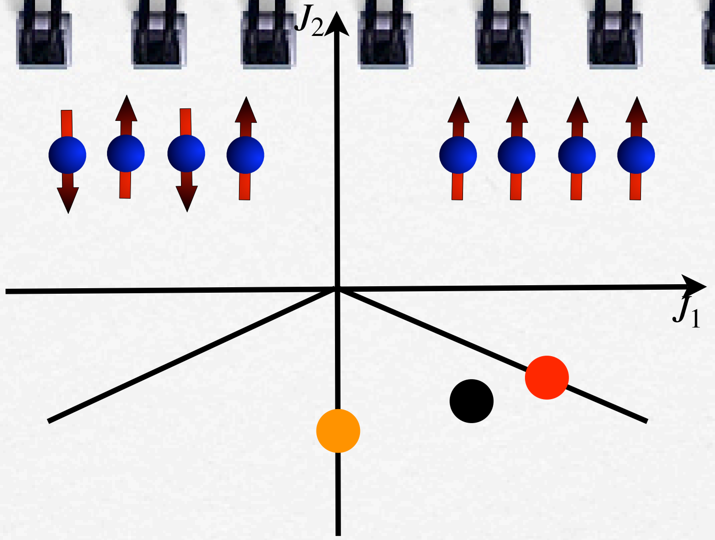

J_2
J_1


Wanted

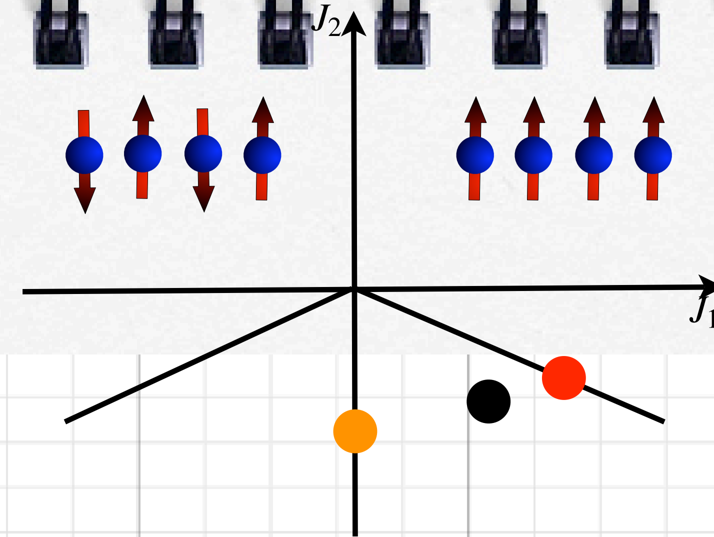
$$J_1 = 1;\ J_2 = -1$$

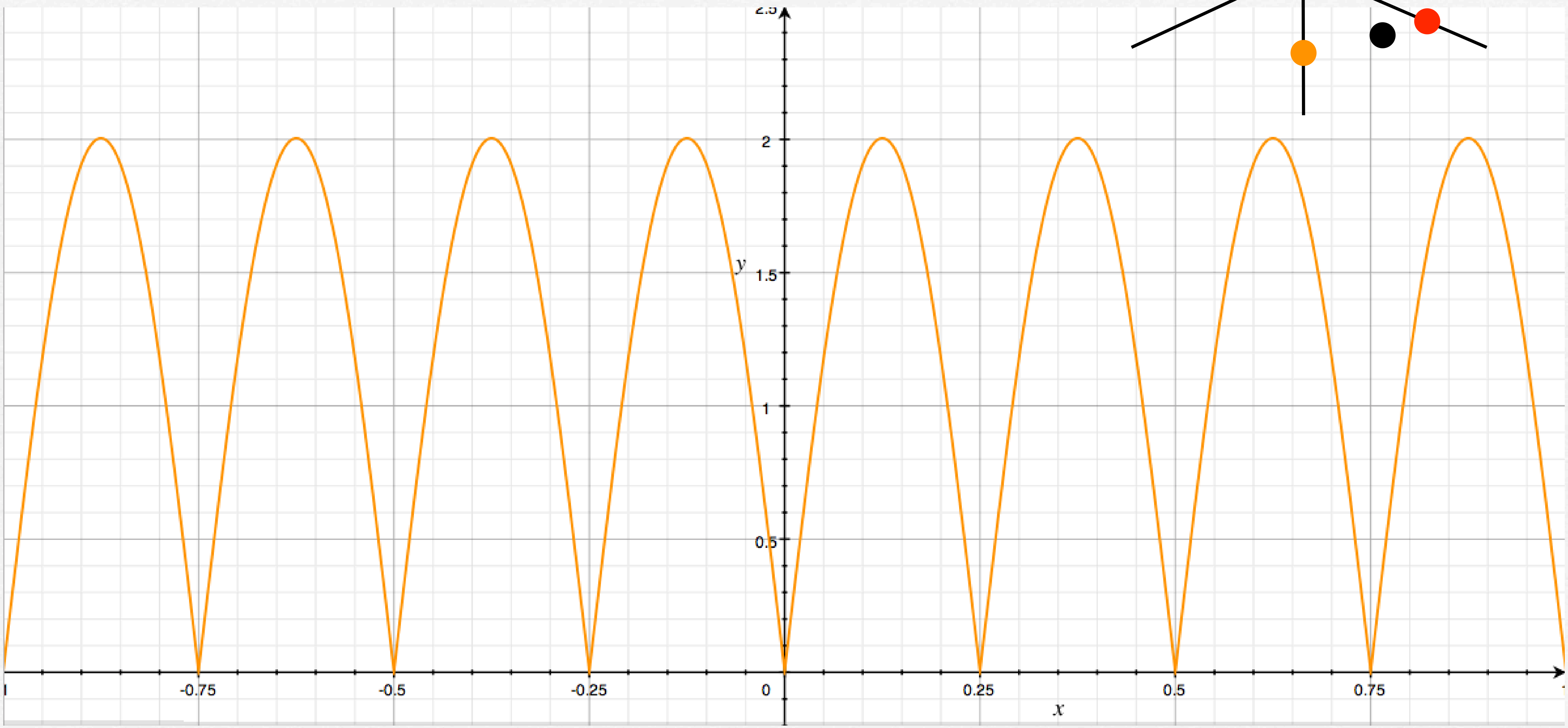


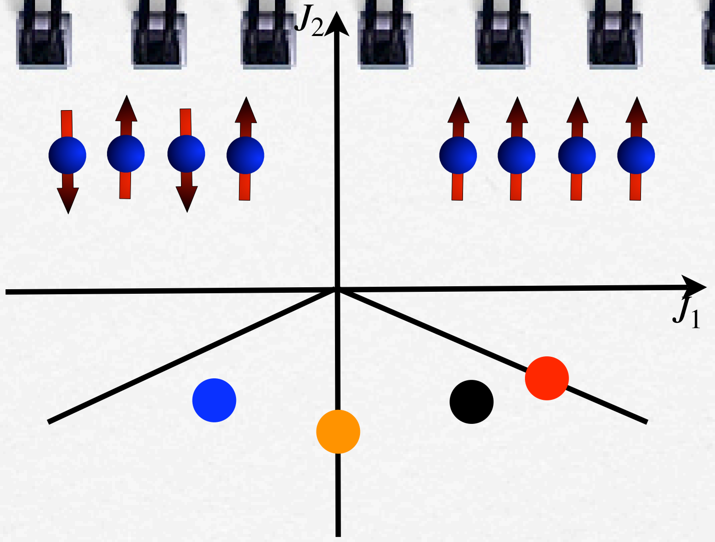



J2
J1

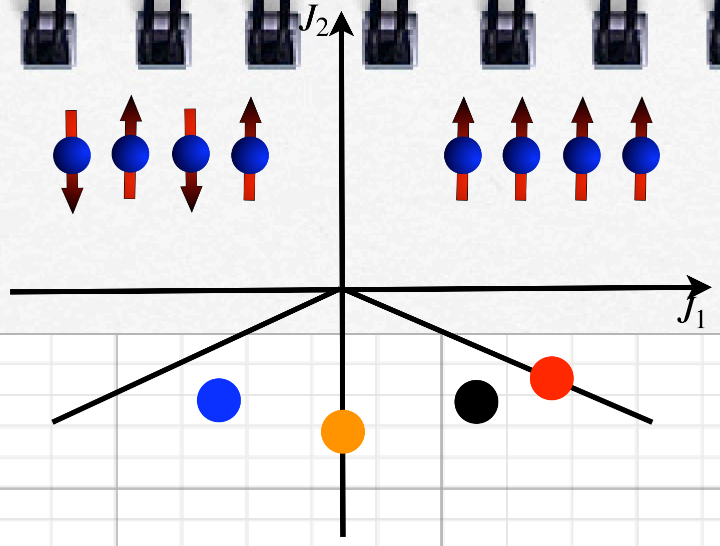
$$J_1 = 0;\ J_2 = -1$$

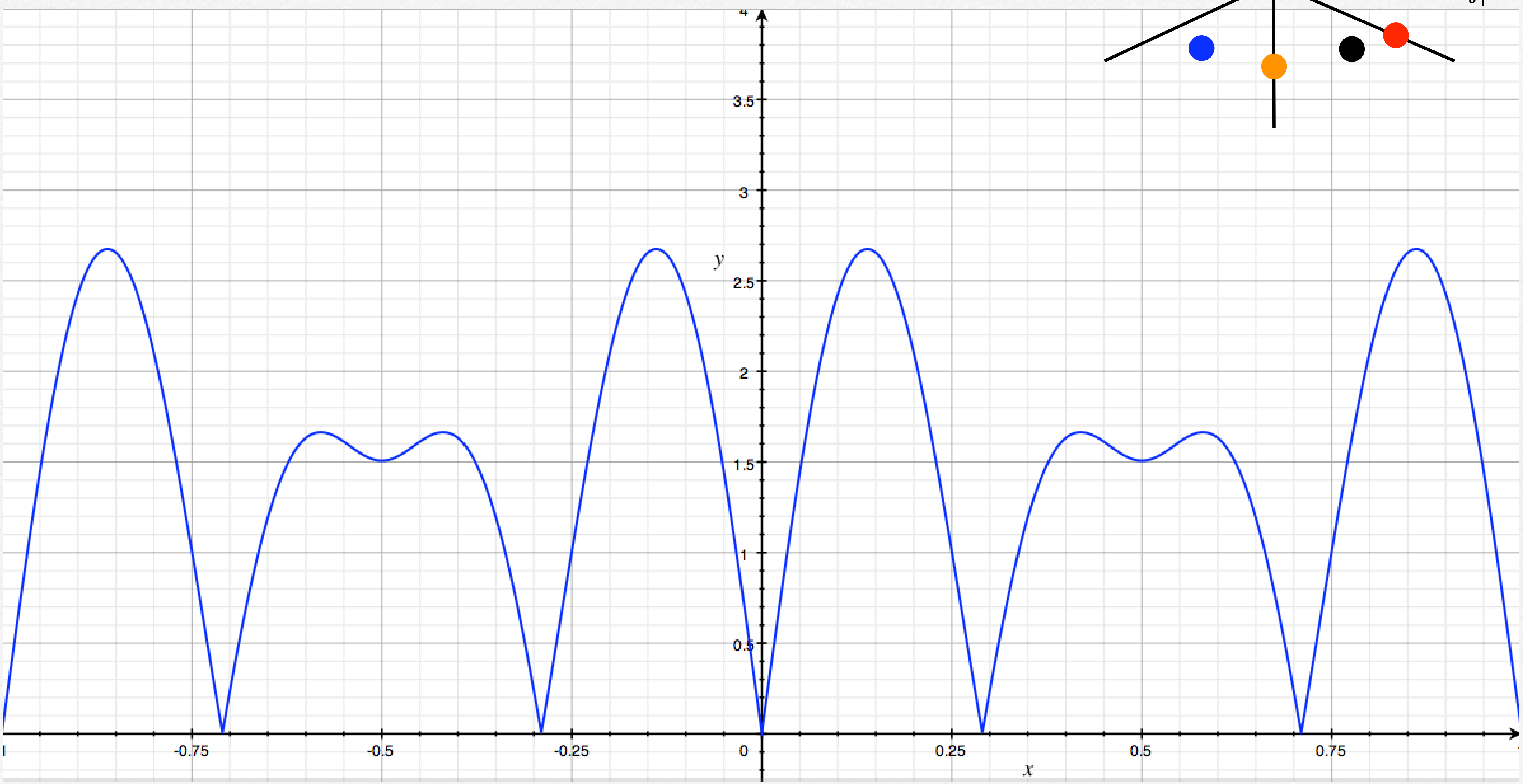


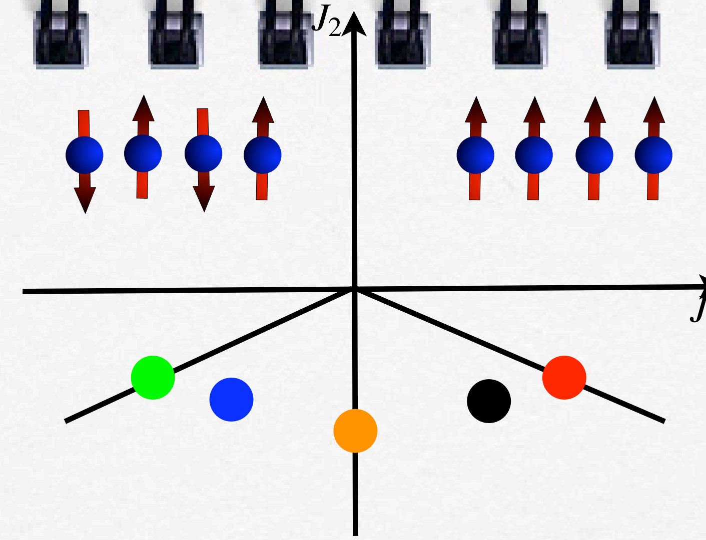



J_2
J_1

$$J_1 = -1;\ J_2 = -1$$

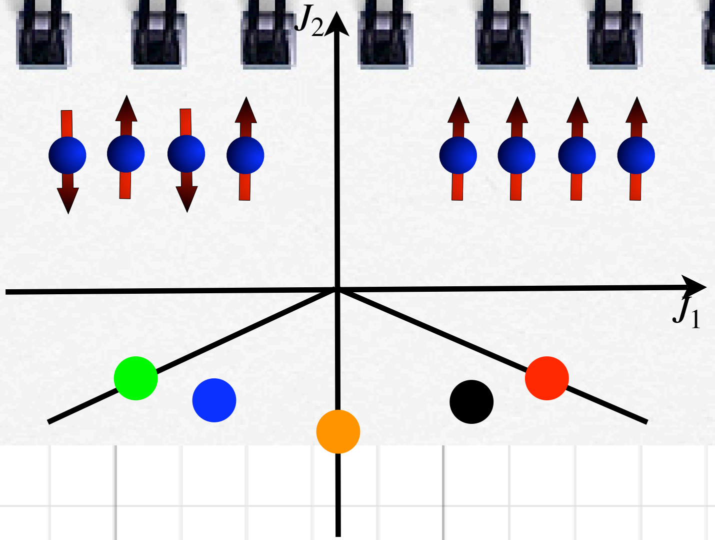


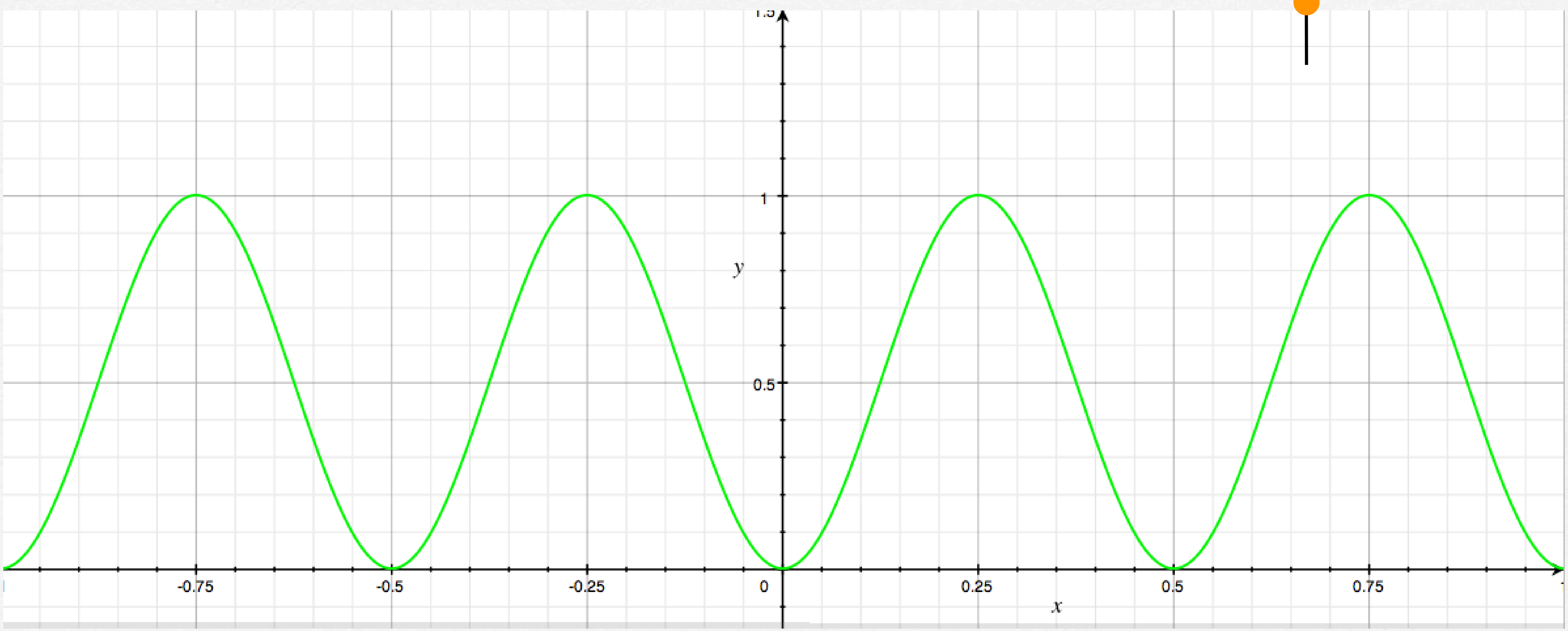



J_2
J_1

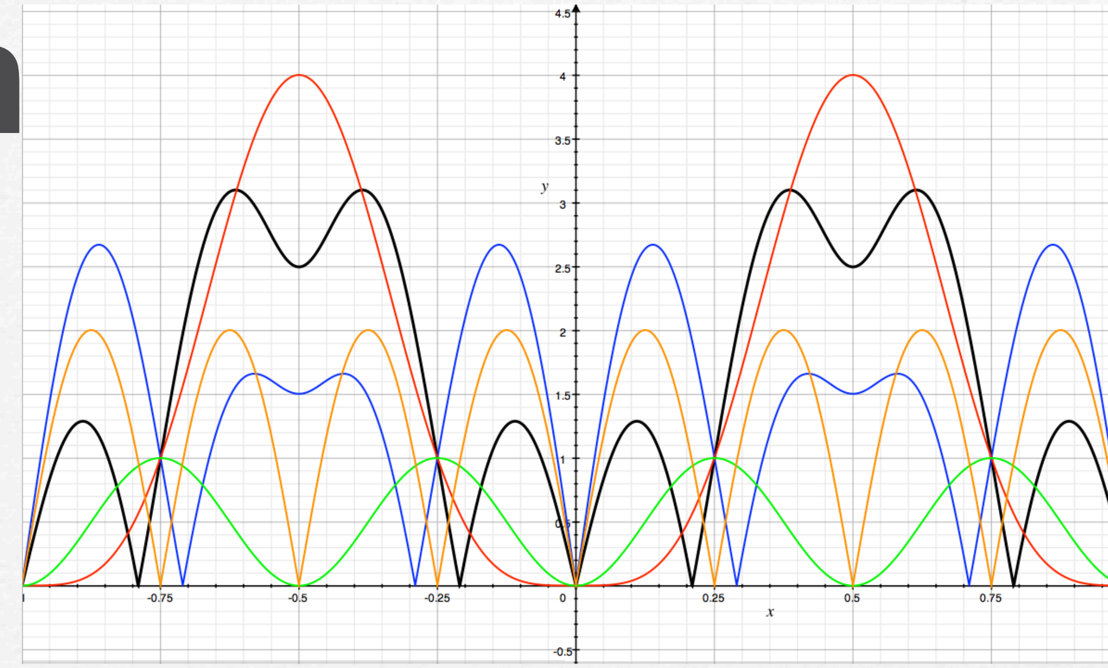

Wanted too

$J_1 = -1;\ J_2 = -1/4$

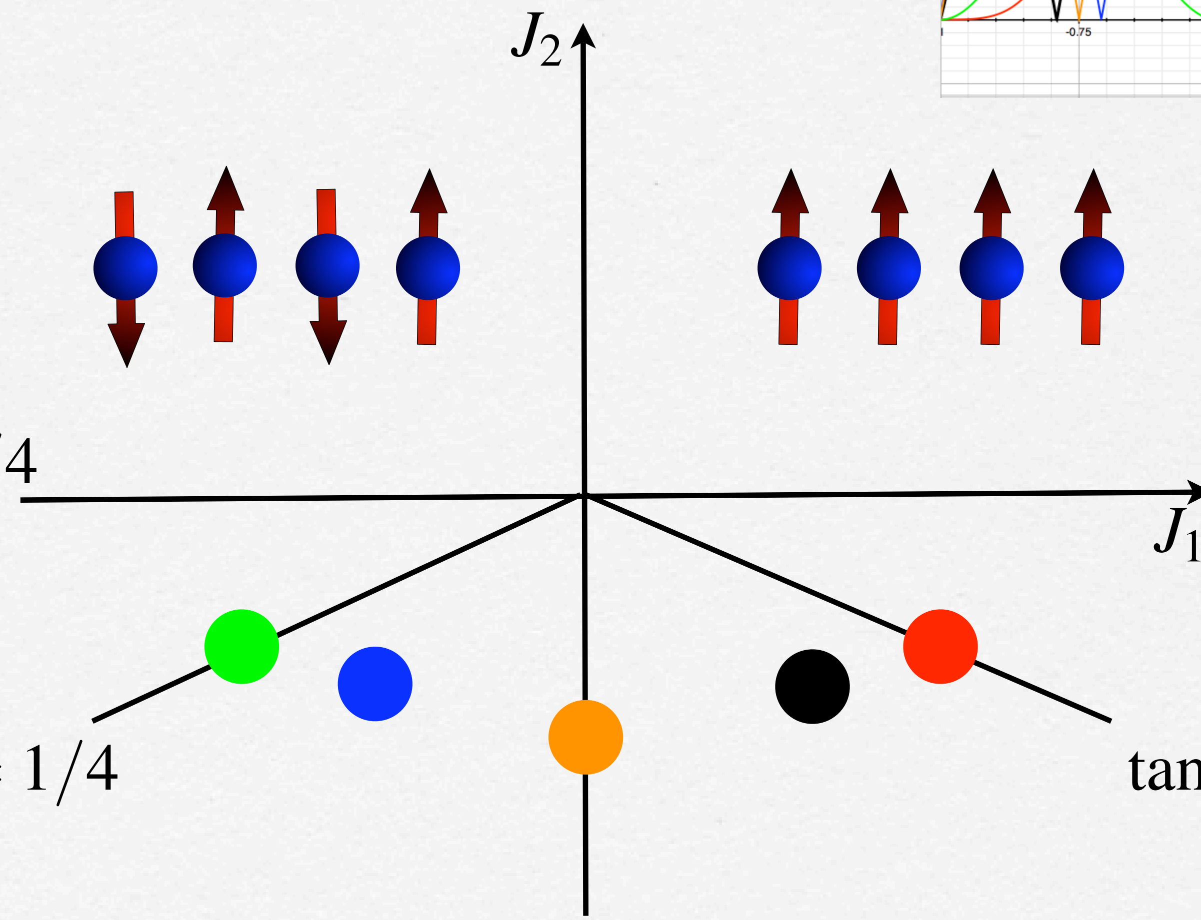




Phase diagram



- $J_1 = 1;\ J_2 = -1/4$ (red)
- $J_1 = 1;\ J_2 = -1$ (black)
- $J_1 = 0;\ J_2 = -1$ (orange)
- $J_1 = -1;\ J_2 = -1$ (blue)
- $J_1 = -1;\ J_2 = -1/4$ (green)



$\tan\phi = 1/4$

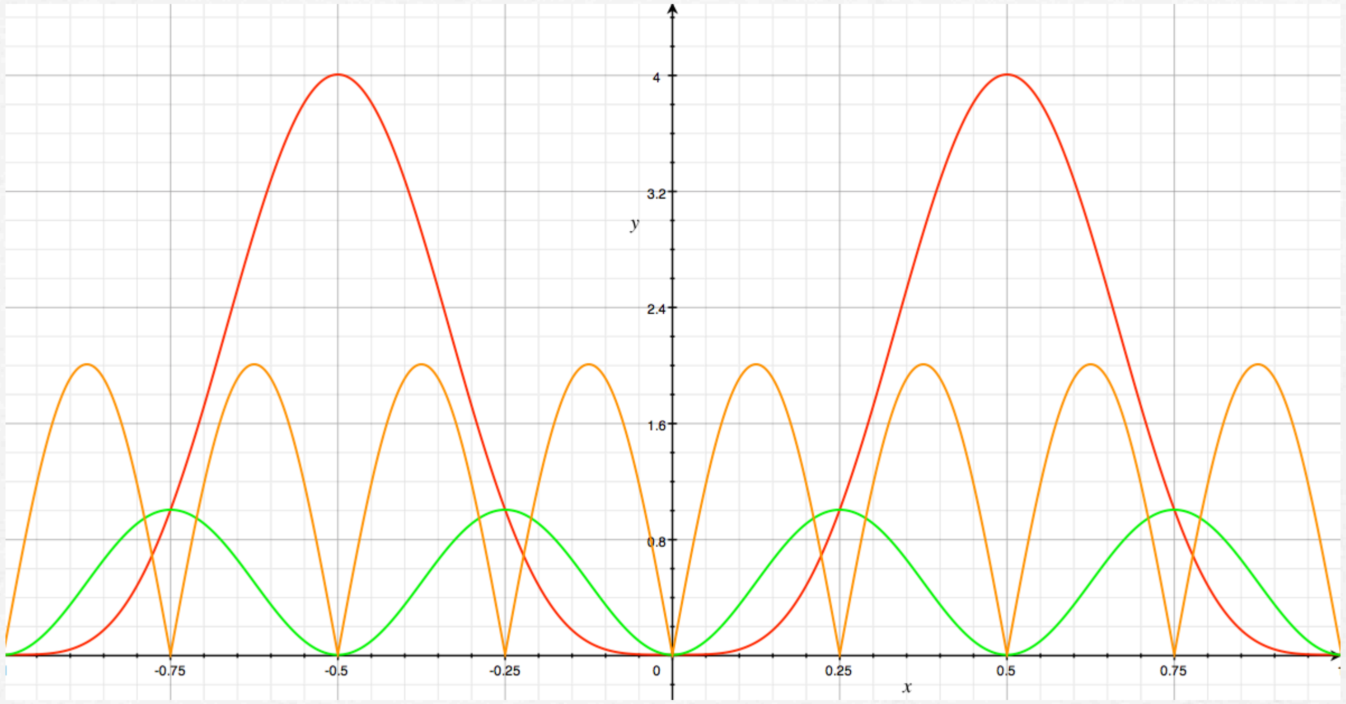
$\tan\phi = -1/4$

$J_1 = cos\phi$

$J_2 = sin\phi$

helicoidal

Conclusions



Concluding observations:

- Development from nearly ferro to nearly antiferro
- Angle between two consecutive interacting spins
- Bragg peaks

Thanks for your attention