Neutron Tomography
At ISIS

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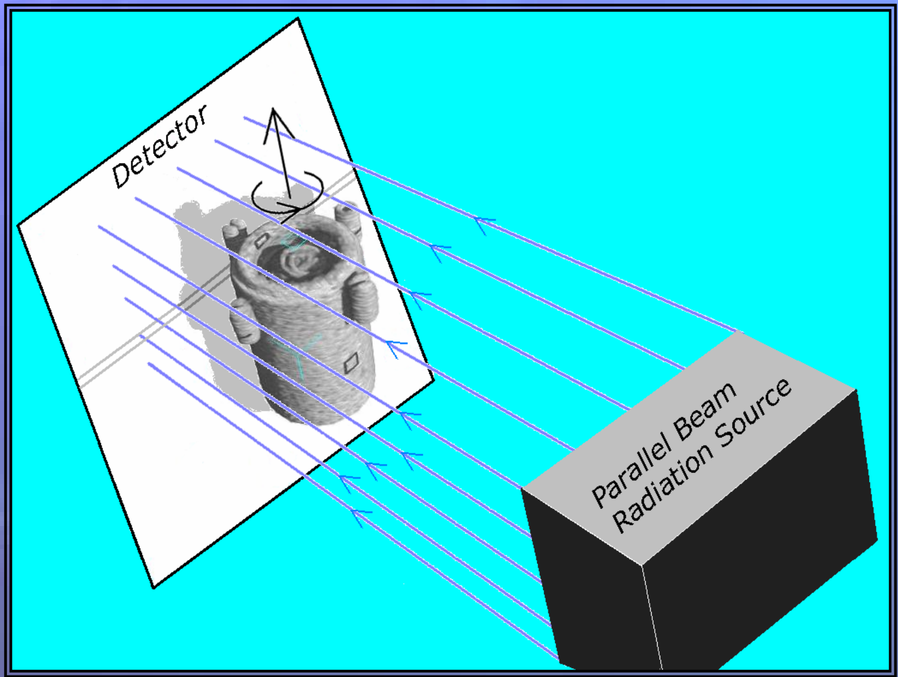
Reference Books:

- A. C. Kak, Malcom Slaney *“Principles of Computerized Tomographic Imaging”* IEEE Press (1988)
  freely available online at:
  http://www.slaney.org/pct/pct-toc.html
- *“Advanced Tomographic Metods in Materials Research and Engineering”* edited by J. Banhart Oxford University Press (2008)

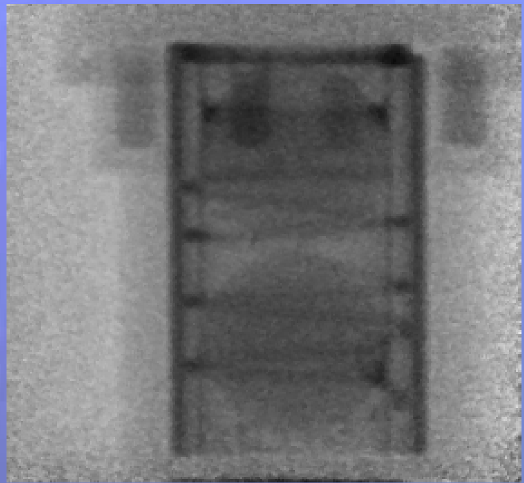
Tomography is a way to reconstruct images of object *"cross-sections"* (slices) from a set of beam attenuation measurements taken at many different angles θ_i.



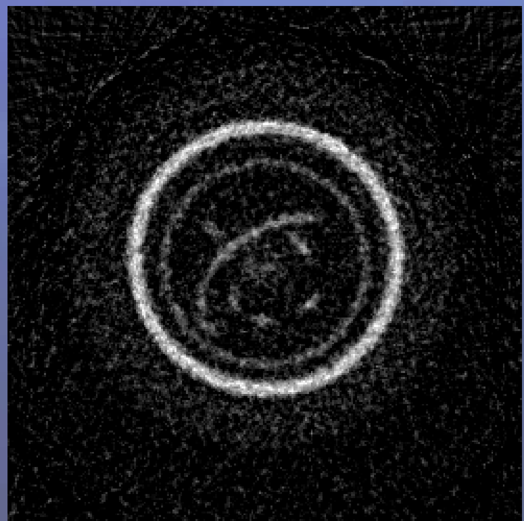
A *"Parallel Beam"* radiation source *"illuminates"* the sample…

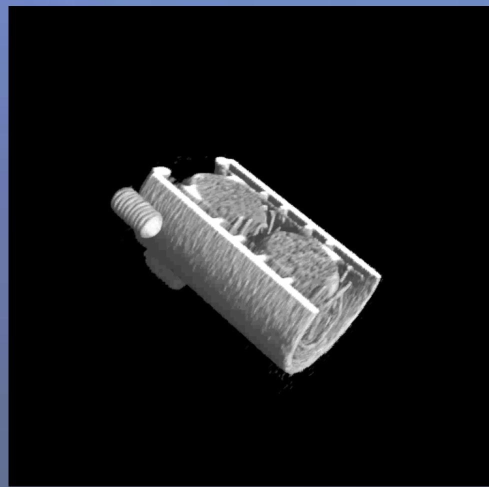
A *"Detector"* measures the intensity of the transmitted beam…

A number of measurements is collected at different angles θ_i ($0 \leq \theta_i < \pi/2$ or π) between sample and beam direction



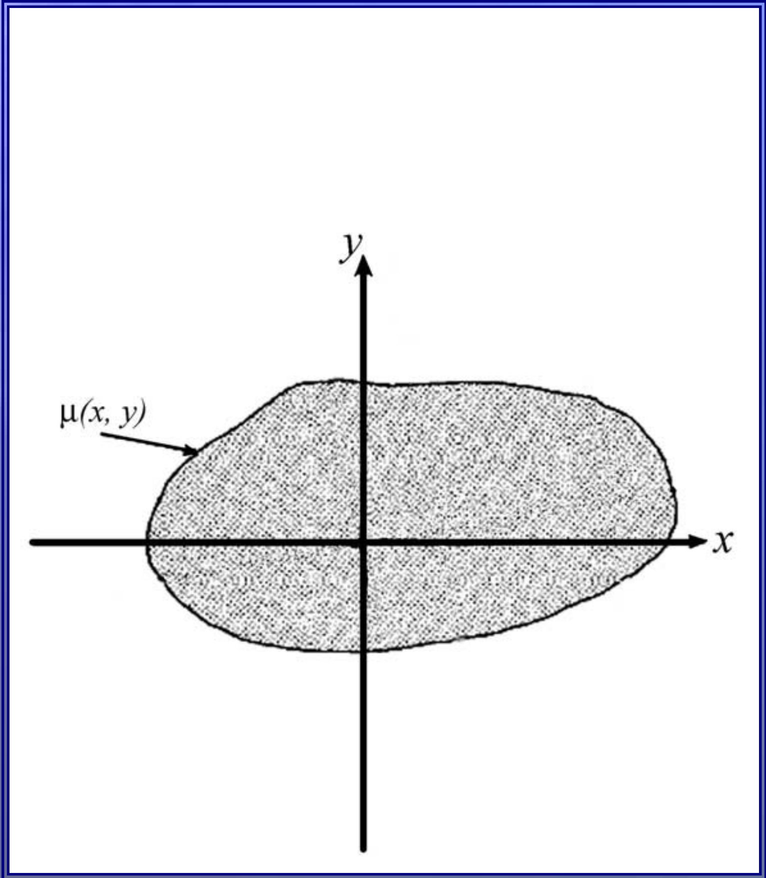
From the obtained data it is possible to reconstruct the sample's cross-sections that reveal its inner structure *in a non-destructive way*





How all this is made ?

Let's consider a thin slice of our sample and a *monochromatic* radiation source

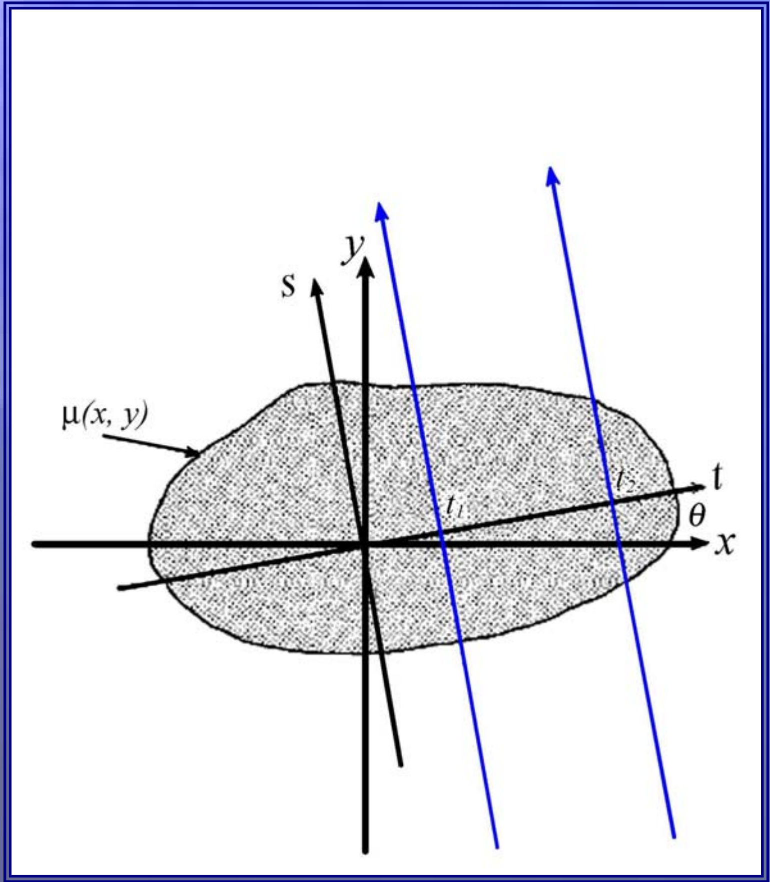


Neglecting other effects (such as scattering, *beam-hardening* etc.) the incident radiation intensity, I_0, is tied to the transmitted (measured) intensity, I_m, by the *Beer-Lambert law*:

$$I_m = I_0 e^{-\int_{along\ the\ ray\ path} \mu(x,y)\,ds}$$

where $\mu(x,y)$ is the *attenuation function*

Let's assume that the incident flux forms an angle, θ, with the reference axes

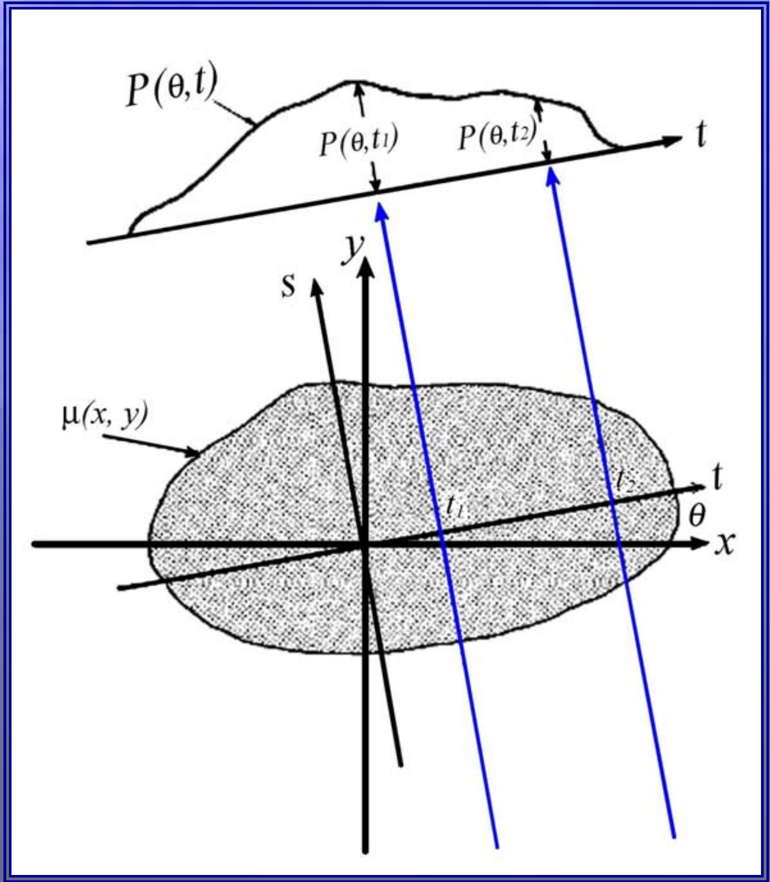


The ray passing through the point t_1 will be attenuated by the interaction with the sample and the measured intensity will be:

$$I_m(\theta,t_1) = I_0 e^{-\int_{line(\theta,t_1)} \mu(x,y)\,dxdy} \Rightarrow$$

$$\int_{line(\theta,t_1)} \mu(x,y)\,dxdy = -\ln\left(\frac{I_m(\theta,t_1)}{I_0}\right)$$

From the measured values (in each detector row) we could calculate a signal proportional to the total attenuation of the radiation.



We call *Parallel Projection,* taken at the angle θ, the quantity $P_\theta(t)$

$$P_\theta(t) = \int_{line(\theta,t)} \mu(x,y)\,ds =$$

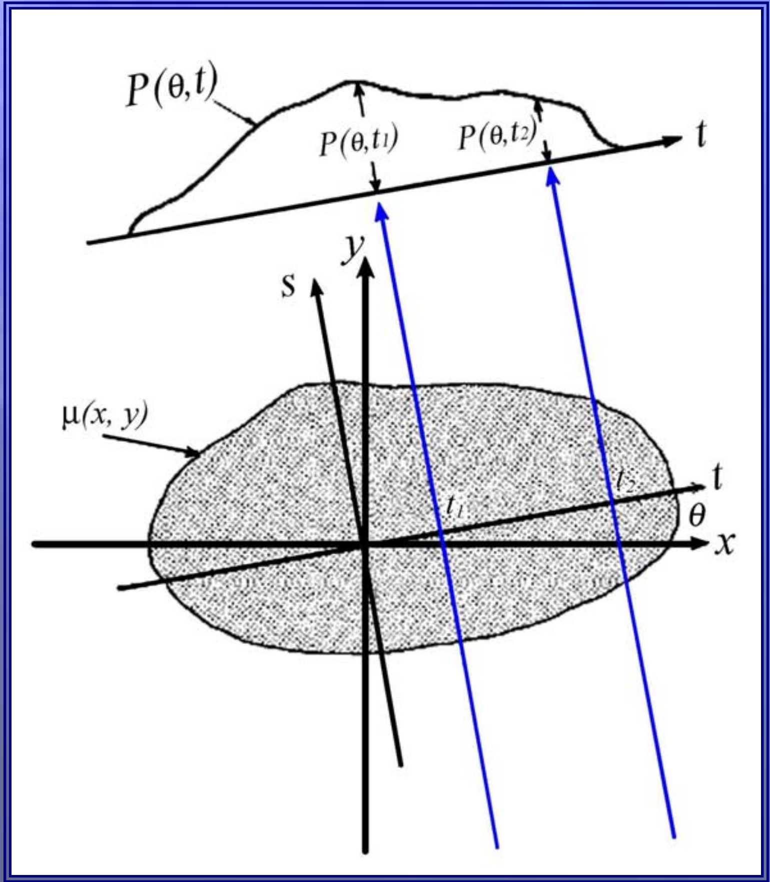
that we may rewrite as:

$$= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mu(x,y)\,\delta(x\cos\theta + y\sin\theta - t)\,dx\,dy$$

being $\delta(x\cos\theta + y\sin\theta - t)$

the *Dirac* delta function

The relation: $P_\theta(t) = \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} \mu(x,y)\delta(x\cos\theta + y\sin\theta - t)\,dxdy$



is called the

"Radon Transform"

of $\mu(x,y)$

In what follows we will need also its Fourier transform that is:

$$S_\theta(\omega) = \int\limits_{-\infty}^{+\infty} \left[P_\theta(x)\right] e^{-i2\pi\omega x}\,dx$$

Let's introduce $F(u,v)$, the *bidimensional Fourier transform* of $\mu(x,y)$,:

$$F(u,v) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mu(x,y) e^{-i2\pi(ux+vy)} dxdy$$

Assuming $v=0$ in the previous integral we have:

$$F(u,0) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mu(x,y) e^{-i2\pi ux} dxdy = \int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty} \mu(x,y) dy\right] e^{-i2\pi ux} dx =$$

$$= \int_{-\infty}^{+\infty} \left[P_{\theta=0^\circ}(x)\right] e^{-i2\pi ux} dx$$

$$F(u,0) = \int_{-\infty}^{+\infty} \left[P_{\theta=0^\circ}(x) \right] e^{-i2\pi ux} dx = S_{\theta=0^\circ}(u)$$

This equation is the simplest form of the so called *Fourier Slice Theorem*

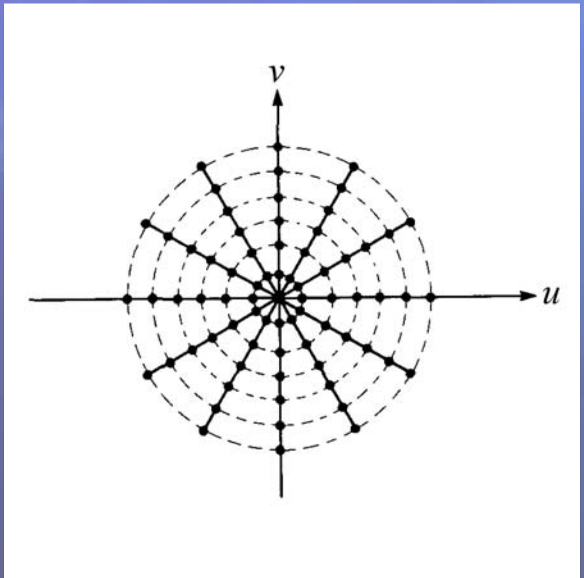
Even if this equation has been obtained with $\theta=0$ it should be clear that the object orientation is altogether arbitrary so we may state that [Kak85]:

Fourier Slice Theorem

"The Fourier Transform of a Parallel Projection of a function $\mu(x,y)$ taken at angle θ gives a slice of the two dimensional transform $F(u,v)$, subtending an angle θ with the u-axis."

With the Fourier slice theorem we have, in theory, a mean to reconstruct the $\mu(x,y)$, if we have a sufficient number of parallel projections at different θ.

In fact, Fourier transforming each projection we may obtain the values of the bidimensional Fourier transform, $F(u,v)$ along the lines shown in the figure below.



By interpolating the complex values of the obtained $F(u,v)$ in the u-v plane and by using the Inverse Fourier Transform we may obtain the searched $\mu(x,y)$:

$$\mu(x,y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(u,v)e^{i2\pi(ux+vy)}dudv$$

In practice other algorithms are employed in the current Tomography apparatus. One of the most often employed is the

"Parallel Beam Filtered Backprojection"

Recalling the inverse Fourier transform of $\mu(x,y)$

$$\mu(x,y)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}F(u,v)e^{i2\pi(ux+vy)}dudv$$

We exchange the rectangular coordinate system (u,v) for a polar coordinate system (ρ,θ):

$$\begin{aligned}u&=\rho\cos\theta\\ v&=\rho\sin\theta\end{aligned}\quad\Rightarrow\quad \mu(x,y)=\int_{0}^{2\pi}\int_{0}^{+\infty}F(\rho,\theta)e^{i2\pi\rho(x\cos\theta+y\sin\theta)}\rho\,d\rho\,d\theta$$

We may split the integral in a sum of two terms:

the first with θ ranging between 0 and π and the second with θ from π to 2π:

$$\mu(x,y)=\int_0^{2\pi}\int_0^{+\infty}F(\rho,\theta)e^{i2\pi\rho(x\cos\theta+y\sin\theta)}\rho\,d\rho\,d\theta=$$

$$=\int_0^{\pi}\int_0^{+\infty}F(\rho,\theta)e^{i2\pi\rho(x\cos\theta+y\sin\theta)}\rho\,d\rho\,d\theta+$$

$$+\int_0^{\pi}\int_0^{+\infty}F(\rho,\theta+\pi)e^{i2\pi\rho[x\cos(\theta+\pi)+y\sin(\theta+\pi)]}\rho\,d\rho\,d\theta$$

Using the property: $F(\rho,\theta+\pi)=F(-\rho,\theta)$

$$\mu(x,y)=\int_0^{\pi}\int_{-\infty}^{+\infty}F(\rho,\theta)|\rho|e^{i2\pi\rho(x\cos\theta+y\sin\theta)}d\rho d\theta=\int_0^{\pi}\left[\int_{-\infty}^{+\infty}F(\rho,\theta)|\rho|e^{i2\pi\rho t}d\rho\right]d\theta$$

$$\mu(x,y) = \int_0^{\pi} \left[\int_{-\infty}^{+\infty} F(\rho,\theta) |\rho| e^{i2\pi\rho t} d\rho \right] d\theta$$

Recalling the

Fourier Slice Theorem:

$$F(\rho,\theta) = S_\theta(\rho) = \int_{-\infty}^{+\infty} P_\theta(t) e^{-i2\pi\rho t} dt$$

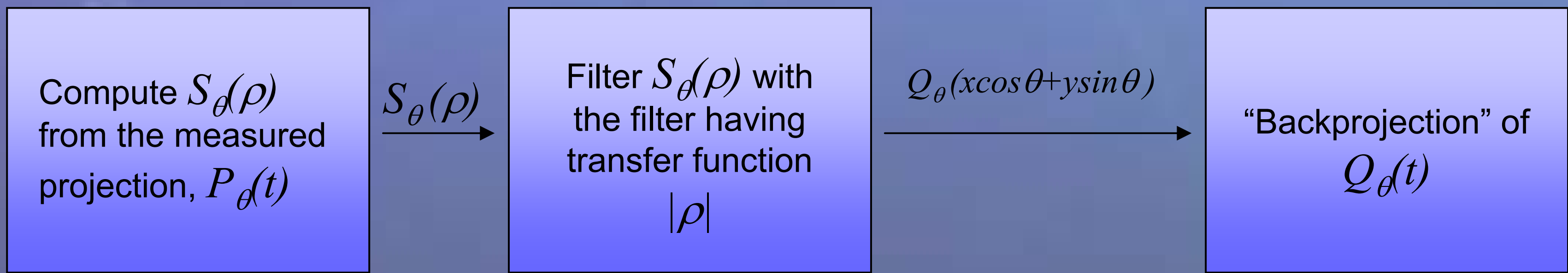
$$\mu(x,y) = \int_0^{\pi} \left[\int_{-\infty}^{+\infty} S_\theta(\rho) |\rho| e^{i2\pi\rho t} d\rho \right] d\theta$$

that could be rewritten as:

$$\begin{cases} \mu(x,y) = \int_0^{\pi} Q_\theta(x\cos\theta + y\sin\theta)\, d\theta \\ Q_\theta(x\cos\theta + y\sin\theta) = \int_{-\infty}^{+\infty} S_\theta(\rho) |\rho| e^{i2\pi\rho t}\, d\rho \end{cases}$$

Now we have the fundamental bricks to reconstruct the given slice of our object:

$$
\begin{cases}
S_\theta(\rho) = \int_{-\infty}^{+\infty} P_\theta(t) e^{-i2\pi\rho t} dt \\
Q_\theta(x\cos\theta + y\sin\theta) = \int_{-\infty}^{+\infty} S_\theta(\rho) |\rho| e^{i2\pi\rho t}\, d\rho \\
\mu(x,y) = \int_{0}^{\pi} Q_\theta(x\cos\theta + y\sin\theta) d\theta
\end{cases}
$$



Many different aspects must be taken into account in order to apply the described algorithm in *"real world"* applications. Let's enumerate some:

1) Signal detection (how to measure the $P_\theta(t)$?)

2) How to obtain the Fourier transforms needed by the algorithm ?

3) How to filter the $S_\theta(\rho)$?

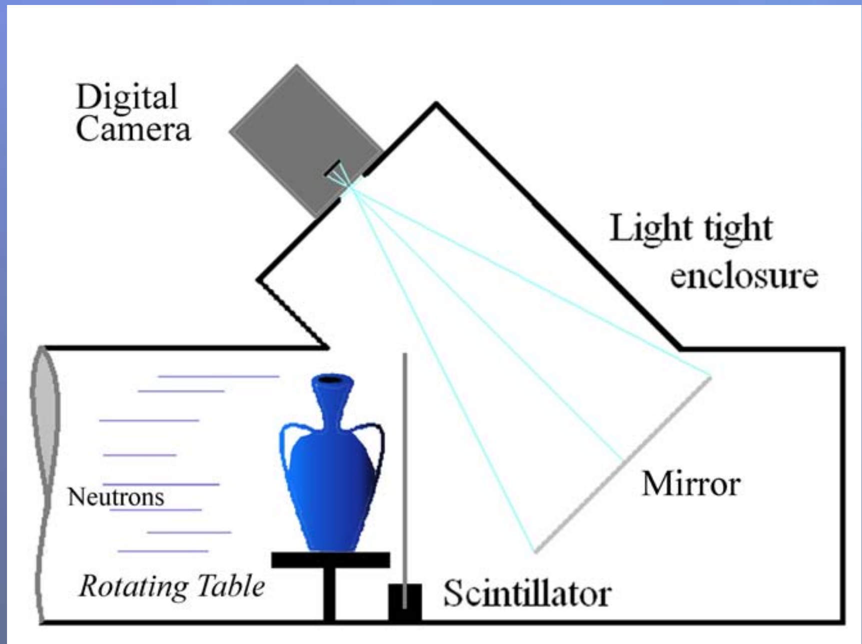
4) How to perform the Backprojection step ?

Signal detection

The common way to detect the intensity of the transmitted neutron flux is to transform the transmitted neutrons in an optically detectable signal whose intensity is proportional to the incoming neutron flux.

There are many ways to obtain such a result.

A commonly used setup is shown on the right



Fourier transforms

Since the detectors give discrete images the common practice is to use FFT algorithms to calculate the required transforms.

So we need to pay attention to the correct sampling of the images in order to avoid *"aliasing" artifacts.*

If the maximum spatial frequency present in the sample is Ω, then we have to sample the projections at spatial intervals ΔX such that

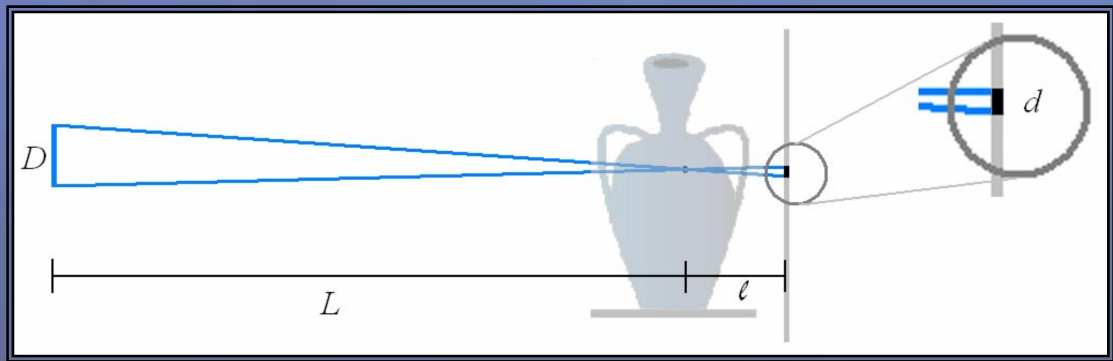
$$\Delta X \leq \frac{1}{2\Omega}$$

Neutron sources are not, usually, point-like sources; in fact they have a finite aperture (usually the dimension of the first diaphragm after the target).

This implies that the maximum spatial resolution is limited.

We may define the *Geometrical Unsharpness* as the diameter of the image produced by the projection of an object single point into the scintillator screen.

$$d = \ell \frac{D}{L} \quad [m]$$



$S_\theta(\rho)$ Filtering

To calculate $Q_\theta(t)$ we have to multiply $S_\theta(\rho)$ by $|\rho|$.

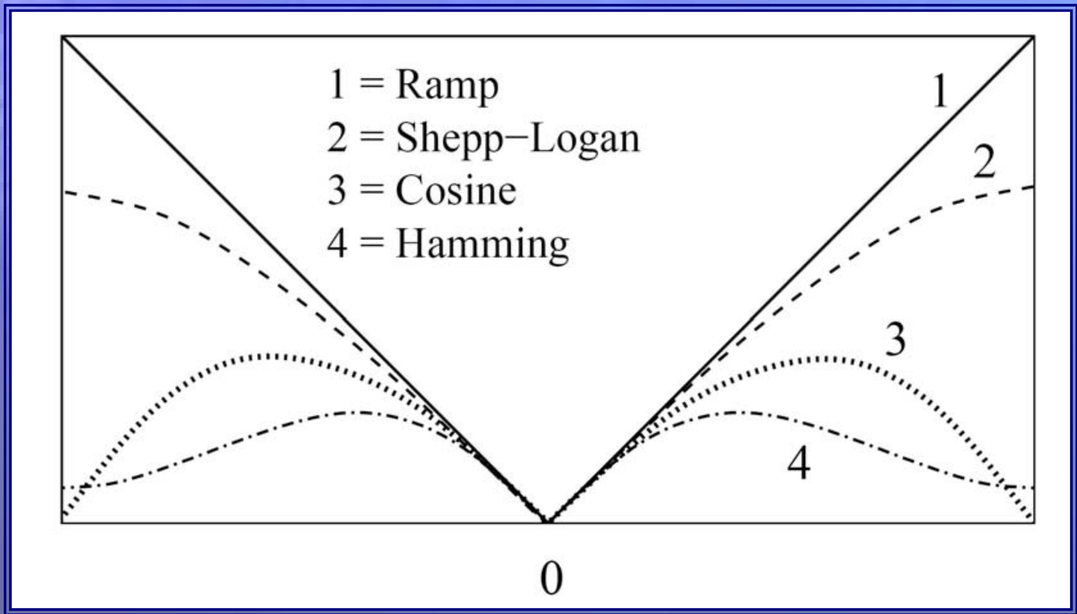
$$Q_\theta(t) = \int_{-\infty}^{+\infty} S_\theta(\rho)|\rho|e^{i2\pi\rho t}\,d\rho$$

This corresponds to a filtering operation that causes an enhancement of the higher frequency contributions present in $S_\theta(\rho)$.
Often these contributions are mainly due to noise.

Furthermore the $|\rho|$ filter is not *Band Limited* causing problems when Discrete Fourier Transform is employed

Better results are obtained if different filters other than $|\rho|$ are used.

Below we show the band limited version of some used filters.



1 $$\begin{cases} |\rho| & |\rho| \leq \rho_{MAX} \\ 0 & |\rho| \leq \rho_{MAX} \end{cases}$$

2 $$\begin{cases} |\rho|\,\mathrm{sinc}\left(\rho / \rho_{MAX}\right) & |\rho| \leq \rho_{MAX} \\ 0 & |\rho| > \rho_{MAX} \end{cases}$$

3 $$\begin{cases} |\rho|\sin\left(\pi\rho / \rho_{MAX}\right) & |\rho| \leq \rho_{MAX} \\ 0 & |\rho| > \rho_{MAX} \end{cases}$$

4 $$\begin{cases} |\rho|\left[0.54 + 0.46 \cdot \cos\left(\pi\rho / \rho_{MAX}\right)\right] & |\rho| \leq \rho_{MAX} \\ 0 & |\rho| > \rho_{MAX} \end{cases}$$

Backprojection

The most obvious way to perform the backprojection step is the *"pixel driven"* one.

```
void
ParBackProj_LI(int N, float teta, float* Q, float * Image) {
    float t, w;
    int m;
    for(int x=0; x<N; x++) {
        for(int y=0; y<N; y++)  {
            t = cos(teta)*x + sin(teta)*y;
            m = int(t);
            w = t - m;
            Image[i*J+j] += (1-w)*Q[m] + w*Q[m+1];
        }
    }
}
```

For each pixel of the reconstructing image we look for the available nearest Q_θ values.

This procedure must be repeated for each θ with

$$0 \leq \theta < \pi$$

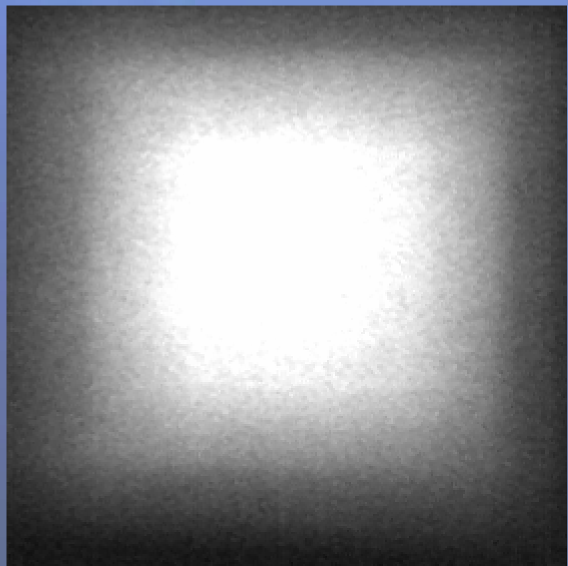
The complexity of the algorithm is $O(N^3)$ and dominates the total complexity of the *Filtered Backprojection* method.

Fast backprojection methods exist that reduce the complexity to $O(N^2 \log N)$ but their study is beyond the scope of this lecture.

INES INSTALLATION

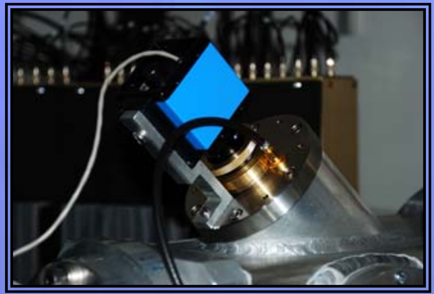
Primary Flight Path
L = 22.8 m

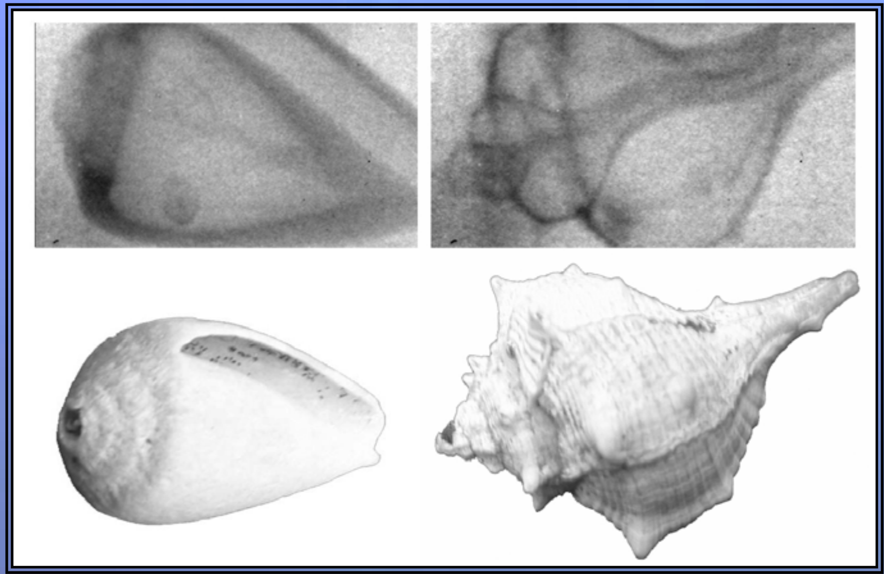
Water moderator
size *D* ~ 10 cm

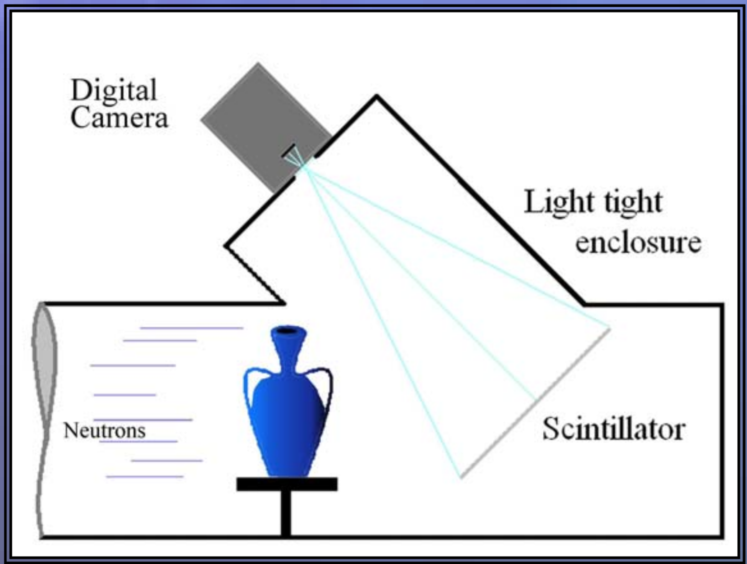


The neutron beam is collimated to give a square cross-section, of ~ 38mm size, at the INES sample position.

INES INSTALLATION







L. Bartoli, F. Aliotta, F. Grazzi, G. Salvato, C.S. Vasi, M. Zoppi
Nucl. Inst. and Meth. A (2008)

INES INSTALLATION

- L = 23.60 m
- D ~ 10 cm

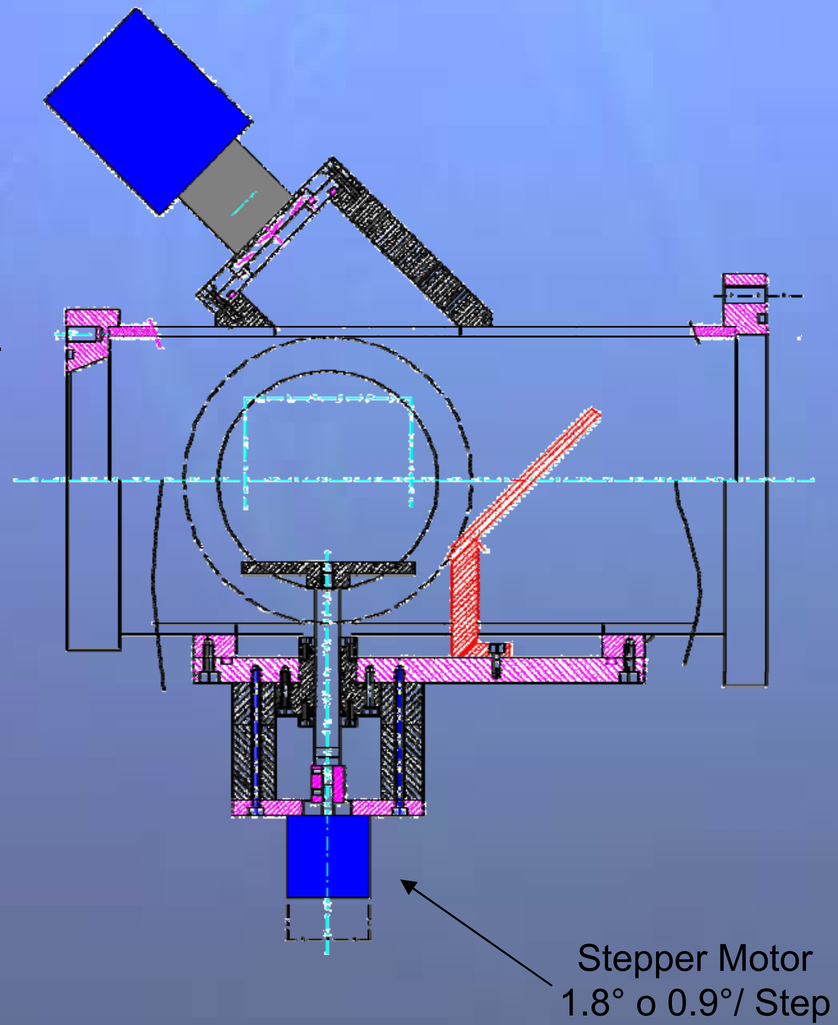
L / D ~ 236

- ℓ ~ 10 cm (Mean) sample-scintillator distance

Geometrical Unsharpness:

0.30 < d < 0.55 [mm]

$$d = \ell \frac{D}{L}$$

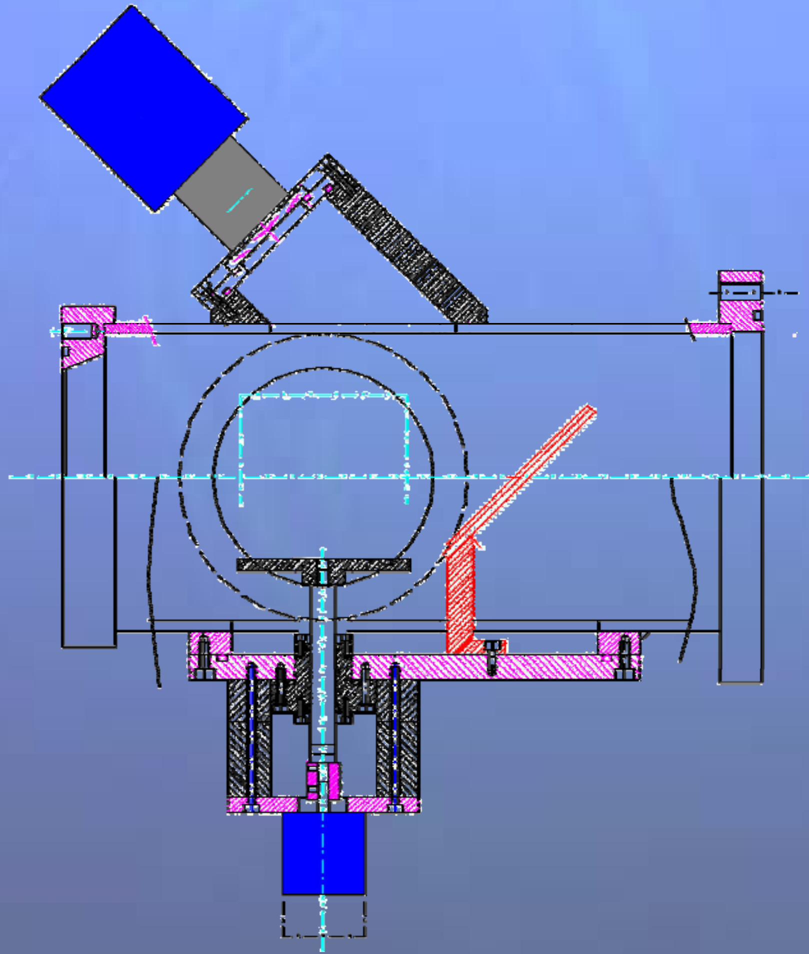


INES INSTALLATION

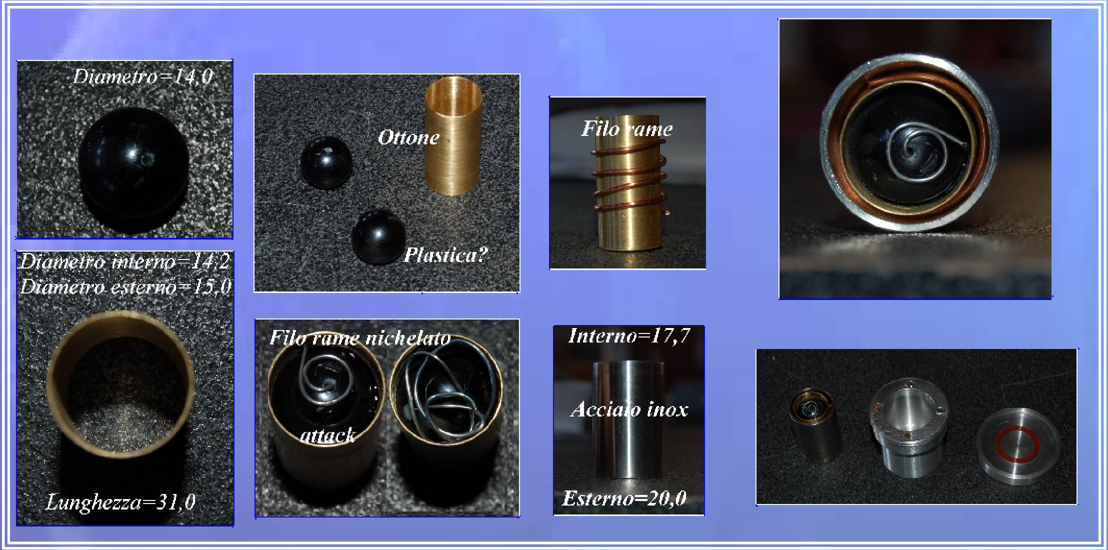
- ***Camera*** CCD *not cooled*
  The Imaging Source DMK 21BF04
  640x480 - 8 bit

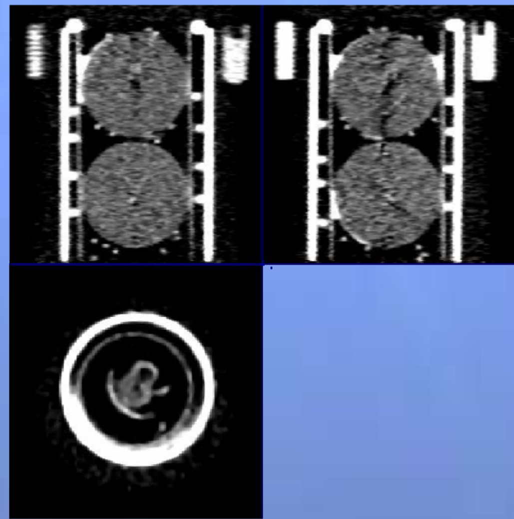
- ***Optics*** 8 mm, f: 1.4
  Pentax C2514M(KP)

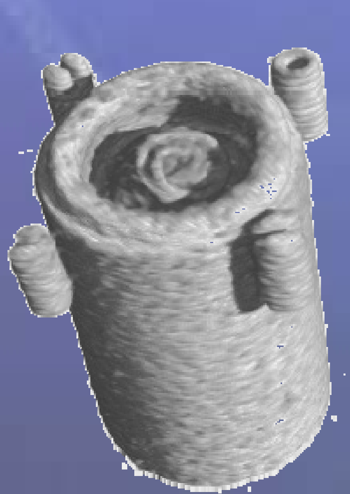
- ***Scintillator*** ZnS / ^{6}LiF on Al substrate *(λ emission~520nm)*

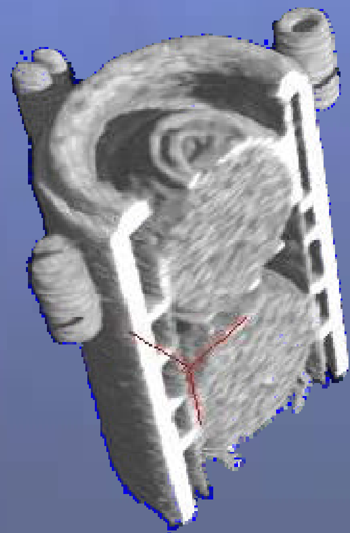


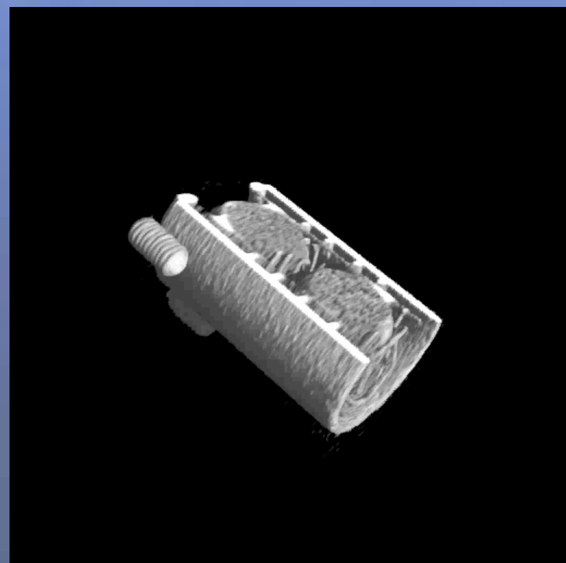
First Results at INES











In this talk we have only *"scratched the surface"* of the complex and very fascinating argument of neutron tomography.

Many very important aspects were left out but I hope at least to have stimulated your curiosity.